Merger Policy with Merger Choice

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PRELIMINARY AND INCOMPLETE

process among the firms. The antitrust authority observes the characteristics of the merger that is proposed, but neither the feasibility nor the characteristics of any mergers that are not proposed. We focus in the main part on an antitrust authority who wishes to maximize expected consumer surplus. Our main result characterizes the form of the antitrust authority's optimal policy, which we show should impose a tougher standard on mergers involving larger acquirers (in terms of their pre-merger share). Specifically, the minimal acceptable level of increase in consumer surplus is strictly positive for all but the smallest acquirer, and is larger the greater is the acquirer's premerger share.

The closest papers to our are Lyons (2003) and Armstrong and Vickers (2010). Lyons first identifies the issue that arises when firms may choose which merger to propose. Armstrong and Vickers (2010) provide an elegant characterization of the optimal policy when mergers (or, more generally, projects that may be proposed by an agent) are ex ante identical in terms of their distributions of possible outcomes. Our paper differs from Armstrong and Vickers (2010) primarily in its focus on the optimal treatment of mergers that differ in this ex ante sense.

The paper is also related to Nocke and Whinston (2008). That paper established conditions under which the optimal dynamic policy for an antitrust authority who wants to maximize discounted expected consumer surplus is a completely myopic policy, in which a merger is approved if and only if it does not lower consumer surplus at the time it is proposed. One of the important assumptions for that result was that potential mergers were "disjoint," in the sense that the set of firms involved in different possible mergers do not overlap. The present paper explores, in a static setting, the implications of relaxing that disjointness assumption, focusing on the polar opposite case in which all potential mergers are mutually exclusive.

The paper proceeds as follows: We describe the model in Section 1. Section 2 derives our main result, which characterizes the optimal policy in the case in which the bargaining between firms proceeds as in the Segal (1999) offer game. In Section 4, we show that our main characterization result extends to some other bargaining models, including the case in which the bargaining is efficient. Section 5 discusses some other extensions of our results, and Section 6 concludes.

We consider a homogeneous goods industry in which firms compete in quantities (Cournot competition). Let N $\{\,;\ ;\ ;\ldots;N\}$ denote the (initial) set of firms. All firms have constant returns to scale; firm i's marginal cost is denoted c. Inverse demand is given by $P\ Q$. We impose standard assumptions on demand:

For all Q such that $P\ Q\ >$, we have:

$P'\ Q\ <\ ;$

$P'\ Q\quad QP''\ Q\ <$

$_{\to\infty} P\ Q$

$_i$ P Q_- q c q satis…es b' Q_- 2 ; , where Q_- $\neq$ q) so that comparative statics are "well behaved" (if a subset of …rms jointly produce less [more] because of a take-it-or-leave-it o¤er t to a single …rm k of its choosing, where k is such that M 2 \ A . If the o¤er is accepted by …rm k, then merger M is proposed to the antitrust authority, who will approve it since M 2 \ A , and …rm k acquires the target in return for the transfer payment t . If the o¤er is rejected, or if no o¤er is made, then no merger is proposed and no payments are made.

Let

$$M \quad M \quad {}^{0}_{0} \quad {}^{0}; k \quad ;$$

denote the change in the bilateral pro…t to the merging parties, …rms 0 and k, induced by merger M . Given the realized set of feasible and approvable mergers, \ A, the proposed merger in the equilibrium of the o¤er game is M^* ; A , where

$$M^* \; ; A \quad \begin{cases} M \; ; A & \text{if} \quad M \; ; A > \\ M_0 & \text{otherwise,} \end{cases}$$

and

$$M \; ; A \quad \underset{k \in (\mathfrak{F} \cap \mathcal{A})}{} \quad M :$$

That is, the proposed merger M is the one that maximizes the induced change in the bilateral pro…t to …rms and k, provided that change is positive; otherwise, no merger is proposed.

In line with legal standards in the U.S. and many other countries, we assume that the antitrust authority acts in the consumers' interests. That is, the antitrust authority selects the approval set A that maximizes expected consumer surplus given that …rms' proposal rule is M^* :

$$\underset{\mathcal{A}}{} \; E_{\mathfrak{F}} \quad CS \; M^* \; ; A \quad ;$$

where the expectation is taken with respect to the set of feasible mergers, . (We discuss aggregate surplus maximization in Section 4.)

We are interested in studying how the optimal approval set depends on the pre-merger characteristics of the alternative mergers. For this reason, we assume that the potential acquirers di¤er in terms of their pre-merger marginal costs. Without loss of generality, let K f $; :::; K$g and re-label …rms through K in decreasing order of their pre-merger marginal costs: $c_1 > c_2 > ::: > c$. Thus, in the pre-merger equilibrium, …rm k 2 K produces more than …rm j 2 K, and has a larger market share, if $k > j$. We will say that merger M is *larger* than merger M if $k > j$ as the combined pre-merger market share of …rms 0 and k is larger than that of …rms 0 and j.

We now investigate the form of the antitrust authority's optimal policy when the bargaining process amongst …rms takes the form of the o¤er game. Given a realized set of feasible mergers and an approval set A, this bargaining process results in the merger M^* ;A , as discussed in the previous section. We begin with some preliminary observations before turning to our main result.

As …rms produce a homogeneous good, a merger M raises [reduces] consumer surplus if and only if it raises [reduces] aggregate output Q. The following lemma summarizes some useful properties of a *CS-neutral* merger M, i.e., a merger that leaves consumer surplus unchanged, $CS\ M$.

Suppose merger M *is CS-neutral. Then*

1. *the merger causes no changes in the output of any nonmerging …rm* $i \,2\, \mathrm{f}\ ; k\mathrm{g}$ *nor in the joint output of the merging …rms and* k*;*
2. *the merged …rm's margin at the pre- and post-merger price* $P\ Q^{\circ}$ *equals the sum of the merging …rms' pre-merger margins:*

$$P\ Q^{\circ} \quad \bar{c} \quad P\ Q^{\circ} \quad c_0 \quad P\ Q^{\circ} \quad c \tag{1}$$

where the inequality follows from Assumption 1.

To see part (2), rewrite firm i's first-order condition to obtain

$$q \quad M \qquad \frac{P \; Q \; M \qquad c}{P' \; Q \; M} :$$

As the RHS is decreasing in $Q \; M$, this implies that $dq \quad M \; = d\bar{c} \quad >$. Next, take the derivative of the merged firm's profit with respect to its post-merger marginal cost:

$$\frac{d}{d\bar{c}} \; P \; Q \; M \qquad \bar{c} \quad q \quad M \qquad q \quad M \qquad P' \; Q \; M \qquad \sum_{\in \mathcal{N} \setminus \{0 \quad \}} \frac{dq \quad M}{d\bar{c}} :$$

To see this, let $\overline{Q} \quad Q \; M \quad Q \; M$ denote the level of aggregate output after either merger. Summing up the N …rst-order conditions of pro…t maximization after merger M , $l \quad j; k$, we obtain

$$NP \; \overline{Q} \qquad \underset{\geq 1 \;\; \neq}{} \; c \quad \bar{c} \qquad \overline{Q} P' \; \overline{Q} \quad :$$

It follows that $c \quad \bar{c}$, $i; l \quad j; k$, i 6 l, is the same under either merger, proving the claim.

Combining these observations, we can re-write equation (3) as

$$q \;\; M \qquad q \;\; M \qquad q^0 \quad q^0 \;\; :$$

Now, as merger M , $l \quad j; k$, is CS-nondecreasing by assumption, the merger induces a weak increase in the joint output of the merger partners and a weak decrease in the output of any other …rm i 6 $\; ; l$. That is,

$$q \;\; M \qquad q_0^0 \quad q^0 > q^0 \quad q \;\; M \;\; ; \; l; r \quad j; k; \; l \; 6 \;\; r;$$

implying that

$$q \;\; M \qquad q \;\; M \quad > \; q^0 \quad q^0 \;\; ;$$

and thus resulting in a contradiction. Hence, equation (3) cannot hold.

