Search, Design, and Market Structure[1]

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Abstract

The Internet has made consumer search easier, with consequences for prices, industry structure and the kinds of products o¤ered. We explore these consequences in a rich but tractable model that allows for strategic design choices. A polarized market structure results, where some …rms choose designs aiming for broad-based audiences, while others target narrow niches. Such an industry structure can arise even when all …rms and consumers are ex-ante identical. We analyze the e¤ect of reduced search costs and …nd results consistent with the reported prevalence of niche goods and the long-tail and superstar phenomena.

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The Internet has dramatically changed the nature of demand and competition. A familiar example is the book-publishing industry. Easier access to information on

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to largely because there was little else on is increasingly being ignored. (*The Economist*, 2009)

In this paper, we allow for a richer choice of …rm strategies than the search literature has typically considered. Speci…cally, …rms choose the "design" of their products in addition to price. Our notion of design is broad and can accommodate not only physical design, but also marketing and information disclosure. In the book publishing example, it is broad enough to accommodate a publisher's decisions both about commissioning particular topics or accepting particular kinds of manuscripts and such marketing decisions as making sample chapters available on line. Our approach allows us to address how designs adapt as search costs fall and to consider the equilibrium e¤ects on market structure, prices and consumer surplus. In particular, our analysis leads naturally to long-tail and superstar e¤ects arising simultaneously, and to prices and industry pro…ts that are non-monotonic in search costs.[4]

Formally, our notion of design choice builds on a recent and growing literature, notably Justin P. Johnson and David P. Myatt (2006), with an important antecedent in Tracy R. Lewis and David E. M. Sappington (1994).[5] While this literature has focused on design choices by monopolists, this paper extends this analysis to a competitive environment. To do so, we introduce product design, along the lines of Johnson and Myatt (2006), into a search model (Asher Wolinsky, 1986; Yannis Bakos, 1997; or Anderson and Renault, 1999). In particular, …rms choose designs ranging from broad market designs that are ino¤ensive to all consumers to more niche or quirky designs that consumers either love or loathe. Meanwhile, each consumer searches

[4] There is a small related literature that considers …rms that vary design in response to falling search costs. Nathan Larson (2008) studies horizontal di¤erentiation in a model of sequential search with a particular emphasis on welfare considerations in what can be viewed as a special case of our model. Dimitri Kuksov (2004) presents a duopoly model where consumers know the varieties available (but not their location) prior to search, and di¤erent designs come with di¤erent costs associated; Simon Anderson and Régis Renault (forthcoming) also consider duopoly and, in a result similar to one in this paper, show that it is the low-quality …rm that has the greater incentives to release information on horizontal characteristics; Gérard Cachon, Christian Terwiesch and Ye Xu (2008) and Randall Watson (2007) focus speci…cally on multi-product …rms' choices of product range. Our model allows for a wide range of designs and a much more general demand speci…cation. It, also, has a di¤erent focus and results from these papers, which, for example, do not consider sales distributions explicitly and so do not address long-tail and superstar e¤ects.

[5] More recently, Heski Bar-Isaac, Guillermo Caruana and Vicente Cuñat (2008, 2010) put more emphasis on consumers' information-gathering decisions and highlight that these are co-determined with the …rm's pricing, design and marketing strategies in equilibrium.

among …rms, paying a small cost to obtain a price quote from an additional …rm and to learn about the extent to which that …rm's product suits his tastes.

The model allows us to address the impact of search engines, the Internet, communication technologies and information technologies in general, by considering these as a fall in search costs. We show, …rst, that …rms choose extremal strategies— that is, either a most-broad or a most-niche design. Second, more-advantaged …rms choose most-broad designs, while disadvantaged …rms prefer most-niche designs. Our central simple condition under which the pro…ts of the worst …rms in the industry increase as search costs fall. This condition has an intuitive economic interpretation: If there is considerable vertical heterogeneity in the industry and the e¤ect of …rms changing designs is relatively limited then the pro…ts of the worst …rms necessarily fall. This is consistent with the evidence of Goldmanis et al. (2010) on bookstores, new auto dealers and travel agencies where di¤erent …rms are often selling identical goods. Instead, if moving from a broad to a niche design has a large e¤ect and there is more limited vertical heterogeneity, pro…ts of the worst …rms increase as search costs fall, as is consistent with the book-publishing industry, where it is easy to imagine that there is more scope for di¤erent book titles to appeal to niches and, as Brynjolfsson et al. (2003) document, product variety has increased dramatically (re‡ecting that small sellers are more pro…table).

