Targeting in Advertising Markets: Implications for O- ine vs. Online Media

Dirk Bergemann[†] Alessandro Bonatti[‡]

August 2010

Abstract

We develop a model with many advertisers (products) and many advertising markets (media). Each advertiser sells to a di¤erent segment of consumers, and each medium has a di¤erent ability to target advertising messages. We characterize the competitive equilibrium in the media markets and evaluate the implications of targeting in advertising markets.

An increase in the targeting ability leads to an increase in the total number of purchases (matches), and hence in the social value of advertising. Yet, an improved targeting ability also increases the concentration of …rms advertising in each market.

1 Introduction

Over the past decade the internet has become an increasingly important medium for advertising. The arrival of the internet has had important consequences on the market position of many traditional media, i.e. o- ine media such as print, audio and television. For some of these media, most notably the daily newspapers, the very business model is under the threat of extinction due to competition from the internet for the placement of advertising. The following chart shows the recent changes in aggregate spending for advertising on di¤erent media.[1]

Figure 1: U.S. Advertising Markets: Revenue Comparison

[redacted]

At the same time, through a variety of technological advances, the internet has allowed many advertisers to address a targeted audience beyond the reach of traditional media. In fact, it has been argued that the distinguishing feature of internet advertising is its ability to convey information to a targeted audience. In particular, targeting improves the quality of the match between the consumer and the advertisement message, and enables smaller businesses to access advertising markets from which they were previously excluded.[2] While this holds for display advertising, it is even more true for sponsored search, where the individual consumer declares her intent or preference directly, by initiating a query.

The objective of this paper is to develop a model of competition between o- ine (traditional) and online (new) media, in which the distinguishing feature of the online media is the ability to (better) target advertisement messages to their intended audience. We

[1] Source: Price Waterhouse Coopers annual reports for the Interactive Advertising Bureau.
[2] Anderson (2006) refers to this phenomenon as the "long tail of advertising."

investigate the role of targeting in the determination of (a) the allocation of advertisements across different media, and (b) the equilibrium price for advertising. For this purpose, we first develop a framework to analyze the role of targeting, and then use this to model to analyze the interaction between off-line and online advertising.

We present a model in which advertising creates awareness for a product. We consider an economy with a continuum of buyers and a continuum of products. Each product has a potential market size which describes the mass of consumers who are contemplating to purchase it. Each consumer is contemplating only one of the available products, and the role of the advertisement is to generate a match between product and consumer. The placement of an advertisement constitutes a message from the advertiser to a group of consumers. If the message happens to be received by a consumer with interest in the advertiser's product, then the potential customer turns into an actual customer and a sale is realized. A message received by a customer who is not in the market for the product in question is irretrievably lost and generates no tangible benefit for the advertiser. At the same time, a potential customer might be reached by multiple and hence redundant messages from the targeting improves. While the marginal product of each message is increasing in the targeting ability, thus potentially increasing the prices for the advertisement, a second and more powerful effect appears. As consumers become more concentrated, the competition among different advertisers becomes weaker. In fact, each advertiser focuses his attention on a few important advertising markets and all but disappears from the other advertising markets. Therefore, the price of advertising is declining in the degree of targeting, even though the value of advertising is increasing. The number of participating advertisers shows a similarly puzzling behavior. While improved targeting increases the total number of advertisers participating across all markets – by allowing smaller advertisers to appear – it reduces the number of actively advertising firms in each specific advertising market.

In the second part of the paper we introduce competition among different media for the attention of the consumer. Thus, while each consumer is still only interested in one product, he can now receive a message from any advertiser through two different advertising media. A single message received in either one of the media is sufficient to create a sale. The "dual-homing" of the consumer across the two media markets may then lead to duplicative efforts by the advertisers, who therefore view messages in the two competing markets as substitutes. We first describe the advertising allocation when the competitors are both traditional media without any targeting ability. In this case, messages on the two media are perfect substitutes, and the equilibrium prices are equalized. Furthermore, the allocation of messages only depends on the total supply, not on its distribution across media.