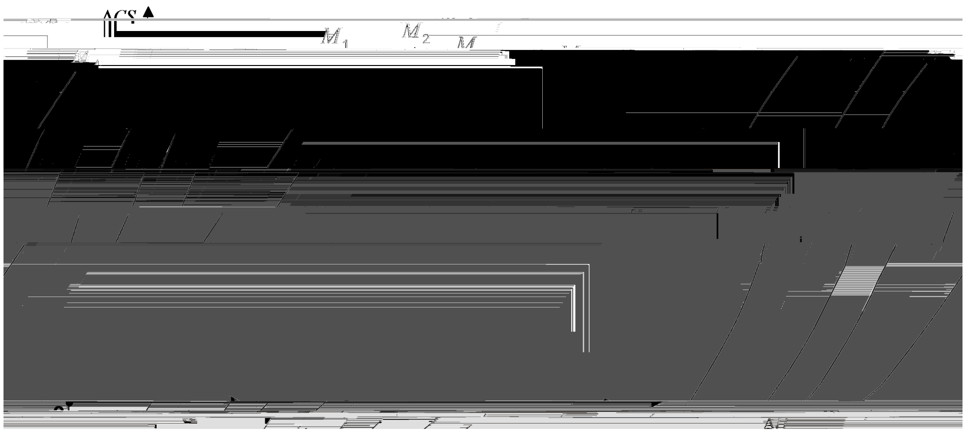


Figure 1: The curves depict the relationship between consumer surplus e¤ect and bilateral pro…t e¤ect of the various mergers, where each point on a curve corresponds to a di¤erent realization of post-merger marginal cost for that merger.

Throughout we restrict attention to such policies.[1] Let $\bar{a}$ f$\bar{c}$ j$\bar{c}$ 2 A g denote the largest allowable post-merger cost level for a merger (i.e., the "marginal merger") between …rms 0 and k. Also let $\underline{CS}$ $CS\ k;\bar{a}$ and ___ $k;\bar{a}$ denote the changes in

Figure 2: Changing the approval set A by blocking all mergers that induce a reduction in consumer surplus, resulting in approval set A^+, raises expected consumer surplus.

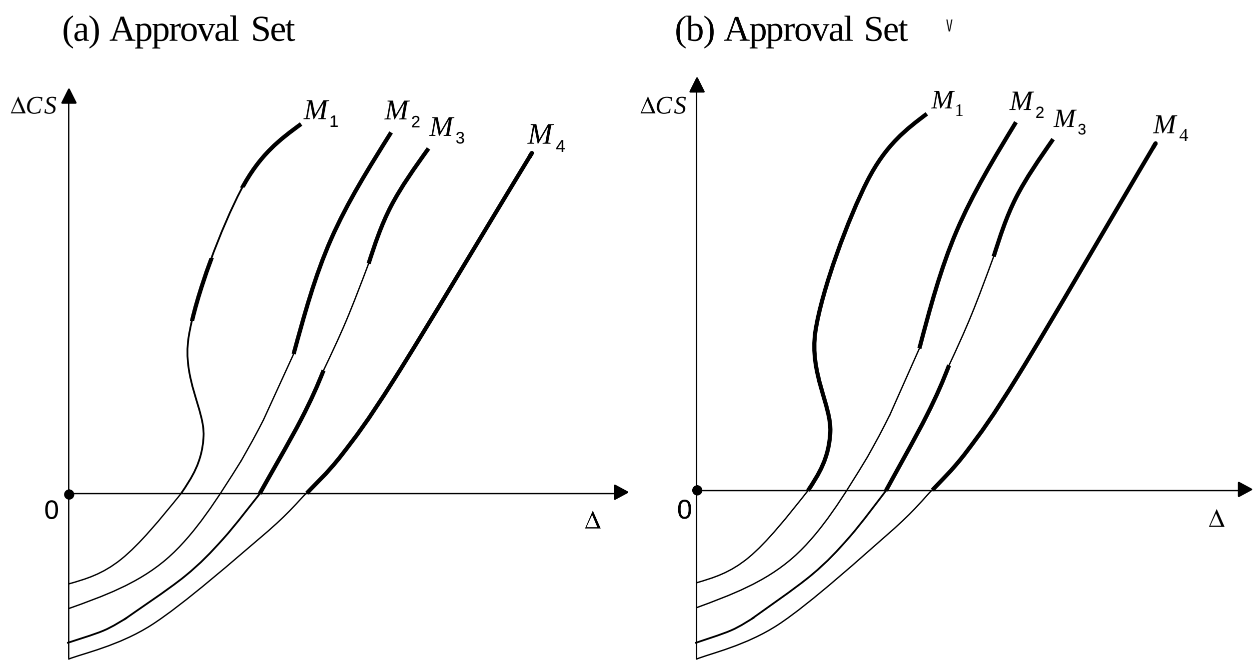


Figure 3: Changing the approval set A by approving the smallest merger $\boldsymbol{M}_1$ whenever it does not reduce consumer surplus, resulting in approval set A^+, raises expected consumer surplus.

(a) ApprovalSet

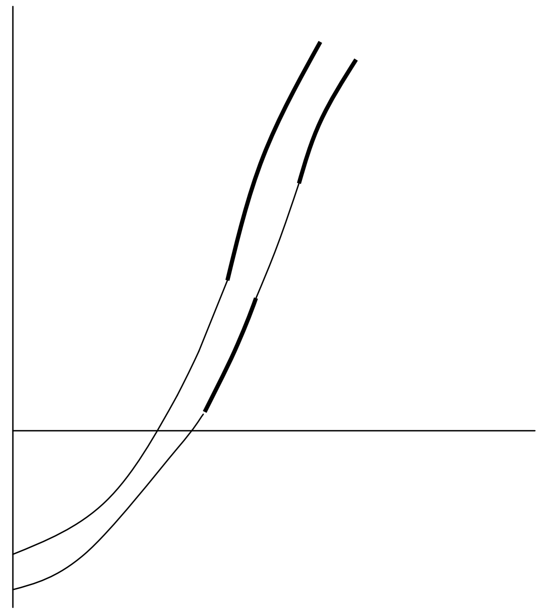


Figure 4: Changing the approval set A by blocking all those mergers other than the smallest that raise consumer surplus by less than ***"***, resulting in approval set A , raises expected consumer surplus for ***"*** su¢ ciently small.

k 2 K$^+$. To see this, consider switching from the policy A to A f***M*** 2 A ***k*** 2 K$^+$ and ***CS M*** **>** ***"***g where ***"*** **>** , as shown in Figure 4. The change in expected consumer surplus equals ***M**** ***;***A 2 AnA times

$$E_{\mathfrak{F}} \quad CS\ M^* \quad ;A \qquad CS\ M^* \quad ;A \ \ jM^* \quad ;A \ \ 2\ AnA \ \ :$$

Now, as ***"*** ! , this conditional expectation approaches

$$E_{\mathfrak{F}} \quad CS\ M^* \quad ;A \quad jM^* \quad ;A \ \ 2\ AnA \ \ ;$$

which is strictly positive given steps 1 and 2.