1 Model

There is a continuum of risk-neutral …rms and consumers of measure 1 and m, respectively. Each …rm i produces a single product. Each consumer l has tastes described by a conditional utility function (net of any search costs) of the form

$$u_{li}(p_i) = v_i + \varepsilon_{li} - p_i \tag{1}$$

if she buys product i at price p_i. The term v_i captures a natural advantage of …rm i. A higher v_i can be thought of as a lower marginal production cost, but it also can be interpreted as better innate vertical quality.[8] Meanwhile, $\varepsilon_{li} \sim F_i$ is a match value between consumer l and product i. It captures idiosyncratic consumer preferences for certain products over others. We assume that realizations of ε_{li} are independent across …rms and individuals.[9]

A consumer incurs a search cost c to learn the price p_i and the match value ε_{li}

[8] Note that our results do not impose that $v_i > 0$; taking $v_i < 0$ and interpretting it as a marginal cost would lead the derived p_i to be interpretted as an absolute mark-up above the marginal cost but otherwise derivations would not change at all, and our results are consistent with either interpretation.

[9] Taking these realizations to be independent, while consistent with previous literature on search (Wolinsky, 1986; and Anderson and Renault, 1999), is not without loss of generality. It does not allow …rms to target speci…c niches. That is, there is no spatial notion of di¤erentiation or product positioning. However, given that we assume a continuum of …rms and no ability for consumers to determine location in advance, this assumption may be more reasonable.

for the product offered by any particular firm i. Consumers search sequentially. The utility of a consumer l is given by

$$u_{lk}(p_k) \quad kc \tag{2}$$

if she buys product k at price p_k at the kth firm she visits. From now on, and for simplicity, we will omit the firm and consumer subscripts, unless there is ambiguity.

Firms cannot affect v, the exogenous quality of the good, which is distributed according to some continuously differentiable distribution $H(v)$ with support $[\underline{v}; \overline{v}]$. In Section 5, we analyze the case where the distribution is degenerate so that, ex-ante, all firms are identical.

We introduce strategic design choice by assuming that the firm can affect the distribution of the match-specific component of consumer tastes F_s by picking a design $s \in S = [B; N]$. That is, designs range from a most-broad (B) to a most-niche (N) design. A design s leads to ε_{li} distributed according to $F_s(\,)$ with support on some bounded interval $(\underline{\;}_s; \overline{\;}_s)$, and with a logconcave density $f_s(\,)$ which is positive everywhere.[10] Regardless of design and intrinsic quality, the firm produces goods at a marginal cost of 0.[11]

The strategy for each firm, therefore, consists of a choice of price p and a product design $s \in S$. We suppose that there are no costs associated with choosing different designs s.[12]

We follow Johnson and Myatt (2006) in assuming that different product designs induce demand rotations. Formally, there is a family of rotation points $\,^{\dagger}_s$ such that $\frac{\partial F_s(\,)}{\partial s} < 0$ for $\; > \;^{\dagger}_s$ and $\frac{\partial F_s(\,)}{\partial s} > 0$ for $\; < \;^{\dagger}_s$; further $\;^{\dagger}_s$ is increasing in s. The

[10] See Mark Bagnoli and Ted Bergstrom (2005) for a broad discussion of logconcavity and functions that do and do not satisfy this condition. The assumption of logconcavity ensures that the failure rate $f_s(\,)/(1 \quad F_s(\,))$ is monotonic, and, so, guarantees existence of a profit-maximizing monopolist price which is continuous and increasing in constant marginal costs.

[11] Assuming constant marginal costs and no fixed costs simplifies the analysis considerably, though it can be relaxed in a similar fashion to Section IIB of Johnson and Myatt (2006). Within a framework of constant marginal costs, setting them to zero is without loss of generality. As already mentioned, differences in marginal costs play an identical role to differences in v.

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concept of a demand rotation is a formal approach to the notion that some designs lead to a wider spread in consumer valuations than others. In particular, a higher value of s should be interpreted as "quirkier" product that appeals more to certain consumers and less to others; the bounds on s correspond to the most broad (B) and the most niche (N) designs. Alternatively, in the marketing interpretation, a higher value of s corresponds to more information that shifts priors (up or down) and leads to more dispersed valuations. The formalization of design choices is general enough to accommodate a wide range of concepts of product design. One of the contributions of Johnson and Myatt (2006) is to show that natural models of physical product design and information-release provide micro-foundations for such demand rotations and to argue that it is natural to focus on instances where information or physical designs lead to more or less dispersed valuations.