The competition among two off-line media markets presents a useful benchmark when we next consider competition between an off-line and an online market. We analyze the interaction of off-line media – such as newspapers or TV – with online media, such as display (banner) and sponsored search advertisements. Display advertisements allow for targeting through superior knowledge of the consumer's preferences (attribute targeting). Sponsored keyword search advertisements allow advertisers to infer the consumer's preferences from her actions (behavioral targeting). As expected, competition lowers the equilibrium revenues of the traditional medium. However, if entry by an online competitor reduces the available advertising space on the traditional media (for example, by reducing the time consumers spend on each channel), then the effect of competition on the equilibrium price of advertising is non monotonic. In particular, as the consumers shift their attention from traditional to new (targeted) media, the price on the traditional channels is first decreasing, then increasing.

This paper is related to several strands in the literature on advertising and media compe-

Similarly, the size of the advertising market a

Finally, the supply of messages M in every advertising market a is proportional to the size s of the advertising market and given by

$$M \; , \; s \; M;$$

for some constant $M > 0$. The constant M can be interpreted as the attention or time that the representative consumer allocates to receiving messages from the advertising outlet.

Firms can purchase advertisement messages at a unit price p in each market a. The total profits of firm x are given by:

$$, \int^{\infty} [s \; f(m \; ; s) \; p \; m \;] \, da: \tag{4}$$

The awareness function $f(\,)$ described above applies literally to the case of display advertising online, where each impression corresponds to a message. However, it can be easily amended for the analysis of broadcast media. In particular, let m be the product of an advertisement's air time t times the audience size s of channel a. The awareness level generated by an ad of firm x placed in market a is then given by $1 \; \exp$

exponential distribution:

$$\frac{s}{s} , \begin{cases} e^{-} \quad {}^{-} ; & \text{if } 0 < a \quad x; \\ 0; & \text{if } \quad a > x; \end{cases} \tag{6}$$

with a mass point

$$\frac{s}{s} , e^{-} \quad \text{if } a = 0.$$

In other words, we model market $a = 0$ as a large advertising market, in which all advertisers are potentially interested (as $s \quad > 0$

may read either one of the publications, but that a consumer with interest in fitness bikes does not read "Velonews," and by extension that a consumer with an interest in comfort bikes does not read "Velonews" nor "Sports Illustrated". In other words, the triangular structure represents a positive but less than perfect correlation of the preference and the audience characteristics of a consumer. The specific feature of the triangular structure, namely the unidirectional diffusion of the consumer x across advertising markets a x, is not essential for the qualitative character of our results, but allow us to represent the strength of the targeting in a single variable, namely the parameter of the exponential distribution.

As we vary the targeting measure from 0 to , we change the distribution and the concentration in each advertising market. The limit values of , namely = 0 and = , represent two special market structures. If = 0, then all consumers are present in advertising market 0 and hence there is a single advertising market. If, on the other hand, , then all consumers of product x are present in advertising market x, and hence we have advertising markets with perfect targeting. More generally, as we increase , an increasing fraction of consumers of product x move away from the large advertising markets (near a = 0) to the smaller advertising markets (near a = x). Figure 2 illustrates the cross section, represented by the conditional distribution s =s , of how the consumer segments of two different advertisers are distributed across the advertising markets (for a low and high degree of targeting in the left and right panel respectively). The mass points indicate the number of consumers of each firm that are present in advertising market 0.