Step 4. Next, we claim that in any optimal policy, for all ***k*** 2 K$^+$, __***CS***__ must equal the expected change in consumer surplus from the next-most-pro…table merger (i.e., from the merger with the second-highest bilateral pro…t change) ***M**** n ***k;a*** ***;***A , conditional on merger $M(k;\bar{a})$ being the most profitable merger in $\mathbb{M}\setminus\mathcal{A}$. Defining the expected change in consumer surplus from the next-most-profitable merger $M^*(\mathbb{M}\setminus M;\mathcal{A})$, conditional on merger $M(k;\bar{c})$ being the most profitable merger in $\mathbb{M}\setminus\mathcal{A}$, to be

$$E^{\mathcal{A}}(\bar{c}) \equiv E_{\mathfrak{F}}[\Delta CS(M^*(\mathbb{M}\setminus M;\mathcal{A})) \mid M = M(k;\bar{c}) \text{ and } M = M^*(\mathbb{M};\mathcal{A})] \quad (5)$$

$$= E_{\mathfrak{F}}[\Delta CS(M^*(\mathbb{M}\setminus M;\mathcal{A})) \mid M = M(k;\bar{c}) \text{ and } \Delta\Pi(M^*(\mathbb{M}\setminus M;\mathcal{A})) < \Delta\Pi(M)], \quad (6)$$

this means that

$$\underline{\Delta CS}(k) = E^{\mathcal{A}}(\bar{a}(k)). \quad (7)$$

In Figure 5 the possible locations of the next-most-profitable merger when the most profitable merger is $M_2 = (2;\bar{a}_2)$ are shown as a shaded set. The quantity $E_2^{\mathcal{A}}(\bar{a}_2)$ is the expectation of the change in consumer surplus for the merger that has the largest change in bilateral profit among mergers other than M_2, conditional on all of these other mergers lying in the shaded region of the figure.

To see that (7) must hold for all $k \in K^+$, suppose first that $\underline{\Delta CS}(k') > E_{k'}^{\mathcal{A}}(\bar{a}(k'))$ for some $k' \in K^+$ and consider the alternative approval set $\mathcal{A}\cup\mathcal{A}_\varepsilon$ where

$$\mathcal{A}_\varepsilon \equiv \{M \mid M = (k';\bar{c}_{k'}) \text{ with } \bar{c}_{k'} \in (\bar{a}_{k'}, \bar{a}_{k'} + \varepsilon)\}.$$

For any $\varepsilon > 0$, the change in expected consumer surplus from changing from $\mathcal{A}$ to $\mathcal{A}\cup\mathcal{A}_\varepsilon$ equals $\Pr(M^*(\mathbb{M};\mathcal{A}\cup\mathcal{A}_\varepsilon) \in \mathcal{A}_\varepsilon)$ times

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathbb{M};\mathcal{A}\cup\mathcal{A}_\varepsilon)) - \Delta CS(M^*(\mathbb{M};\mathcal{A})) \mid M^*(\mathbb{M};\mathcal{A}\cup\mathcal{A}_\varepsilon) \in \mathcal{A}_\varepsilon]. \quad (8)$$

This conditional expectation can be rewritten as

$$E_{\mathfrak{F}}[\Delta CS(M^*(\mathbb{M};\mathcal{A}\cup\mathcal{A}_\varepsilon)) - E_{k'}^{\mathcal{A}}(\bar{c}_{k'}) \mid M^*(\mathbb{M};\mathcal{A}\cup\mathcal{A}_\varepsilon) \in \mathcal{A}_\varepsilon], \quad (9)$$

where $\bar{c}_{k'}$ is the realized cost level in the bilateral profit-maximizing merger $M^*(\mathbb{M};\mathcal{A}\cup\mathcal{A}_\varepsilon)$, which is a merger of firms 0 and k' when the conditioning statement is satisfied. By continuity of $\Delta CS(k';\bar{c}_{k'})$ and $E^{\mathcal{A}}(\bar{c}_{k'})$ in $\bar{c}_{k'}$, there exists an $\bar{\varepsilon} > 0$ such that $\Delta CS(M_{k'}) > E^{\mathcal{A}}(\bar{c}_{k'})$ for all $M_{k'} \in \mathcal{A}_\varepsilon$ provided $\varepsilon \in (0;\bar{\varepsilon})$. For all such ε, the conditional expectation (9) is strictly positive so this change in the approval set would strictly increase expected consumer surplus. A similar argument applies if $\underline{\Delta CS}(k') < E_{k'}^{\mathcal{A}}(\bar{a}(k'))$.

Step 5. Next, we argue that for all $j < k$ such that $j, k \in$

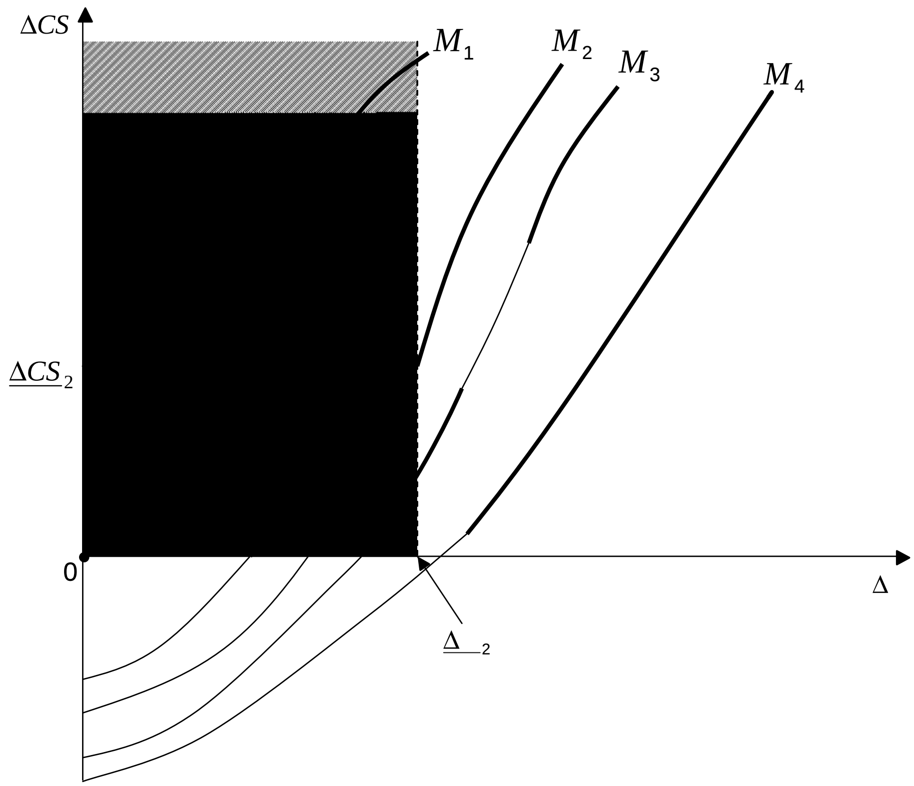
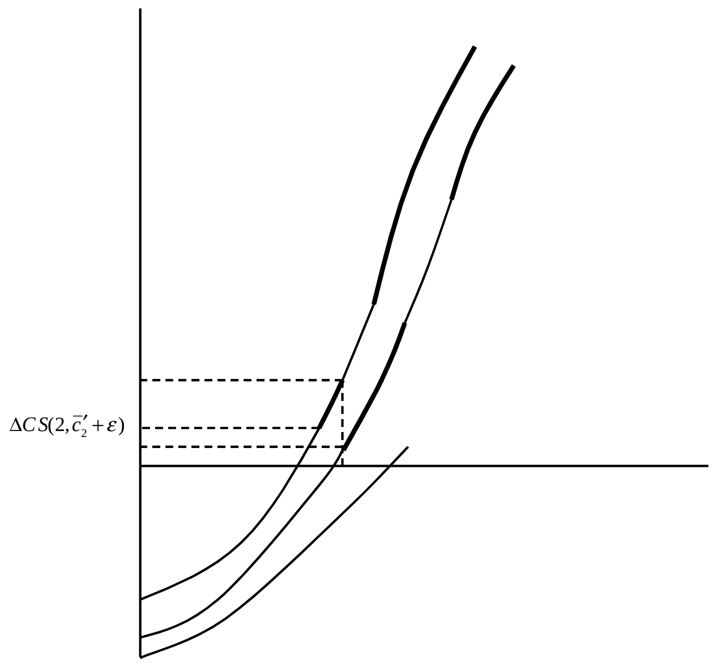


Figure 5: The optimal approval policy is such that the increase in consumer surplus induced by the worst allowable merger ***M*** is equal to the expected consumer surplus change from the next-most pro…table merger, conditional on the marginal merger being the most pro…table merger in the set of feasible and allowable mergers.