As is standard in the search literature, we assume that consumers keep the same (passive) expectations about the distribution of future prices and design, independent of today's observed realization. This implies that a consumer's search and purchase behavior can be described by a threshold rule U: She buys the current product, obtaining $u_{li}(p_i)$, if this is more than or equal to U, and continues searching otherwise. This allows us to use Nash as our equilibrium concept. Consumers choose a threshold U and each v firm a pair $(p; s)$.[13] One advantage to this notation is that U also represents the consumer surplus from participating in the market. Note that there always exist equilibria where consumers do not search and firms choose prohibitively high prices. We do not consider such equilibria if others exist.

2 Equilibrium

2.1 Consumer behavior

Suppose that a consumer expects each firm of type v to choose strategy $(p_v; s_v)$.[14] Consider a consumer who can stop searching and obtain utility u. If the consumer, instead, samples an additional firm of type v, she will prefer the new product if $v + \varepsilon - p_v > u$. In this case, the additional utility obtained is $v + \varepsilon - (u + p_v)$, and

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so the expected incremental utility from searching at one more firm that is expected to have design s_v and price p_v and to be of quality v is

$$E_\varepsilon(\max\{v+\varepsilon-p_v-u;0\}) = \int_{u+p_v-v}^{\infty}(v+\varepsilon-p_v-u)f_{s_v}(\varepsilon)d\varepsilon. \tag{3}$$

It is worth visiting one more firm if and only if the expected value of the visit is worth more than the cost, where the final expectation is taken over v (with an implicit firm strategy for both price and design); that is, as long as $E_v[E_\varepsilon(\max\{v+\varepsilon-p_v-u;0\})] \geq c$, or, equivalently, if $u < U$ where U is implicitly defined by:

$$\int_{-\infty}^{\infty}\int_{U+p_v-v}^{\infty}(v+\varepsilon-p_v-U)f_{s_v}(\varepsilon)d\varepsilon\, h(v)dv = c. \tag{4}$$

There is, at most, one solution to (4) since the left-hand side is strictly decreasing in U (as the integrand is decreasing in U and the lower limit of the inner integral is increasing in U). For large enough c, there is no feasible positive U that satisfies (4): No consumer would ever continue searching, and firms would have full monopoly power (as in Peter Diamond, 1971). In other words, the consumer initiates search if and only if $U \geq 0$.

2.2 Firm profit maximization

Consumers who visit a firm of type v buy as long as they receive a match ε such that $v+\varepsilon-p > U$ and, thus, purchase with probability $1-F_s(p+U-v)$.

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and its profits as

$$\Pi = \frac{m}{\ }p(1 - F_s(p + U - v)). \tag{6}$$

It is useful to define $p_{vs}(U)$ as firm v's profit-maximizing price when the consumer's threshold is U and the design strategy is s. This price is implicitly determined by

$$p_{vs}(U) = \frac{1 - F_s(p_{vs}(U) + U - v)}{f_s(p_{vs}(U) + U - v)}. \tag{7}$$

Our first result, a consequence of the logconcavity assumption, ensures that p_{vs} is well-defined and behaves in a way that is intuitive: Higher-quality firms charge

To gain some intuition for this result, …rst consider the case when the optimal price at a given design s is below the point of rotation, so that the pro…t-maximizing quantity is greater than the quantity at the point of rotation $1 \quad F_s(\,^{y}_{s})$. Then, decreasing s (and so "‡attening" out demand) will lead to a greater quantity being sold even if the price is kept …xed. Therefore, decreasing s must lead to higher pro…ts. A similar argument applies when the optimal price is above the point of rotation.[16]

Proposition 1 allows us to restrict attention to equilibrium strategies in which …rm v chooses either a broad design $(p_{vB}; B)$ or a niche one $(p_{vN}; N)$, where p_{vB} and p_{vN} are de…ned by (7) for $s = B; N$, respectively.

Next, de…ne $V(U)$ as the solution to

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chance that this happens increases with a design that leads to dispersed valuations—a niche design. Instead, a high-value firm can appeal to many consumers by adopting the broad strategy and, thereby, minimize the chance that a well-disposed consumer observes that a product is such a bad match that she would prefer not to purchase.