Figure 2: Conditional Distribution of Consumers across Advertising Markets

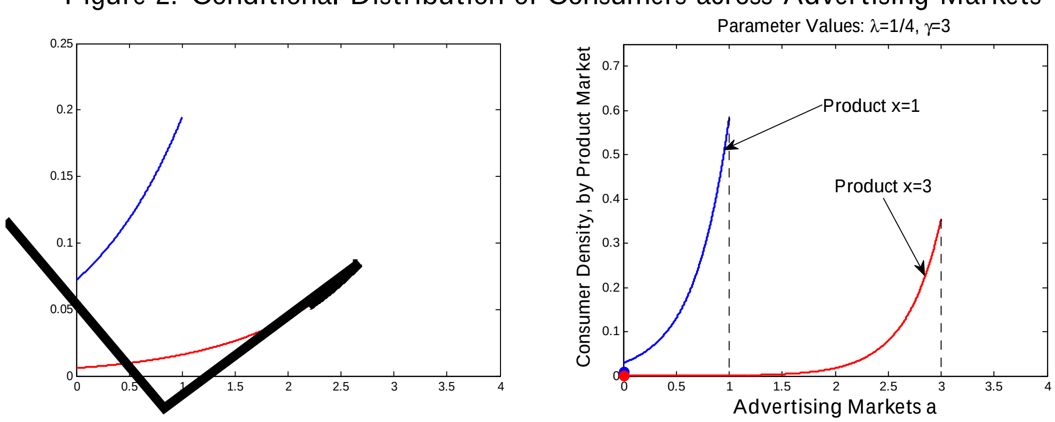

Figure 3 shows how two advertising media host consumers across different product categories, represented by the conditional distribution s =s

It is an implication of the above optimality conditions that …rms with a larger market share s choose to send more messages to the consumers. In consequence, at the equilibrium price, the …rms with the largest market share choose to advertise. Let $[0; X]$ be the set of participating …rms, where X is the marginal …rm, and let M be the total supply of messages. The equilibrium price p for messages is then determined by the market clearing condition:

$$\int m \, dx = M:$$

Using the optimal demand of …rm x and the formula for product market shares (5), we obtain

$$\int (\ln(\ /p) \quad x) \, dx = M. \tag{7}$$

The equilibrium price and participation are determined by imposing $m \ \ = 0$ and the market clearing condition in (7). The competitive equilibrium is then characterized by $(p^*; X$

the competitive equilibrium implements the socially efficient allocation of advertisement messages (given). An easy way to see this is that with a uniform unit price of messages, the marginal returns to the messages bought by different firms are equalized. A natural question is how does the social value of advertising depend on the product market concentration. Consider holding the allocation m^* fixed, and increasing . Now the total market share of the as the market share of the large firms is already substantial, the increase in their demand is not sufficient to pick up the decrease in demand coming from the marginal firms, and consequently the equilibrium price falls. The additional demand of the large firms is weak because of decreasing marginal returns: an increase in the already large advertising volume leads to many more redundant messages, which generate few additional sales. Figure 4 shows the market demand and supply for different values of the concentration measure .

Figure 4: Equilibrium Demand for Different Concentration Measures

We can view the dichotomy in the comparative statics as driven by the determination of the marginal demand for advertising. For high enough , the source of the marginal demand is the marginal firm, and the price goes down with an increase in . Likewise for low , the marginal demand is driven by the inframarginal firms, and then the advertising price is increasing with . In this sense, the non monotonic behavior of prices is not specific to the exponential distribution of firms' market shares. On the contrary, it is a consequence of the natural tension between competition and concentration.

It is useful to recast the equilibrium of the model in hedonic terms. In this respect, Proposition 2 shows that larger firms pay a decreasing amount per consumer reached as increases. This result is driven by the concentration of the equilibrium messages in the hands of a few firms, who make large profit levels on the inframarginal units. Conversely, the price per consumer reached is increasing in for firms smaller than the median advertising firm. For these firms, the price per consumer reached increases until it attains a value of one (which is the marginal return to the first message $f'(0)$). In particular, for all , the marginal …rm $X^*(\ )$, which pays a price per consumer reached equal to one.