(a) Approval Set



where $\bar{c}$ is the realized cost level in the aggregate pro…t-maximizing merger M^* ;A [$\overline{A}$, which is a merger of …rms 0 and j when the conditioning statement is satis…ed. As ε ! , the expected change in (10) converges to

$$CS\ j;\bar{c}' \quad E^{\mathcal{A}}\ \bar{c}' \qquad CS\ j;\bar{c}' \quad E^{\mathcal{A}}_0\ \bar{a}\ _0$$
$$> \quad \underline{CS}\ _0 \quad E^{\mathcal{A}}_0\ \bar{a}\ _0$$
$$;$$

where the inequality follows from Corollary 1 since $j;\bar{c}'$ ___ $_0$.

Step 6. We next argue that $\underline{CS} < \underline{CS}$ for all $j;k$ 2 K$^+$ with $j < k$. Suppose otherwise; i.e., for some $j;h$ 2 K$^+$ with $h > j$ we have $\underline{CS}$ $\underline{CS}$. De…ne k h 2 K$^+$ $h > j$ and $\underline{CS}$ $\underline{CS}$. Figure 7 depicts such a situation where j and k .

By Step 4, we must have $E^{\mathcal{A}}\ \bar{a}$ $\underline{CS}$ $\underline{CS}$ $E^{\mathcal{A}}\ \bar{a}$. But recalling (6); $E^{\mathcal{A}}\ \bar{a}$ can be written as a weighted average of two conditional expectations:

$$E_{\mathfrak{F}} \quad CS\ M^* \ \ nM\ ;A\ \ jM \quad k;\bar{c}\ ,\ M \quad M^*\ ;A\ , \text{and} \quad M^*\ \ nM\ ;A \ < \_\_\_ \tag{11}$$

and

$$E_{\mathfrak{F}} \quad CS\ M^* \ \ nM\ ;A\ \ jM \quad k;\bar{c}\ ,\ M \quad M^*\ ;A\ , \text{and} \quad M^*\ \ nM\ ;A \ \ 2\ \_\_\_\ ;\_\_\_\ : \tag{12}$$

Expectation (11) conditions on the event that the next-most-pro…table merger other than $k;\bar{a}$ induces a bilateral pro…t change less than ___

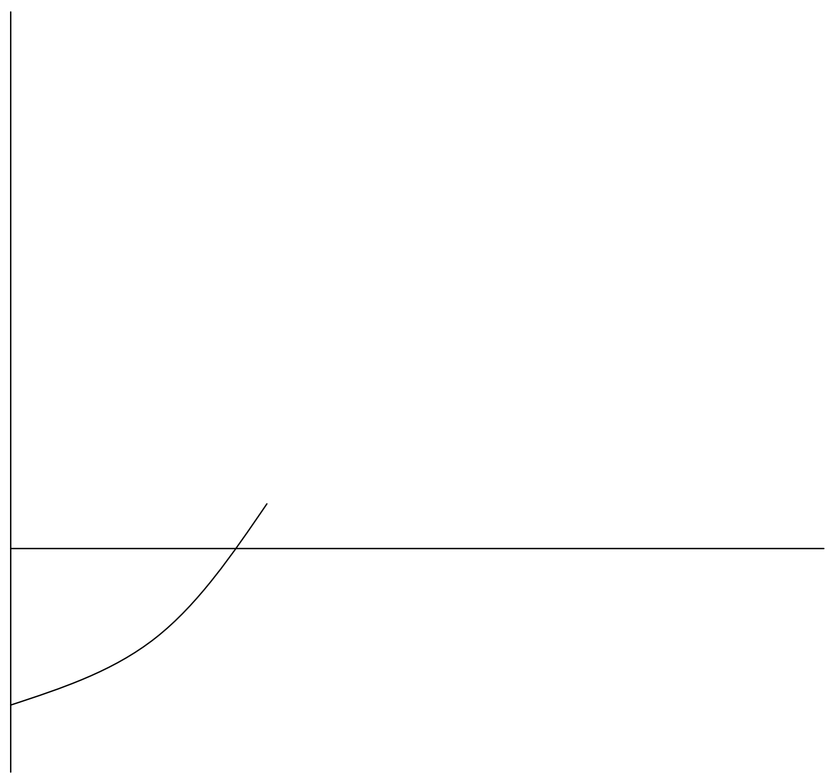


Figure 7: The optimal approval set is such that the consumer surplus increase induced by the worst allowable merger M , is less than that by the worst allowable larger merger M , $k > j$, i.e., $CS < CS$. In the figure, $CS_2 > CS_3$, which is a violation of that property.

is larger (in terms of the pre-merger size of the merger partner) than the one that would maximize consumer surplus. To compensate for this intrinsic bias in ...rms' proposal incentives, the antitrust authority should optimally adopt a higher minimum CS-standard the larger is the proposed merger.

Does the optimal policy have a cut-o¤ structure so that A $\boldsymbol{l; \bar{a}}$ *? The answer is no, as the following example illustrates. (For simplicity, the example considers the case where, contrary to the assumption of the model, one of the mergers has a ...nite support of post-merger marginal costs. But the same insight would obtain if we perturbed the example and assumed that the support is continuous with no atoms.)*

Suppose that there are two possible mergers, $\boldsymbol{M}_1$ *and* $\boldsymbol{M}_2$ *. The smaller merger,* $\boldsymbol{M}_1$*, is always feasible. Its post-merger marginal cost is either* $\bar{c}_1$ $\boldsymbol{l}$ *or* $\bar{c}_1$ $\boldsymbol{h}_1$*, where the probability on the latter is 0.9. The corresponding changes in consumer surplus and bilateral pro...t are given by* $\boldsymbol{CS}$ $\boldsymbol{;l;}$ $\boldsymbol{;l}$ $\boldsymbol{;}$ *and* $\boldsymbol{CS}$ $\boldsymbol{;h_1;}$ $\boldsymbol{;h_1}$ $\boldsymbol{;}$ *. The unconditional expected increase in consumer surplus from approving* $\boldsymbol{M}_1$ *is thus equal to* $\boldsymbol{:}$ *. The post-merger marginal cost of the larger merger,* $\boldsymbol{M}_2$*, has a continuous support* $\boldsymbol{l; h_2}$ *with no atoms, satisfying* $\boldsymbol{CS}$ $\boldsymbol{;h_2}$ $\boldsymbol{<}$ *and* $\boldsymbol{<}$ $\boldsymbol{CS}$ $\boldsymbol{;l}$ *. It is straightforward to verify that the optimal approval policy* A^* *is such that* A_1 f$\boldsymbol{l; h_1}$g *and* A_2 $\boldsymbol{l; c'_2}$ [$\boldsymbol{c''_2; \bar{a}_2}$ *, where* $\boldsymbol{c'_2}$ *and* $\boldsymbol{c''_2 > c'_2}$ *are implicitly de...ned by* $\boldsymbol{CS}$ $\boldsymbol{;c'_2}$ $\boldsymbol{:}$ *and* $\boldsymbol{CS}$ $\boldsymbol{;c''_2}$ *. This situation is illustrated in Figure 8. To see why the optimal approval policy for* $\boldsymbol{M}_2$ *does not have a cut-o¤ structure, note that for any post-merger marginal cost* $\bar{c}_2$ 2 $\boldsymbol{c'_2; c''_2}$ *, the induced change in consumer surplus is less than 5 (which is the induced change in consumer surplus of the best realization of* $\boldsymbol{M}_1$*). But, if approved, the ...rms would propose the larger merger even if the realized* $\boldsymbol{M}_1$ *is better for consumers as, for* $\bar{c}_2$ 2 $\boldsymbol{c'_2; c''_2}$ *,* $\boldsymbol{;\bar{c}_2}$ $\boldsymbol{>}$ $\boldsymbol{;l}$ *. The optimal policy corrects for this bias in ...rms' proposal policies by not approving* $\boldsymbol{M}_2$ *whenever* $\bar{c}_2$ 2 $\boldsymbol{c'_2; c''_2}$ *.*