This result is economically rich and appealing. First, when interpreting v as relating to marginal costs, it states that low-cost firms try to attract a broad market, while high-cost firms, who must charge higher prices to be profitable, target niches. Second, as an example of the quality interpretation, consider five-star hotels competing in a city. Although they are in the same category, they differ in an important dimension: location. Our model predicts that hotels that are well located (center of the city, close to the airport or other facilities) are more likely to deliver standard services. Meanwhile, those with less-desirable locations are more likely to be specialized— for example, boutique hotels with distinctive styling or those catering to specific groups, such as customers with pets.

2.3 Equilibrium Summary

Given the analysis above, we can express an equilibrium as a pair $(U; V)$, where U

visits a random firm. This is given by

$$(U;V) \quad \int_{1}^{V} (1 \quad F_N(p_{vN}(U)+U \quad v))h(v)dv + \int_{V}^{1} (1 \quad F_B(p_{vB}(U)+U \quad v))\, h(v)dv. \tag{10}$$

Note there always exist equilibria where consumers prefer not to search (and firms charge sufficiently high prices that this is optimal behavior for consumers). When search costs are sufficiently high, that is for $c > c_0$; this is the unique equilibrium. For lower search costs, equilibria involve some firms choosing niche designs and others broad designs, or all firms choosing niche or broad designs. The latter case is, relatively easy to characterize and we summarize results in the Proposition below. The other, more interesting case— where different firms choose different kind of designs— is the focus of our analysis is more involved and analyzed below.

Proposition 3 *Let* U_B

is, having higher ***U***). First, there is a direct effect that leads firms to drop prices and sell less per consumer-visit. But, second, there is a countervailing effect: More consumers will visit any given firm (i.e., is lower), not only because consumers are

$U < \overline{U}$, all …rms prefer a broad design. It is only at $U = \overline{U}$ that …rms might mix. However, a mixed-strategy equilibrium can exist over a wide range of search costs. This is immediate, by noting that at $U = \overline{U}$, expression (11) can be rewritten as

$$c = \quad c_N + (1 \qquad) c_B, \tag{14}$$

where c_B and c_N are the search cost thresholds introduced in Proposition 3 and are formally characterized in Equations (22) and (24) in the Appendix. Note that c_B and c_N have interpretations as the expected consumer surplus from visiting a broad or a niche …rm, respectively, when the reservation utility $\overline{U}$ is such that a …rm makes

First, note that although a fall in search costs represents a direct benefit to consumers, this gain is exactly offset by the negative impact from searching more (decreases) and from the increased preponderance of niche firms that provide less surplus in expectation ($c_B > c_N$). Next, since the consumer threshold is constant throughout the region, a firm's expected profit per consumer visit does not change. However, given that there are more consumer visits (decreases), profits increase.[24]

Finally, we turn to market structure. Consistent with "long-tail" stories, we observe that as search costs fall, each niche firm sells more. In addition, there are more niche firms and, since the total volume of sales is constant, it follows that the niche firms account for a greater proportion of overall sales. Note, also, that superstar effects are present. The "top" firm chooses a broad design and sells more as c goes down. The tail is niche throughout and also sells more as c goes down. The middle region, where the mix of broad and niche is changing, is the one that loses sales to both the head and the tail of the sales distribution. This is illustrated in Figure 1 below.

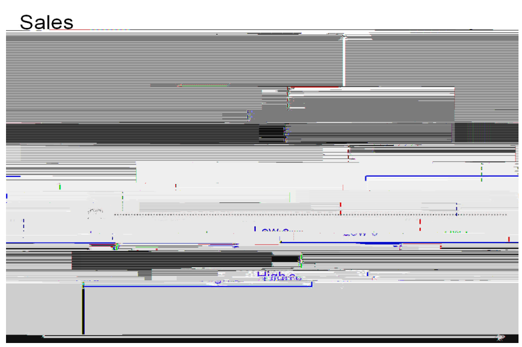

Fig 1: Distribution of sales at different search costs.

When search costs are low enough or high enough, all firms choose the same design and all of them sell m. Thus, as Figure 1 shows, sales are non-monotonic. Profits are also non-monotonic: They decrease in search costs when these are low or

[24] Although the probability of making a sale for any given visit stays constant for any given type of firm, this is consistent with more consumers visiting since the composition of firms changes. There are more niche firms as c falls, and niche firms sell less than broad firms.

high, but increase in search costs in the intermediate region (as shown in Propositions 5 and 8).