One may wonder how relaxing the assumption of perfectly inelastic supply a¤ects the comparative statics result in Proposition 2. For the case of constant supply elasticity $q =$ Mp

The model with a single advertising market was described by = 0, while the case of perfect targeting is described by =

$m_{*} = 0$: In particular, we have:

$$\int m_{*}\, dx = s\, M,$$

and

$$s_{*} = s = p.$$

We can now characterize the equilibrium prices p^*, the number of active firms X^*

Proposition 3 (Many Markets, Efficiency)
The social value of advertising is strictly increasing in the targeting ability .

To understand the implications of targeting on social welfare, consider the relative size of consumer segment x in advertising market $a = x$:

$$\frac{s}{s} = \quad + \quad .$$

We observe that better targeting increases the value that firm x assigns to a message in the advertising market $a = x$. Now let us consider holding the allocation of messages m constant, and increasing the degree of targeting . The volume of matched consumers and firms is increasing because of the shift in the relative sizes of advertising markets. Since we know that the competitive allocation of messages is Pareto efficient, the equilibrium (for the new) has unambiguously improved the social value of advertising.

The comparative statics results with respect to the concentration measure and message volume M do not differ qualitatively from the case of a single advertising market. More importantly, the effect of targeting ability and product market concentration on the equilibrium allocation is remarkably similar. In particular, prices are increasing in if and only if both the concentration and the targeting parameter are low enough. We now focus on the comparative statics with respect to , where a higher means more precise targeting. We define the equilibrium advertising revenues on each advertising market a as $R^*\ , s\ p^*$.

Proposition 4 (Role of Targeting)

1. The number of messages per capita $m^* = s$ is increasing in for $x \quad (a + X^*) = 2$, and decreasing for $x > (a + X^*) = 2$:

2. The number of participating firms $X^* \quad a$ is decreasing in .

3. The equilibrium price p^* is increasing in if and only if $+$ $< 2 = M$.

4. The equilibrium revenue R^* is decreasing in . The revenues R^* are increasing in if and only if $< (1 + \quad \sqrt{1 + 2M}\ ) = M$.

The equilibrium number of messages m^* is increasing in for the participating firms larger than the median firm active on each market a. Furthermore, more precise targeting implies a lower number of active firms. Notice that the relationship between targeting ability and equilibrium price is generally hump-shaped. However, if either M or are large, then p^* is decreasing in for all values of . In other words, despite the increased social value of advertising, the equilibrium price of advertising is decreasing in the targeting ability over a large range of parameter values. In terms of revenues, it is immediate to see from equations (11) and (12) that an increase in leads to an increase in the size of markets $a > 0$ and to a decrease in the size of market 0. Since prices are constant, revenues in market 0 are decreasing in . Finally, targeting has the same qualitative effect on the equilibrium revenues in all markets $a > 0$.

We now come back to the similar effects of concentration and targeting. In particular, as with product market concentration, an increase in targeting reduces the demand of the marginal firm on each advertising market a. At the same time, better targeting increases the demand of the inframarginal firms. The underlying tension is the one between identifying a consumer segment precisely, and finding several advertisers who are interested in it. The resulting trade-off between competition and inframarginal willingness to pay applies to a number of contexts, such as generic vs. specific keyword searches, and more or less precise attributes targeting on social networks.

For example, Goldfarb and Tucker (2010) analyze bidding data for "personal injury" Google keywords, and report the prices paid by advertisers (law firms) in several locations. The variation in prices across locations is considerable, ranging from over $50 per click to nearly zero. We can reinterpret these results in light of our comparative statics results. In particular, fix at a high value, reflecting the precise targeting ability of a specific Google keyword. The different markets (zip codes) considered by Goldfarb and Tucker (2010) differ by product market concentration ( ), measured by the number of personal injury lawyers, and presumably also by the average exposure (M) of consumers to online advertising. In our Proposition 4.3, we show how these market conditions affect the profitability of a high level of targeting. In particular, the effect of targeting on the equilibrium price is positive if both and M are low enough. Therefore, variation in concentration and supply across different advertising markets can lead to a wide dispersion in prices for precisely targeted advertisement messages.