In our analysis so far, we have focused on the case where the bargaining process between ...rms is given by the o¤er game, resulting in the proposal of the merger that maximizes the change in the bilateral pro...t of the merger partners in the realized set of feasible and approvable mergers. In this section, we explore two alternative bargaining processes. First, we consider the benchmark case of e¢ cient bargaining. Second, we consider the case where there is (e¢ cient) bargaining only between a subset of ...rms (including all of those ...rms that are involved in potential mergers). We show that, in both cases, the main result continues to hold: the optimal approval policy has the property that the minimum CS-standard is increasing in the size of the proposed merger.

Suppose the outcome of the bargaining processes is e¢ cient for the ...rms in the industry in the sense that it maximizes aggregate pro...t. That is, we assume that, from the realized set of feasible and approvable mergers, \ A, ...rms choose to propose merger

$$\boldsymbol{M}^* \quad \boldsymbol{;} \mathrm{A} \qquad {}_{k \in (\mathfrak{F} \cap \mathcal{A})} \qquad \boldsymbol{M} \quad \boldsymbol{;}$$

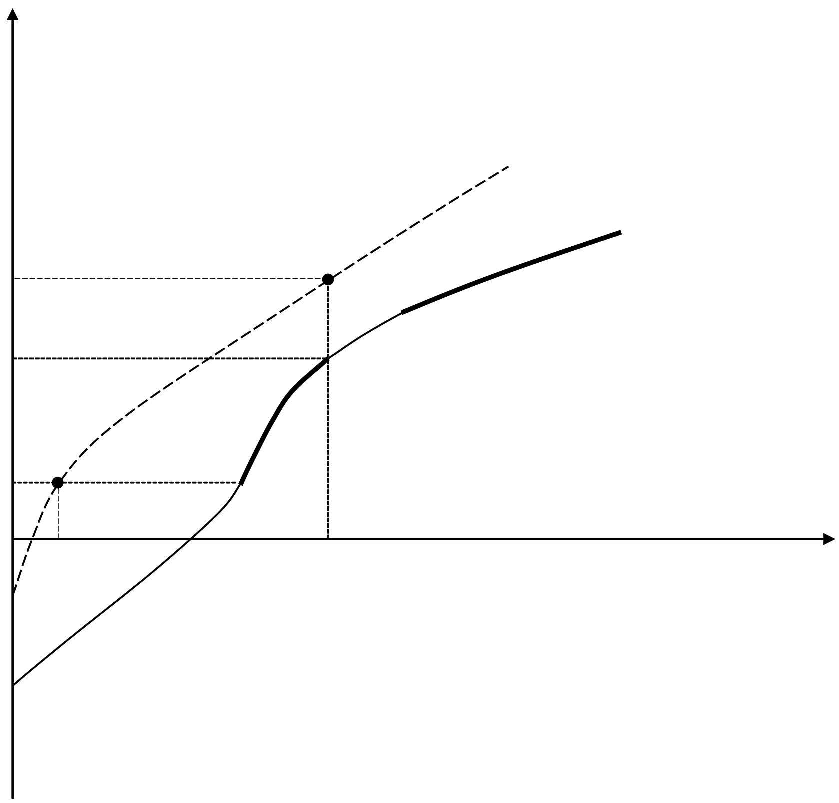


Figure 8: The ﬁgure depicts an example where the optimal approval set does not have a cutoﬀ structure.

where M now denotes the change in aggregate pro…t induced by merger M,

$$M \quad \in\mathcal{N}\backslash\{0\} \quad M \quad \in\mathcal{N} \quad ^0:$$

There are several bargaining processes which could lead to aggregate pro…t maximization:

1. Multilateral "Coasian bargaining" under complete information amongst all …rms would lead to an e¢ cient (aggregate-pro…t maximizing) outcome.

2. Suppose the auctioneer (here, …rm 0) conducts a "menu auction" in which each …rm i submits a nonnegative bid $b \; M$ for each merger $M \; 2 \quad \backslash \; A$ with k. Firm 0 then selects the merger that maximizes its pro…t, where the pro…t from selecting merger M is given by the sum of all bids for that merger, $\in\mathcal{N}\backslash\{0\} \; b \; M$, and the pro…t from selecting the null merger M_0 is $_0 \; M_0$. Bernheim and Whinston (1996) show that there is an e¢ cient equilibrium which, in this setting, implements the merger that maximizes aggregate pro…t.

3. Suppose the target (…rm 0) can commit to any sales mechanism. Jehiel, Moldovanu and Stacchetti (1996) show that one such optimal mechanism has the following structure: The target proposes to implement merger $M \; 2 \quad \backslash \; A$ and requires payment M $\underline{M}$ from each …rm i, where $\underline{M} \; 2 \quad \backslash \; A$ is the merger in set $\backslash \; A$ nM that minimizes …rm i's pro…t. If a …rm i does not accept participation in the mechanism when all other …rms do, then the principal commits to proposing merger $\underline{M}$ to the antitrust authority [who will then approve it since $\underline{M} \; 2 \quad \backslash \; A$].[2] Jehiel, Moldovanu and Stacchetti show that there exists an equilibrium in which all …rms participate in the mechanism. Given the set of feasible and approvable mergers, $\backslash$ A, the resulting outcome maximizes aggregate pro…t; that is, merger $M^* \quad ; A$ is proposed.[3]

We claim that Proposition 1 carries over to this bargaining process: the optimal approval policy A is such that the minimum CS-standard is zero for the smallest merger and increasing in the size of the proposed merger, $\underline{CS}_1 < \underline{CS}_2 < \quad < \underline{CS}_{\mathrm{b}}$, where K is the largest merger that is approved with positive probability. The key steps in the argument are the following. First, note that Lemma 1 states that a CS-neutral merger M, k, raises not only the bilateral pro…t of the merger partners but also aggregate pro…t, $M \; >$. Second, part (2) of Lemma 2 does not extend to the case of aggregate pro…t without imposing some condition. We therefore *assume* that a reduction in post-merger marginal cost increases aggregate pro…t if the merger is CS-nondecreasing, and then discuss when this condition does indeed hold true.

If merger M, k, is CS-nondecreasing [i.e., $\overline{c} \quad c \; Q^0$], then reducing its post-merger marginal cost $\overline{c}$

As we now show, this assumption must hold for merger M if whenever it is CS-nondecreasing we have $\bar{c}$ $\neq$ c; i.e., the merged firm has the lowest marginal cost. Since this would always be true were the firms in set N n f g to have identical initial marginal costs, it clearly holds provided their initial marginal costs are sufficiently close. To see why Assumption 3 holds in this case, note that summing up the post-merger first-order conditions for profit maximization yields

$$P\ Q \quad c\ q \quad Q^2 P'\ Q \quad H; \qquad (14)$$
$$\in \mathcal{N} \setminus \{0\}$$

where H $\in \mathcal{N}\setminus\{0\}$ s ^{2} is the post-merger industry Herfindahl Index. Assumption 1 ensures that the first term, $Q^2 P'\ Q$, is increasing in Q. By part (1) of Lemma 2, a reduction in post-merger marginal cost leads to a larger Q, so that a sufficient condition for the claim to hold is that reducing the merged firm's marginal cost induces an increase in H. But this is indeed the case if the merged firm has lower costs, and hence a larger market share, than any of its (unmerged) rivals, since then a further reduction in its marginal cost increases its share and lowers the shares of all of its rivals, increasing H (see Lemma 5 in the Appendix).