6 Uniformly distributed quality and linear demands

We once again consider heterogeneous firms, but impose further structure that allows us to derive additional analytic results. These highlight that the results of Section 5 extend naturally to more-general settings. We analyze the case where the distribution of firm quality is uniform $v \sim U[L; H]$, and the distributions $F_s(\cdot)$ are uniform, leading to linear demand functions. In particular, the niche and broad product competition-softening effect of firms switching to niche designs more than compensates for the intensified vertical competition that arises as search costs fall.

Note that if firms' types are very dispersed then a low quality firm must be forced out of the market when search costs are sufficiently low; following our definition, trivially, in such circumstances, long tail effects cannot arise. Proposition 6, therefore, focuses on parameter ranges where all firms remain active even for low values of c.

We illustrate some results of Proposition 6 in the case that $^{-}{}_{N} \quad ^{-}{}_{B} > H \quad L$.

Fig 2: Price against search cost.

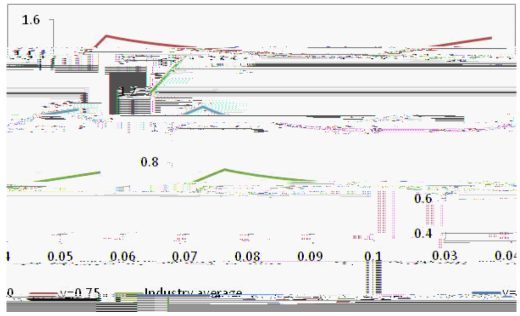

Fig 3: Profits against search costs.

Finally, we consider sales distributions. Figure 4 is the analogue of Figure 1 and plots the distribution of sales for two different search costs. Naturally, higher-quality firms sell more than low-quality firms, regardless of the search costs. Comparing sales at different search costs, both the highest- and lowest-quality firms sell more at the lower level of search costs, illustrating that superstar and long-tail effects arise simultaneously. These are also illustrated at intermediate levels of search costs (where there is dispersion in designs offered) in Figure 5, which plots sales against search costs for the best and worst firms.

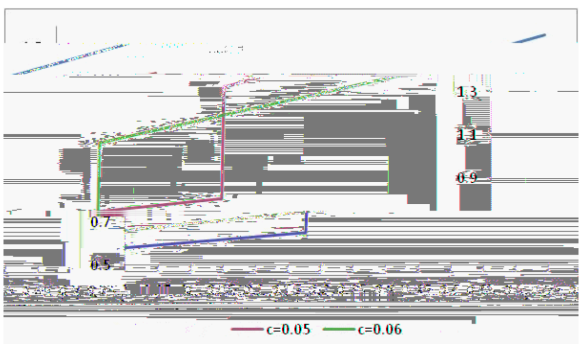

Fig 4: Sales against quality (v) at $c = 0.05$ and $c = 0.06$.

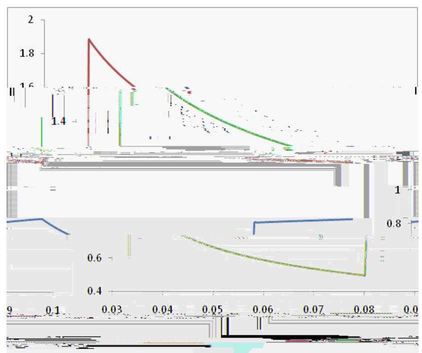

Fig 5: Sales against search cost for best and worst firms.

7 Conclusions

There has been considerable attention on the influence of the Internet on the kind of products offered and the distribution of their sales. In particular, academic and popular commentators have highlighted both long-tail and superstar e¤ects for various industries (including publishing, media, and travel destinations, among others). This paper presents a simple and tractable model integrating consumer search and …rms' strategic product-design choices that is useful to analyze these phenomena.

We show that, in equilibrium, di¤erent product designs coexist. More-advantaged …rms prefer broad-market strategies, seeking a very broad design and choosing a relatively low price, while less-advantaged …rms take a niche strategy with quirky products priced high to take advantage of the (relatively few) consumers who are well-matched to the product. Such design diversity arises even when all …rms are homogeneous.

Prices and pro…ts can be non-monotonic in consumer search costs. There is an intuitive rationale for this: As search costs fall, and as long as the product designs remain unchanged, prices fall. However, at ever lower prices, the broad-market strategy becomes less appealing to …rms, some of whom adopt a niche strategy, charging a high price to the (few) consumers who are well-matched for the product. Moreover, the …rms' decision to adopt a niche strategy acts as a form of di¤erentiation that softens price competition, and e¤ectively create a positive externality on other …rms. Indeed, this observation suggests a rationale for industry coordination: since pro…ts can be non-monotonic in search costs, as search costs fall exogenously, industries might bene…t from reducing them further (for example, through industry-sponsored comparison sites).