To conclude this section, we should point out that the exponential distributions over advertising and product markets provide particularly tractable expressions. The insights about the non monotonic behavior of the equilibrium price of advertising extend to more general production and distribution functions. In the working paper version, Bergemann and Bonatti (2010), we present a set of su¢ cient conditions for the comparative static to remain true beyond the exponential model presented here. The general conditions are stated in terms of the matching function and distribution over advertising and product markets: (i) the production function $f(m)$ is strictly concave, the marginal returns to the

5 Media Competition

In this section, we deploy our model of general and targeted advertising markets as a framework to provide insights into the effects of competition between new and established media. For this reason, we shall weaken the single-homing assumption to allow each consumer to be present in multiple markets. A first effect of competition is then to multiply the opportunities for matching an advertiser with a customer. At the same time, we maintain all the assumptions of the previous sections, namely that each buyer is only interested in one product, and that one message is sufficient to generate a sale.

We initially consider competition between traditional media, i.e. sellers of non-targeted messages, where each medium is described by a single advertising market. For example, this may represent the competition between nation-wide TV broadcasting and nation-wide newspaper publishers. We initially abstract away from the role of targeting, in order to trace out the implications of (a) the number of consumers present on each market, and (b) the distribution of consumer characteristics in each market. The analysis of competition between traditional advertising markets can shed light on the interaction of new and established (off-line and online) media along at least two dimensions.

First, new media are likely to have an initially smaller user base. As a consequence, advertisement messages have a more narrow reach, though a smaller market makes it easier to reach a large fraction of the audience. Our results show that only the largest advertisers buy a positive number of messages in both markets. Furthermore, these firms purchase a constant number of advertising messages in the (new) smaller market. Therefore, media competition allows medium-sized firms to have a relatively larger presence on the new advertising market, compared to the case of a single medium.

Second, the main feature of a targeted, online advertising market is a higher concentration of consumers of a particular product, compared to a traditional market. Therefore, the degree of product market concentration, which we focus on here, plays a similar role to the degree of advertising market targeting of Section 4. In particular, differences in market concentration lead firms to sort into those markets where their messages have a higher probability of forming a match with the desired customer segment.

5.1 Competition by Symmetric O- ine Media

We begin the analysis with a model of competition between two traditional media. The two market a until the critical level at which the value of advertising in a falls below p . This level depends on the amount of advertising in the other market. We denote by m , P m the total number of messages demanded by …rm x, and we describe the equilibrium allocation in the following proposition.

Proposition 5 (O- ine Media)

B also visit the established medium A. For example, one may think of the early days of online advertising, or more recently about new online advertising channels (such as social networks).

We normalize the supply of messages per capita to M in each market a. Since each …rm x can reach a subset of its customers on the new market B

Proposition 6 (New Advertising Medium)

1. The equilibrium allocation of messages in the established market A is

$$m^* \quad = \quad \sqrt{2\ M} \qquad x, \quad \text{for } x \qquad \sqrt{2M\ =\ } .$$

2. The equilibrium allocation of messages in the new market B is given by

$$m^* \quad = \begin{cases} (\sqrt{2(M \quad + M\ )} \qquad \overline{2M}\ ); & \text{for} \qquad x \quad \sqrt{2M\ =\ }, \\ (\sqrt{2(M \quad + M\ )} \qquad x); & \text{for} \quad \sqrt{2M\ =\ } < x \quad \sqrt{2(M \quad + M\ )=\ } . \end{cases}$$

3. The equilibrium prices are given by

$$p^* \quad = \quad e^{-} \overline{\qquad\qquad} + (1 \quad)\ e^{-\sqrt{\qquad}} ,$$

$$p^* \quad = \quad e^{-} \overline{\qquad\qquad} .$$

Figure 5 illustrates the allocation for M = M = 1; = 2; and several values of : When = 1, we return to the case of symmetric advertising markets, and the specific allocation displayed below is just one of the possible equilibrium allocations. The displayed allocation for = 1 is however the unique limit for the equilibrium allocations as 1.