Third, the systematic misalignment of interests between firms and the antitrust authority, as stated in Lemma 3, is also present when bargaining is efficient:

Suppose two mergers, M and M, with $j < k$, induce the same non-negative change in consumer surplus, $CS\ M$ $CS\ M$. Then, the larger merger M induces a greater increase in aggregate profit: M $>$ M $>$.

Proof. From the discussion above, the post-merger aggregate profit is given by (14). As both mergers induce the same level of consumer surplus (and thus the same Q), the first term on the right-hand side of (14) is the same for both mergers. It thus suffices to show that the larger merger M induces a larger value of H than the smaller merger M.

Now, as both mergers induce the same Q, Assumption 1 implies that the output of any firm not involved in M or M is the same under both mergers. Hence,

$$s\ M \quad s\ M \quad s\ M \quad s\ M: \qquad (15)$$

Next, recall that a CS-nondecreasing merger increases the share of the merging firms and reduces the share of all nonmerging firms. Thus, we have $s\ M$ s $s_0 > s\ M$ and $s\ M$ s $s_0 > s\ M$. In addition, since total output is the same after both mergers and $c < c$, we also have $s\ M < s\ M$. By (15), this in turn implies that $s\ M$

In the CES model, the utility function of the representative consumer is given by

$$U \quad X^{p} \quad Z^{C};$$

where 2 ; and > are parameters, X is consumption of di¤erentiated good i, and Z is consumption of the numeraire. Utility maximization implies that the representative consumer spends a constant fraction = of his income Y on the N di¤erentiated goods (and the remainder on the numeraire). Using the normalization $Y=$, the resulting demand for di¤erentiated good i is

$$X \quad p^{-i-1}$$

From the indirect utility (17), it follows that consumer surplus is an increasing function of .
Finally, in the multinomial model, we have $a \quad p =$ and $a \quad p =$, so that pro…t from product i can be written as

$$;c \qquad a \qquad\qquad c \;—:$$

From the indirect utility (18), it follows that consumer surplus is an increasing function of .

In the Appendix, we show that the equilibrium pro…t functions of these three models share some important properties. Using this common structure, we show in the Appendix that if merger M is CS-neutral, then it raises the joint pro…t of the merging …rms as well as aggregate pro…t. Moreover, a reduction in post-merger marginal cost increases the merged …rm's pro…t and, provided pre-merger di¤erences between …rms are not too large, aggregate pro…t. Moreover, if any two mergers M and M , $k > j$, induce the same nonnegative change in consumer surplus, then the larger merger M induces a greater increase in aggregate pro…t than the smaller merger M . In sum, in the two di¤erentiated goods models, the merger curves have the same features in $CS;$ -space as in the Cournot model. Our main result, Proposition 1, therefore carries over as well.

In our baseline model, we have assumed that the antitrust authority seeks to maximize conmust involve synergies in that $\bar{c} < c$.[5] Hence, if M is W-nondecreasing, the merged …rm is the …rm with the lowest marginal cost post merger. Reducing the merged …rm's marginal cost $\bar{c}$ induces an increase in aggregate output Q, thereby raising $|Q^2 P' Q|$, and a further increase in the Her…ndahl index H. From equation (14), a lower level of post-merger marginal cost $\bar{c}$ thus results in a greater level of aggregate pro…t . By continuity of consumer and producer surplus in marginal costs, it follows that $W M$ implies that $\bar{c} <$ f$c_0; c$ g; and that is decreasing in $\bar{c}$, if pre-merger marginal cost di¤erences are su¢ ciently small.

We also impose the following analog of Assumption 2:

For all k 2 K, the probability that the merger M is W-increasing is positive but less than one: $W\ k; h < < W\ k; l$.

Assumption 3' allows us to obtain a slightly stronger version of Lemma 4:

Suppose two W-nondecreasing mergers, M and M, with $k > j$, induce the same change in consumer surplus, $CS\ M$ $CS\ M$. Then the larger merger M induces a greater increase in aggregate pro…t: $M >$ $M >$.

Proof. The proof proceeds exactly as that of Lemma 4, except that the inequalities $s\ M >$ $s\ M$ and $s\ M > s\ M$ in equation (16) now hold since any W-nondecreasing merger involves synergies, $\bar{c} < c$ and $\bar{c}$

ΔCS

M_1

M_2

M

$\Delta\Pi$

Figure 9: The merger curves in ; ***CS*** -space. The downward-sloping lines are the iso-welfare curves.

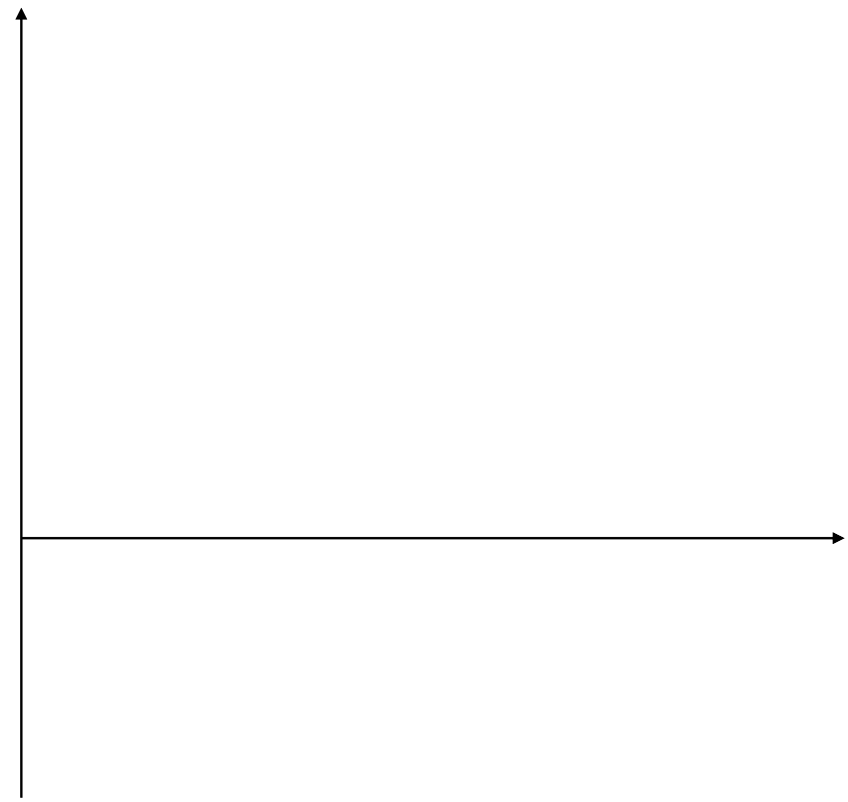


Figure 10: The merger curves in $;\ \boldsymbol{W}$ -space.

curve is upward-sloping in the positive orthant (except possibly for the curve corresponding to $\boldsymbol{M}_1$). Finally, in the positive orthant, the curve of a larger merger lies everywhere to the right of that of a smaller merger.