Finally, our comparative statics analysis provides a demand-side explanation of the long-tail e¤ect. As search costs fall, a greater proportion of …rms choose the niche strategy. Consumers search to a much greater extent and, consequently, niche …rms may account for a larger proportion of the industry's sales. In addition, lower search costs can simutrche J(r)11(yT3J(r)11(yT3J()11(e)-3i))8(t)8(s)-33J(r)11(y1)36((i)6(n)12(d)11(u)79(s would remain unchanged. Further, the Internet has had broader impacts that go beyond search costs, and long-tail and superstar phenomena may reflect changes to production costs.[26] In this paper we have focused on changes to the demand-side to isolate their effects, as we believe they are economically significant.

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A Proofs

Proof of Lemma 1 First, note that since $f_S(x)$ is logconcave, $\frac{1-F_S(x)}{f_S(x)}$ is strictly decreasing in x (See, for example, Corollary 2 of Bagnoli and Bergstrom, 2005). Suppose (for contradiction) that at some value of U, $p_{VS}(U)$ is increasing in U; then, $p_{VS}(U)+U$ is also increasing in U, and so $\frac{1-F_S(p_{VS}(U)+U-v)}{f_S(p_{VS}(U)+U-v)} = p_{VS}(U)$ is decreasing in U, which provides the requisite contradiction. A similar argument ensures that $p_{VS}(U)+U$ is increasing in U, that $p_{VS}(U)$ is increasing in v; and that $p_{VS}(U)-v$ is decreasing in v. ■

Proof of Proposition 1 The optimal design is chosen to maximize $p_{VS}(U)(1-F_S(p_{VS}(U)+U-v))$. Now, given that $p_{VS}+U-v$ is an affine transformation of p_S, it follows that $D_V(p_{VS};S)$, as in (5), are rotation-ordered. The proof then follows immediately from Proposition 1 in Johnson and Myatt (2006), p. 761. ■

Proof of Proposition 2 For a fixed value of U; in principle, there may be more than one V solving equation (8). We show later that this is not the case. Consider one such solution and notice that

$$p_{VB}(U)(1-F_B(p_{VB}(U)+U-V)) = p_{VN}(U)(1-F_N(p_{VN}(U)+U-V)) \tag{15}$$

$$\geq p_{VB}(U))(1-F_N(p_{VB}(U)+U-V)). \tag{16}$$

It follows that

$$1-F_B(p_{VB}(U)+U-V) \geq 1-F_N(p_{VB}(U)+U-V). \tag{17}$$

Similarly,

$$1-F_N(p_{VN}(U)+U-V) \geq 1-F_B(p_{VN}(U)+U-V). \tag{18}$$

We use these facts to show that $p_{VN}(U) > p_{VB}(U)$. Suppose (for contradiction) that

$p_{VN}(U) < p_{VB}(U)$. Note that since N and B are drawn from a family of demand rotations, it follows that there is some $\hat{x}$ such that $1 - F_N(x) > 1 - F_B(x)$ if and only if $x > \hat{x}$. If $p_{VB}(U) + U - V > \hat{x}$, then, $1 - F_N(p_{VB}(U) + U - V) > 1 - F_B(p_{VB}(U) + U - V)$ in contradiction to (17). If, instead, $\hat{x} - p_{VB}(U) + U - V > p_{VN}(U) + U - V$, then (18) is contradicted. Thus, $p_{VN}(U) > p_{VB}(U)$ and from (8), trivially,

$$1 - F_B(p_{VB}(U) + U - V) > 1 - F_N(p_{VN}(U) + U - V). \quad (19)$$

Define $\pi_{vs} := p_{vs}(1 - F_s(p_{vs} + U - v))$ with $s = B; N$. Since the price is chosen to maximize profits, by the envelope theorem, we have that $\frac{d\pi_{vs}}{dv} = p_{vs} f_s(p_{vs} + U - v) = 1 - F_s(p_{vs}(U) + U - v)$ where the second equality follows from (7). Now, given (19), it follows that $\frac{d\pi_{VN}}{dv} < \frac{d\pi_{VB}}{}$

c_0, there is no positive search equilibrium with $c > c_0$. Take, now, $c \in [c_B; c_0)$. Using the definitions of c_B and c_0 and by looking at condition (9), one can easily see that there exists a $U \in (0; U_B]$ such that $(U; \underline{v})$ constitutes an equilibrium. Finally, for $c < c_B$, there cannot be an all-broad equilibrium. By looking at (22), note that the induced U had to be bigger than U_B; but this would imply that the type $\underline{v}$ firm prefers a niche strategy, providing a contradiction.