Figure 5: Equilibrium Demand for Different Market Sizes

Proposition 6 shows that the number of active firms in market A is determined by the
+

M + M . Finally, the equilibrium price on the larger market p is decreasing in the size of the smaller market , while the price on the smaller market p is independent of . Both results can be traced back to changes in the supply of messages in the new market. Indeed as increases, demand by the larger advertisers also increases. This would drive the price up and reduce the number of active …rms, but this e¤ect is o¤set by a proportional increase in supply.

5.3 Media Markets with Di¤erent Distributions

As we saw in Section 4, the key advantage of more targeted advertising markets is to allow fewer …rms to deliver messages to a more concentrated consumer population. We now shift our attention to the role of the distribution of consumer characteristics for the competition between di¤erent media markets.

We consider two advertising markets, a A; B and let the distribution of consumers in market a be given by s , exp (x). We assume that the advertising market A has a more concentrated distribution over consumer characteristics than advertising market B, or > . As the distribution of consumers across advertising markets is now assumed to be di¤erent, it follows that not all consumer will be dual-homing. In particular, if a …rm x has a larger presence in market A, then all its potential customers are present in market A, but only a subset of them is present in market B. Given that > , this is the case for the larger …rms, for which s > s . The converse holds for the smaller …rms, which

1. The largest …rms x $[0; X]$ only buy on market A:
2. A set of “medium-sized” …rms x $[X; Y]$ buy on both markets. These …rms divide

When this is the case, the marginal …rm $X = Y$ has an identical share of consumers in each of the two distributions.

The results in this section provide two kinds of insights into the interaction of online and o- ine advertising markets. Indeed, we can view each online advertising market as a separate medium with a higher concentration of consumers. With this interpretation, the prediction of the model is that Internet advertising induces the largest, most pro…table advertisers to switch away from the o- ine medium, and to advertise only on the more concentrated online markets.

In this sense, competition by a more concentrated (targeted) market is very di¤erent from an (identical) emerging market with a smaller user base. In the former case, the established media lose the most valuable …rms, as these …rms …nd a more pro…table market where to reach their customers. In the latter case, the established media share the largest buyers with the new media, and actually hold a relatively favorable position (in terms of the allocation of messages purchased by the largest …rms).

In an alternative interpretation, we can view market B as the newer medium, such as the Internet, with a relatively larger presence of consumers of small (long tail) …rms. Competition with a more concentrated (established) market then causes the demand for messages by smaller …rms to completely crowd out the demand of larger …rms, and to partially o¤set the demand of medium-size …rms. In this sense, online advertising increases the number of …rms that have access to messages in equilibrium, and allows for a more signi…cant participation of smaller …rms.

6 O- ine vs. Online Media

The internet has introduced at least two technological innovations in advertising, namely (a) the ability to relate payments and performance (e.g. pay per click), and (b) an improved ability to target advertisement messages to users. We focus on the latter aspect, and in particular on the equilibrium allocation of advertising when both traditional and targeted media are present.

In our model, the targeted markets represent specialized websites, and messages can be thought of as display advertisements. We therefore refer to the traditional medium as "o- ine," and to the many targeted markets as "online." We then consider a population of dual-homing consumers, who spend a total time of M on the o- ine medium, and M on a single market a in the online, targeted, medium. More speci…cally, s M denotes the supply of messages on each targeted market.

Because of the risk of duplication, messages sent online and o- ine are viewed as substitutes by each …rm. This is not the case for messages sent on two di¤erent online markets, since each consumer only visits one website (in addition to the o- ine market). Therefore, if …rm x sends a total of m non targeted messages and m messages on each online market a, its pro…t function is given by

$$= \int (s \quad (1 \quad e^{-}$$

that the more …rm x advertises o- ine, the lower the price on the corresponding online market a = x. This is again a consequence of the substitutability of messages across media.