Let $\underline{\boldsymbol{W}}$ $\boldsymbol{W}\ \boldsymbol{k;\bar{a}}$ denote the welfare level of the "marginal merger," i.e., the lowest welfare level in any allowable merger between …rms 0 and $\boldsymbol{k}$. The following proposition shows that our main result (Proposition 1) extends to the case where the antitrust authority maximizes an arbitrary convex combination of consumer surplus and aggregate surplus:

Any optimal approval policy $\boldsymbol{A}$ *approves the smallest merger if and only if it is W-nondecreasing, and satis…es* $\underline{\boldsymbol{W}}_1 < \underline{\boldsymbol{W}} < \underline{\boldsymbol{W}}$ *for all* $\boldsymbol{j;k}$ 2 K^+, $< \boldsymbol{j} < \boldsymbol{k}$*, where* K^+ K *is the set of mergers that is approved with positive probability. Moreover, if* $\boldsymbol{j}$ 2̸ K^+ *and* $\boldsymbol{k}$ 2 K^+, $\boldsymbol{j} < \boldsymbol{k}$*, then* $\boldsymbol{W}\ \boldsymbol{j;l} < \underline{\boldsymbol{W}}$ *. That is, the lowest level of welfare change that is acceptable to the antitrust authority equals zero for the smallest merger*

M_1, is strictly positive for every other merger M with $k >$, and is monotonically increasing in the size of the merger.

Proof. The proof proceeds in seven steps. Steps 1 through 6 are as in the proof of Proposition 1 but with the welfare criterion replacing the consumer surplus criterion. Step 7 does not carry over as we cannot guarantee that $W\ k;l > W\ k\ ;l$. But the same type of argument can be used to show that if $j \in K^+$ and $k \in K^+$, $j < k$, then $W\ j;l < \underline{W}$. □

So far, we have assumed that firms have constant returns, implying that all merger-specific efficiencies involve marginal cost savings. We now consider the case where firms have to incur a fixed cost, a part of which may be saved by merging, and show that our main result carries over to this setting.

Let f denote the fixed cost of firm i.[6] A feasible merger M is described by $M\ \ k;\bar{c}\ ;\bar{f}$, and no mass points. Assume also that when merger M is proposed, the antitrust authority can observe and separately (and condition the approval set on both components separately).[7] Using the same arguments as above, it is straightforward to show that the optimal approval set is constant in . For notational simplicity, we will from now on assume that there is no common component, , so that $\overline{f}$ f_0 f .

Graphically, the possibility of …xed cost savings implies that the merger curves in $; \ CS$ -space are "broad bands" rather than curves, with each point in the band of merger M corresponding to a di¤erent realization of $\overline{c}\ ;\overline{f}$, and with the horizontal width of the band given by f f at any $CS\ M$. We assume that f f is su¢ ciently small so that the bands of the di¤erent mergers are non-overlapping in the positive orthant. From Lemma 3 it follows that if any two mergers M and M , $j < k$, induce the same nonnegative change in consumer surplus, $CS\ M$ $CS\ M$, then the larger merger is more pro…table, independently of the realized …xed cost savings. As …xed cost savings are nonnegative by assumption, the conclusion of Lemma 1 – that a CS-neutral merger is pro…table – continues to hold.

Our main result, Proposition 1, carries over to this setting:

In the model with …xed cost savings, any optimal approval policy A *approves the smallest merger if and only if it is CS-nondecreasing, approves only mergers* k 2 K^{+} f $; :::; K$g *with positive probability* (K *may equal* K) *and satis…es* $\underline{CS}_1 < \underline{CS}_2 < ::: < \underline{CS}_b$ *for all* k K.

Proof. Steps 1-3 proceed along the same lines as those in the proof of Proposition 1.

Step 4. As in the absence of …xed cost savings, any optimal policy has the property that, for all k 2 K^{+}, $\underline{CS}$ $\overline{f}$ is equal to the expected change in consumer surplus from the next-most pro…table merger M^{*} n $k; \overline{c}\ ; \overline{f}$; A , conditional on merger M $k; \overline{c}\ ; \overline{f}$ maximizing the change in the merging …rms' bilateral pro…t in \ A. That is,

$$\underline{CS}\ \overline{f}\qquad E^{\mathcal{A}}\ \overline{a}\ \ \overline{f}\ \ ; \overline{f}$$

$$E_{\mathfrak{F}}\ \ CS\ M^{*}\ \ \text{n}M\ ;\text{A}\ \ \text{j}M\qquad k; \overline{a}\ \ \overline{f}\ \ ; \overline{f}\ \ \text{ and }\ \ M^{*}\ \ \text{n}M\ ;\text{A}\qquad M\ \ :$$

To see that this equation must hold for all k 2 K^{+}, suppose …rst that $\underline{CS}_{\ 0}\ \overline{f}'_{\ 0} > E^{\mathcal{A}}_{0}\ \overline{a}_{\ 0}\ \overline{f}'_{\ 0}\ ; \overline{f}'_{\ 0}$ for some …rm k' 2 K^{+} and …xed cost realization $\overline{f}'_{\ 0}$, and consider the alternative approval set A [A $_0$, where

$$\text{A}_{\ 0}\qquad M\quad M\qquad k'; \overline{c}_{\ 0}; \overline{f}_{\ 0}\ \text{ with } \overline{c}_{\ 0}\ 2\ \ \overline{a}_{\ 0}\ \overline{f}'_{\ 0}\ ; \overline{a}_{\ 0}\ \overline{f}'_{\ 0}\qquad "\ \text{ and } \overline{f}_{\ 0}\ 2\ \ \overline{f}'_{\ 0}\ \ "; \overline{f}'_{\ 0}\qquad "\ \ :$$

Using the same type of argument as in the proof of Proposition 1, it is straightforward to show that, for $" >$ small enough, the change in expected consumer surplus from changing the approval set from A to A [A $_0$ is strictly positive. A similar logic can be used to show that we cannot have $\underline{CS}_{\ 0}\ \overline{f}'_{\ 0} < E^{\mathcal{A}}_{0}\ \overline{a}_{\ 0}\ \overline{f}'_{\ 0}\ ; \overline{f}'_{\ 0}$.

Step 5. Let M fM $CS\ M$ $\underline{CS}$ and M 2 A g denote the set of marginal mergers M that induce a change in consumer surplus of $\underline{CS}$, and let M 2 M denote the most pro…table amongst these mergers, i.e., M M' for all M' 2 M . An

n [7] That is, a feasible$_0$merger M_k is described by M_k $k; \overline{c}_k;\ ; \overline{f}_k$, and the approval set by $\mathcal{A}$ $\equiv$ M_k $\overline{c}_k;\ ; \overline{f}_k$ $\in \mathcal{A}_k$ $\cup M_0$, where $\mathcal{A}_k \subseteq$ $l; h_k$ $\times$ $^l;$ ^{h} $\times$ $\overline{f}^l_k; \overline{f}^h_k$.

optimal approval set must have the property that, for all $j < k$ such that $j;k$ 2 K^+, we have M ___ . The argument proceeds in two parts.

Part (i). For all $j < k$ such that $j;k$ 2 K^+, we must have ___ and $\underline{\quad CS}$ $\underline{\quad CS}$ g. By Step 4, we must have $E^{\mathcal{A}}$ a $\bar{f}$; $\bar{f}$ $\underline{\quad CS}$ $\underline{\quad CS}$ $E^{\mathcal{A}}$ a $\bar{f}$; $\bar{f}$. Now, $E^{\mathcal{A}}$ a $\bar{f}$; $\bar{f}$ can be written as a weighted average of two conditional expectations:

$$E_{\mathfrak{F}} \quad CS\ M^* \quad \text{n}M\ ;A \quad \text{j}M \quad k; c \quad ; \bar{f} \quad , M \quad M^* \quad ;A \text{ , and } \quad M^* \quad \text{n}M\ ;A \quad < \underline{\quad} \tag{21}$$

and

$$E_{\mathfrak{F}} \quad CS\ M^* \quad \text{n}M\ ;A \quad \text{j}M \quad k; c \quad ; \bar{f} \quad , M \quad M^* \quad ;A \text{ ,}$$
$$\text{and} \quad M^* \quad \text{n}M\ ;A \quad 2 \ \underline{\quad} \ ; \quad M \quad : \tag{22}$$

Using the same arguments as in the proof of Proposition 1, we obtain that the term in (21) is equal to $CS\ M$, which weakly exceeds $\underline{\quad CS}$ by de…nition, and that the second term strictly exceeds $E^{\mathcal{A}}$ a $\bar{f}$; $\bar{f}$ $\underline{\quad CS}$, which leads to a contradiction.