Analogous to the all-broad case, we can consider all firms choosing the niche design, so that $V = \bar{v}$, together with the consumer stopping rule that makes the highest-quality firm indifferent in its design choice, U_N, and the associated search cost, c_N. These are defined by the following conditions:

$$p_{\bar{v}B}(U_N)(1 - F_B(p_{\bar{v}B}(U_N) + U_N - \bar{v})) = p_{\bar{v}N}(U_N)(1 - F_N(p_{\bar{v}N}(U_N) + U_N - \bar{v})), \quad (23)$$

$$c_N = \int_{-\infty}^{\infty} \int_{p_{vN}(U) + U_N + v}^{\bar{\varepsilon}_N} (v + \varepsilon - p_{vN}(U_N) - U_N) f_N(\varepsilon)$$

that $A \to \infty$ and as $V \to \infty$ then $A \to -\infty$. Consider

$$\frac{dA}{dV} = \frac{1}{8}\frac{\left(Lb + \beta_B n - Vb - Ln - \beta_N n + Vn\right)^2}{n^2(H-L)(n-b)^2} - \frac{1}{8}\frac{\left(\beta_B b + Hb - \beta_N b - Hn - Vb + Vn\right)^2}{b^2(H-L)(n-b)^2}$$ and

$$\frac{d^2A}{dV^2} = \frac{1}{4}\frac{Hn^2 - Lb^2 + bn(\beta_N - \beta_B)}{b^2n^2(H-L)} - \frac{1}{4}\frac{n^2-b^2}{b^2n^2(H-L)}V.$$

Now $V \in (\min\{K; L\}; H)$. Note that $\frac{d^2A}{dV^2}|_{V=H} = \frac{1}{4}\frac{(H-L)b+n(\beta_H-\beta_B)}{bn^2(H-L)} > 0$, and since $\frac{d^3A}{dV^3} < 0$, this means that $\frac{d^2A}{dV^2} > 0$ throughout the relevant region. Consider $\frac{dA}{dV}|_H = \frac{1}{8}\frac{2n(\beta_N-\beta_B)-(H-L)(n-b)}{n^2(n-b)}$. If $\frac{dA}{dV}|_H = \frac{1}{8}\frac{2n(\beta_N-\beta_B)-(H-L)(n-b)}{n^2(n-b)} < 0$, then, since $\frac{d^2A}{dV^2} > 0$ through the region, $\frac{dA}{dV} < 0$ and there can be, at most, one solution to $A = 0$. This is the case if and only if

$$2n\frac{\beta_N - \beta_B}{n-b} > H - L. \tag{28}$$

Note that, throughout, we assumed that all firms are active. Consider, now, the limiting

Taking the derivative of each with respect to U, we obtain

$$\frac{d \quad_{HB}(U)}{dU} = 2m(H \quad L)(\bar{\quad}_N \quad \underline{\quad}_N) \; \bar{\quad}_B \quad \underline{\quad}_B \; \frac{\bar{\quad}_B + H \quad U \quad \bar{\quad}_N + L \quad U \; (\bar{\quad}_N \quad \bar{\quad}_B \quad (H \quad L))}{((\bar{\quad}_B + H \quad U)^2(\bar{\quad}_N \quad \underline{\quad}_N) \quad (\bar{\quad}_N + L \quad U)^2(\bar{\quad}_B \quad \underline{\quad}_B))^2}, \text{ and}$$

$$\frac{d \quad_{LN}(U)}{dU} = 2m(H \quad L)(\bar{\quad}_B \quad \underline{\quad}_B)(\bar{\quad}_N \quad \underline{\quad}_N) \frac{\bar{\quad}_B + H \quad U \quad \bar{\quad}_N + L \quad U \; (\bar{\quad}_N \quad \bar{\quad}_B \quad (H \quad L))}{((\bar{\quad}_B + H \quad U)^2(\bar{\quad}_N \quad \underline{\quad}_N) \quad (\bar{\quad}_N + L \quad U)^2(\bar{\quad}_B \quad \underline{\quad}_B))^2}.$$

Note that since all ...rms are active, $p_{HB}(U)$ and $p_{LN}(U)$ must be positive. Following that prices must be non-negative and using (26), it follows that $\frac{d \quad_{HB}(U)}{dU}$ and $\frac{d \quad_{LN}(U)}{dU}$ have the same sign as $(\bar{\quad}_N \quad \bar{\quad}_B) \quad (H \quad L)$.