We now turn to the message demands o- ine. Since each …rm reaches a constant fraction 1 exp (M) of its customers online, the supply of messages online simply acts as a scaling factor for each …rm's demand function o- ine. Intuitively, each …rm now has s exp (M) potential customers o- ine. The equilibrium allocation is then given by

$$X^* = \sqrt{2M\ /\ }; \qquad (19)$$

$$m^* = \sqrt{2\ M} \quad x: \qquad (20)$$

Equations (19) and (20) show that the equilibrium distribution of o- ine messages across the participating …rms, as well as the number of active …rms, are both identical to the single market case. However, competition has a clear e¤ect on the equilibrium prices and revenues, as we show in the next proposition.

Proposition 7 (Equilibrium Prices)

1. The equilibrium price on the o- ine medium is given by

$$p^* = \ \exp(\ M \quad \sqrt{2\ M}\ ):$$

2. The equilibrium prices on the online markets are given by

$$p^* = \begin{cases} \exp(\ a \quad M \quad \overline{2\ M}\ ); & \text{for } a \quad X^*, \\ \exp(\ M\ ); & \text{for } a > X^*. \end{cases}$$

Consistent with intuition, the o- ine price p^* is decreasing in M . This re‡ects the decline in each …rm's willingness to pay for regular advertisements when an alternative, better targeted market is present. In other words, a targeted online market does not modify the composition of the o- ine market, but lowers the equilibrium pro…ts. The prices in the online markets are initially increasing in a, and then constant. This re‡ects the allocation of messages o- ine, where relatively smaller …rms buy a lower number of messages, and are willing to pay more for M messages per capita online. F03ictio2824ted327-18.9l-441(s)-1(m)360lly (m)367c

two models are very close, as the concentration p [redacted] sumers on market 0 is equal to + . As a result, the l [redacted] um and advertise exclusively online, in the largest marke [redacted] price of the o- ine medium.

This e¤ect is somewhat mitigated if the onli [redacted] As in the case of competition between o- ine media of [redacted] ll …rms larger than a critical x^* advertise both o- ine an [redacted] ts (i.e., each …rm x buys messages on markets a $[0; x]$ [redacted] better targeting reduces the demand for online messages [redacted] higher concentration in the o- ine medium. At the sam [redacted] o reach a larger fraction of their customers online, and t [redacted] mn2(duqc1.)-34tabiers)-t-1(s)-

equilibrium model. While the competitive equilibrium is the natural benchmark, it is of interest to consider the pricing of advertising in strategic environments. In the working paper version (Bergemann and Bonatti (2010)), we investigate the equilibrium pricing when each advertising market is owned by a single or a few publishers, each one maximizing his revenues. For the case of a single publisher, we analyze the optimal nonlinear pricing tari¤ in order to discriminate across advertising …rms of di¤erent segment size. We then extend the model to a small number of publishers in which each publisher determines the supply of messages in his outlet. In particular, we establish that for a su¢ ciently large, but …nite number of publishers, the Nash equilibrium yields the competitive equilibrium outcome analyzed here. Clearly, extending our model to incorporate the auctions for keywords in the sponsored search environment, or the emerging ad exchange model, might o¤er valuable additional insights in this respect.

In our model, the advertisers were competing for messages but they were not competing for consumers. In other words, competition among …rms for advertising messages did not interact with their competition in the product market. A natural next step therefore might

Appendix

Proof of Proposition 1. The average probability of a match, which is equal to the total fraction of consumers reached, is given by

$$W(\alpha; M) = \int_0^{X^*} s(1 - e^{-\alpha\tau^*})dx = 1 - \frac{1+\sqrt{2\alpha M}}{e^{\sqrt{2\alpha M}}},$$

which is increasing in α. ■

Proof of Proposition 2. (1.)–(4.) The comparative statics results can be derived directly by differentiating expressions (8), (9), and (10) in the text.