Step;

Proof. Without loss of generality, take r and de…ne s' s'' for $n >$. Observe that for all $n >$ and $>$ for some $n >$. De…ne as well the vectors s s'_1 $_{=2}$ $; s'_2$ $_2; :::; s'$ $; s'_{+1}; :::; s'$ for $n >$ and s^1 s'. Note that s s''. Then

$$H\ s'' \quad H\ s' \quad \sum_{=1}^{-1} H\ s^{+1} \quad H\ s \; :$$

Now letting $\bar{s}^1_1$ s'_1 and $\bar{s}_1$ s'_1 $_{=2}$ s'_1 for all $n >$, each term in this sum is nonnegative,

$$H\ s^{+1} \quad H\ s \quad \bar{s}_1 \quad _{+1}{}^2 \quad s' \quad _{+1}{}^2 \quad \bar{s}_1{}^2 \quad s'{}^2$$
$$_{+1}\ \bar{s}_1 \quad s' \quad _{+1}{}^2 \quad ;$$

and strictly positive if $_{+1} >$. Since $_{+1} >$ for some n , the result follows. □

Suppose an unmerged …rm i's pro…t can be written as

$$; c \quad ;$$

where is …rm i's strategic variable, c the …rm's constant marginal cost, and an aggregator summarizing the "aggregate outcome." The …rm's cumulative best response, $r\ c$ $_i$ $; c$ $_{\neq}$ is assumed to be decreasing in its marginal cost c . Similarly, a merged …rm k's pro…t is given by $; \bar{c}$, and its cumulative best response, $\bar{r}\ \bar{c}$ $_k$ $; \bar{c}$ $_{\neq 0}$, is decreasing in $\bar{c}$. Consumer surplus, denoted V , is an increasing function of the aggregator and does not depend on the composition of the aggregator.

Suppose that there exists a unique stable equilibrium. Let M denote …rm i's equilibrium action under market structure M , and M M . Further, suppose that …rm i's equilibrium pro…t can be written as

$$g \quad M \quad M \quad _i \quad ; c \quad M \quad \text{if …rm } i \text{ is unmerged;}$$
$$g \quad M \quad M \quad _i \quad ; \bar{}$$

as in the main text. The pro…t maximization problem of a single-plant …rm i with marginal cost c can be written as

$$P \qquad c \quad :$$
$$_{i} \qquad \neq$$

From the …rst-order condition of pro…t maximization, $P \quad c \quad P'$, we can write the equilibrium pro…t under merger M as

$$g \quad M \quad M \qquad M \; ^2 P' \quad M \; :$$

The pro…t maximization problem of a merged …rm k with marginal cost $\bar{c}$ (and two plants) can be written as

$$P \qquad \bar{c} \quad :$$
$$_{k} \qquad \neq 0$$

From the …rst-order condition of pro…t maximization, $P \quad \bar{c} \quad P'$, so that we can write the merged …rm's equilibrium pro…t under merger M as

$$g$$

we obtain the merged ...rm's equilibrium pro...t under merger M:

$$g \quad M \quad M \quad \frac{M}{M} \quad ^{-1} :$$

It can easily be veri...ed that our assumptions hold in the CES model. In particular, the equilibrium pro...t function g has all of the required properties (it takes the value of zero if its ...rst argument is zero and is increasing and convex in its ...rst argument). Consider a reduction in post-merger marginal cost $\bar{c}$. For a given level of , the merged ...rm wants to choose a higher value of , and every other ...rm i wants to choose a higher level of , as can be seen from the ...rst-order conditions. In any stable equilibrium, the reduction in $\bar{c}$ thus induces a higher value of . Rewrite the ...rst-order condition of an unmerged ...rm i:

$$c \quad ^{1 \ i} \quad —$$

can be rewritten to obtain …rm k's equilibrium pro…t under merger M :

$$g \quad M \quad M \qquad \frac{M}{\quad} \qquad ^{-1} :$$

It can easily be veri…ed that our assumptions hold in the CES model. In particular, the equilibrium pro…t function g has all of the required properties (it takes the value of zero if its …rst argument is zero and is increasing and convex in its …rst argument). Consider a reduction in post-merger marginal cost $\bar{c}$. For a given level of , the merged …rm wants to choose a higher value of , and every other …rm i wants to choose a higher level of , as can be seen from the …rst-order conditions. In any stable equilibrium, the reduction in $\bar{c}$ thus induces a higher value of . Rewrite the …rst-order condition of an unmerged …rm i as

$$\frac{a \qquad c}{a \qquad c} \quad \frac{}{} :$$

It can easily be checked that the l.h.s. of this equation is decreasing in . As the induced increase in induces an increase in (i.e., prices are strategic complements), the ratio $=$ must fall as otherwise the l.h.s. of the equation would decrease. But as

$$\frac{}{\quad} \qquad \frac{\neq 0}{\quad} \qquad ;$$

it follows that the same ratio for the merged …rm, $=$, must increase. From the expression for the equilibrium pro…ts, we thus obtain that the pro…t of the merged …rm, g M $) \ ) \ =$

As a reduction in post-merger marginal cost increases the merged …rm's pro…t, any CS-nondecreasing merger is pro…table. Let us assume that a reduction in post-merger marginal cost (of a CS-nondecreasing merger) also increases aggregate pro…t. In the Cournot model, we have seen that this assumption holds if pre-merger cost di¤erences are not too large. This observation also holds in the CES and multinomial logit models:

(CES) In the CES model, if pre-merger marginal cost di¤erences are not too large so that for any CS-nondecreasing merger M we have M $>$ M, then the reduction in post-merger marginal cost $\bar{c}$ increases aggregate pro…t. To see this, note that from the argument given in our exposition of the CES model above, the reduction in $\bar{c}$ induces a change from $=$ to $=$, i 6 $;k$, $>$, and from $=$ to $=$ $\neq 0$. It thus su¢ ces to show that the joint pro…t of the merged …rm k and any other …rm i,

$$h \quad \frac{r}{r} \quad \frac{r}{r} \; ;$$

where , r and r $=$, is increasing in . But this holds as we have

$$h' \quad \frac{}{r^2} \quad \frac{}{r^2} > \; ;$$

where the inequality follows as $r > r$ by assumption.

(Multinomial Logit) In the multinomial logit model, if pre-merger marginal cost di¤erences are not too large so that for any CS-nondecreasing merger M we have M $>$ M, then the reduction in post-merger marginal cost $\bar{c}$ increases aggregate pro…t. To see this, note that from the argument given in our exposition of the multinomial logit model above, the reduction in $\bar{c}$ induces a change from $=$ to $=$, i 6 $;k$, $>$, and from $=$ to $=$ $\neq 0$. It thus su¢ ces to show that the joint pro…t of the merged …rm k and any other …rm i,

$$h \quad \frac{r}{r} \quad \frac{r}{r} \; ;$$

where , r and r $=$, is increasing in . But this holds as we have

$$h' \quad \frac{}{r^2} \quad \frac{}{r^2} > \; ;$$

where the inequality follows as $r > r$ by assumption.

We are now in the position to extend Lemma 4 to this larger class of models:

Suppose mergers M *and* M , $k > j$, *induce the same nonnegative change in consumer surplus so that* M M ^{0}. *Then, the larger merger* M *induces a greater increase in aggregate pro…t than the smaller merger* M .

Proof. As the aggregate outcome is the same under both mergers, the pro…t of each …rm not participating in either merger is also the same under both mergers. We thus only need to show that

$$g \quad M$$

$$g$$

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