Finally, turning to sales, we can write the sales of the highest-quality and lowest-quality ...rms as

$$S_{HB}(U) = m\frac{(H \quad L)(\bar{\quad}_N \quad \underline{\quad}_N)}{2} \qquad (\bar{\quad}_B + H \text{ad} p_{LNN}$$

B Omitted results

B.1 Results related to Section 3

Existence of Equilibria: Consider $V(\ )$ and $U(\ )$ which are, respectively detemined as the solution for V to Equation (8) as a function of U and the solution for U to Equation (9) as a function of V. These are well-behaved continuous functions. The composition $V(U(\ ))$ is, therefore, a continuous function of $[\underline{v}; \overline{v}]$ into itself. Given that $[\underline{v}; \overline{v}]$ is compact, $V(U(\ ))$ has a …xed point V . It is immediate that $(U(V\ ); V\ )$ constitutes a Nash equilibrium of the game.

Concept of Stability: We de…ne the following di¤erential dynamic system

$$V = V(U) \quad V$$

$$U = U(V) \quad U.$$

One can immediately see that the Nash equilibrium of our game coincides with the steady states of this system. Now, a steady state $(V\ ; U\ )$ of this system is asymptotically stable if

The superstar e¤ects arise if and only if

$$\frac{\partial\left(\frac{m(1-F(p_{\overline{v}}(U))+U-\overline{v}))}{\Pi(U)}\right)}{\partial U} = m\frac{\partial}{\partial U}\left(\frac{[1-F(p_{\overline{v}}(U)+U-\overline{v})]}{\int_{\underline{v}}^{\overline{v}}[1-F(p_v(U)+U-v)]h(v)dv}\right) > 0.$$

A su¢ cient condition, therefore, is that

$$\frac{\partial}{\partial U}\left(\frac{1-F(p_{\overline{v}}(U)+U-\overline{v})}{1-F(p_v(U)+U-v)}\right) > 0 \text{ for all } v < \overline{v}. \tag{31}$$

Similarly, a su¢ cient condition to ensure that no long-tail e¤ect arises is

$$\frac{\partial}{\partial U}\left(\frac{1-F(p_{\underline{v}}(U)+U-\underline{v})}{1-F(p_v(U)+U-v)}\right) < 0 \text{ for all } v > \underline{v}. \tag{32}$$

Writing $W = U - v$ (and the corresponding $\overline{W}$ and $\underline{W}$), we can write $1 - F(p_v(U) + U - v) = q(W)$. Then, (31) is equivalent to $\frac{d}{dW}(\frac{q(\overline{W})}{q(W)}) > 0$ and (32) to $\frac{d}{dW}(\frac{q(\underline{W})}{q(W)}) < 0$: Note that Lemma 1 shows that $q(\overline{W}) > q(W)$ and that $\frac{d}{dW}q(W) < 0$. But neither of these conditions is enough to guarantee (31) and (32). A su¢ cient condition, though, is

$$\frac{d^2}{dW^2}q(W) < 0 \text{ for } W \in (\underline{W};\overline{W}). \tag{33}$$

It remains to verify this condition. Consider the …rm's maximization problem $p[1 - F_s(p + U - v)]$; this is equivalent to maximizing $(P - W)(1 - F(P))$ and $q(W) = 1 - F(P)$. It follows that we can write:

$$\frac{d^2q}{dW^2} = -f\frac{d^2P}{dW^2} - f'(\frac{dP}{dW})^2$$

This is necessarily the case when $f'(\cdot) > 0$ or, more generally, when $F(\cdot)$ is not too concave. ■

B.2 Results related to Section 5

Proposition 8 *In the homogeneous firms model of Section 5, if $c \le c_N$ or $c > c_B$, then as c falls: (i) consumer surplus U is increasing; (ii) consumers search more (decreases); (iii) every firm's profits decrease; and (iv) every firm's sales stays constant.*

Proof. Consider the case $c \le c_N$ (the other case is analogous). Then, (11) and (13) can be written simply as

$$c = \int_{p_N(U)+U}^{\infty} (\varepsilon - p_N(U) - U) f_N(\varepsilon) d\varepsilon,$$