(5.) The total expenditure of firm $x \leq X^*$ is given by

$$p^*m^* = e^{-\sqrt{2\alpha M}}(\sqrt{2\alpha M} - x),$$

and the total number of consumers reached is

$$s(1 - e^{-\alpha\tau^*}) = e^{-x}(1 - e^{-\sqrt{2\alpha M}+x}):$$

Therefore, the price paid by firm x per consumer reached is given by

$$\frac{p^*m^*}{s(1 - e^{-\alpha\tau^*})} = \frac{\sqrt{2\alpha M} - x}{e^{\sqrt{2\alpha M}-x} - 1} = \frac{z}{e^z - 1},$$

which is decreasing in z (with $z = \sqrt{2\alpha M} - x$), and therefore increasing in x. It is also decreasing in α if $x < \sqrt{\alpha M/2}$ (which represents the median active firm). ■

Proof of Proposition 3. The average probability of a match now takes into account which is increasing in and . ■

Proof of Proposition 4. (1.)–(4.) These statements follow from di¤erentiation of expressions (14), (15), and (16) in the text. ■

Proof of Proposition 5. From the …rst order conditions for …rm x, we obtain

$$1 \quad f \quad = \quad e^{-} \quad = e \quad \frac{p}{(1 \quad f \quad)},$$
$$1 \quad f \quad = \quad e^{-} \quad = e \quad \frac{p}{(1 \quad f \quad)}.$$

It follows that in equilibrium we must have p = p = p, and that the sum of the demands is given by

$$m \quad + m \quad = \ln \frac{}{p} \quad x:$$

Consider the market clearing condition for A and B combined,

$$\int (m \quad + m \quad) \, dx = M \quad + M \quad ;$$

This expression is always negative for < 8: If 8 then the relevant root is given by

$$\max \left(1 \quad \frac{\bar{\ } + \sqrt{\quad 8}}{2\sqrt{2M}} ; 0 \right) ;$$

which concludes the proof. ■

References

Ambrus, A., and M. Reisinger (2006): "Exclusive vs Overlapping Viewers in Media Markets," Discussion paper, University of Munich.

Anderson, C. (2006): The Long Tail. Hyperion New York.

Anderson, S., and S. Coate (2005): "Market Provision of Broadcasting: A Welfare Analysis," Review of Economic Studies, 72, 947–972.

Athey, S., and G. Ellison (2008): "Position Auctions with Consumer Search," Discussion paper, Harvard University and MIT.

Athey, S., and J. Gans (2010): "The Impact of Targeting Technology on Advertising Markets and Media Competition," American Economic Review, Papers and Proceedings, 100, 608–613.

Bagwell, K. (2007): "The economic analysis of advertising," Handbook of Industrial Organization, 3, 1701–1844.

Bergemann, D., and A. Bonatti (2010): "Targeting in Advertising Markets: Implications for O- ine Vs. Online Media," Discussion Paper 1758R, Cowles Foundation for Research in Economics, Yale University.

Chandra, A. (2009): "Targeted Advertising: the Role of Subscriber Characteristics in Media Markets," Journal of Industrial Economics, 57(1), 58–84.

Chandra, F., and U. Kaiser (2010): "Targeted Advertising in Magazine Markets," Discussion paper, University of British Columbia.

Chen, Y., and C. He (2006): "Paid placement: Advertising and search on the internet," Discussion paper, University of Colorado at Boulder.

de Cornière, A. (2010): "Targeted Advertising with Consumer Search: an Economic Analysis of Keywords Advertising," Working Paper, Paris School of Economics.

Edelman, B., M. Ostrovsky, and M. Schwarz (2007): "Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords," American Economic Review, 97(1), 242–259.

Esteban, L., A. Gil, and J. Hernandez (2001): "Informative advertising and optimal targeting in a monopoly,"