The First-Order Approach to Merger Analysis

Sonia Ja e[z]   E. Glen Weyl[x]

October 16, 2011

Abstract

Using only information local to the pre-merger equilibrium, we derive approximations of the expected changes in prices and welfare generated by a merger. We extend the pricing pressure approach of recent work to allow for non-Bertrand conduct, ad-

The logic of FOAM is intuitive: when companies A and B merge, company A (and similarly, B) has an additional opportunity cost of selling its products: it now internalizes the loss of pro table sales by company B that occurs when company A lowers its price. The per-unit magnitude of this opportunity cost is the value of the sales diverted from B for each (marginal) sale by A: the fraction of sales gained by A that are cannibalized from B (typically called the *diversion ratio*), multiplied by the pro t-value of those sales (*rm B's mark-up*). This quantity, typically called \Upward Pricing Pressure" (UPP), is discussed explicitly in the new guidelines as being critical to determining merger e ects; Werden (1996) and Farrell and Shapiro (2010a) advocate using thresholds for UPP to determine merger approval.[1]

However, some signi cant objections have been raised against the use of FOAM, in its current form, for evaluating mergers:

1. Coate and Simons (2009) object to its near-universal assumption of Nash-in-prices (Bertrand) competition and its reliance, in some settings, on constant marginal costs.

2. Schmalensee (2009) and Hausman et al. (2010) are skeptical of its assumption of default e ciencies and argue that providing only a directional indication of price e ects is insu cient.

3. Carlton (2010) emphasizes the di culty of applying the FOAM approach to mergers between multi-product rms.

While many of these critiques apply to one or all available alternative approaches, there is clearly room for improvement; this paper attempts to address these issues. We consider the most general oligopoly model we are aware of, in which rms have a single strategic variable per product, encompassing Bertrand, Cournot and most supply function equilibrium or conjectural variations models. From this we derive a generalized version of FOAM that -33emphasizesemphasizes).

product firms 1 and 2 merge, a first entry of the form

$$g_1 = \underbrace{\overbrace{\mathcal{D}_{12}}^{\text{Conjectured diversion ratio}}\overbrace{m_2}^{\text{Mark-up}}}_{\text{Generalized UPP}} \quad Q_1 \underbrace{\left(\overbrace{\frac{1}{\frac{d^M Q_1}{dP_1}}}^{\text{Post-merger (inverse) derivative of demand}} \quad \overbrace{\frac{1}{\frac{dQ_1}{dP_1}}}^{\text{Pre-merger}} \right)}_{\text{End of (accommodating) reactions}} \qquad (2)$$

and an analogous second entry.[2] The first term in equation (2) generalizes the basic Bertrand UPP logic by replacing the Bertrand diversion ratio, D_{12} with the *conjectured diversion ratio* $\mathcal{D}_{12}$. This is the diversion ratio from good 1 to good 2 (the fraction of a unit of good 2 that goes unsold when one more unit of good 1 is sold) when the impetus for the change in sales is a reduction in the price of good 1 *holding fixed the price of good 2* but *allowing all other prices to adjust as they are conjectured to* by the merged firm.[3] The price of good 2 is now held fixed because it has become, as a result of the merger, one of the quantities over which the merged firm optimizes. The second term in (2) is the quantity of good 1 multiplied by the change in the inverse of the slope of demand induced by the merger: now that the firms are merged, firm 1 no longer anticipates a reaction from firm 2 and thus expects the elasticity of its own demand to be higher (assuming accommodation pre-merger).

Anticipated accommodating reactions will have two effects. First, they will increase the (conjectured) diversion ratio, as they both reduce the number of sales lost by firm 1 and increase those gained by firm 2, whose price is held fixed. Second, they will increase the end of accommodating reactions (EAR) term as the larger are such reactions the more impact their end has on the elasticity of demand. Which of these effects dominates will depend on whether anticipated accommodation between the merging firms and other firms in the industry (first effect) or accommodation between the merging partners (second effect) is stronger. Thus the size of GePP may not differ as much across alternative conduct assumptions as it might at first appear.[4] Thus GePP under assumptions (such as consistent conjectures) that make identification easier can approximate GePP under other (possibly more realistic) assumptions.

The second term in equation (1), , is the *merger pass-through* matrix, the rate at which the changes in opportunity cost, the GePP, created by the merger are passed through to changes in prices. As we show in Section III, this quantity, which is a function of local second-order properties of the (conjectured) demand and cost system, converts GePP into a quantitative approximation of the price effects of the merger. In Section IV we argue that in many relevant cases merger pass-through is close to both pre-merger and post-merger similar approach may be used to estimate social surplus impacts. Furthermore, this broad approach allows for the incorporation of impacts of mergers on consumer welfare not directly mediated by prices, such as changes in network size or product quality.

Section VI discusses an extension of our formula to allow marginal costs e ciencies and thus the calculation of \compensating marginal cost reductions" (Werden, 1996), as well exploring some salient special cases. Section VII discusses the practical implications of our work, including various assumptions that greatly simplify the calculations our formula requires, the comparison of our approach to MS and the stage of merger analysis at which we see our tools applying. At a theoretical level, our approach shows how changes discontinuous in one space (viz. market structure) but local in another (viz. pricing incentives) can be estimated by standard comparative statics techniques, as we emphasize in Subsection VI.D and our conclusion in Section VIII. A companion policy piece (Ja e and Weyl, 2011) proposes a few potential reforms to the merger guidelines based on our analysis.

I Background

During the 1970's the \Chicago School" of law and economics, culminating in Posner (1976), played a leading role in the growing importance of formal economics in antitrust analysis. The 1982 U.S. Merger Guidelines (United States Department of Justice and Federal Trade Commission, 1982) re ected this growing in uence in its move towards more detailed quantitative measures in the delineation of, and measurement of concentration within, antitrust product markets. These standards began with techniques based on market de nition (MD) and Her ndahl (1950)-Hirschman (1945) Index (HHI) calculations; they were based on Stigler (1964)'s construction of a model in which the likelihood of collusion is mediated by HHI. However, emphasis during the late 1970's and 1980's on the di erentiated nature of most product markets led to increasing concern (Werden, 1982) with the unilateral (non-cooperative) effects of mergers.[5] Farrell and Shapiro (1990) challenged the relationship between MD and the unilateral harms from mergers in the basic undi erentiated Cournot models.[6] Thus, many economists have argued for approaches to merger analysis based more explicitly on di erentiated product models.[7]

To help supply this need, Werden and Froeb (1994) proposed a logit demand system, which made merger simulation (MS) techniques practical for policy analysis. During the 1990's merger simulation achieved widespread success in academic circles, exploiting the advances in techniques for demand estimation pioneered by Berry et al. (1995), and culminating in the seminal MS analysis of Nevo (2000). However, Shapiro (1996) and Crooke et al. (1999) argued that the e ects of mergers predicted by simulations could di er by an

[5] 1990

order of magnitude or more based on properties of the curvature of demand not typically measured empirically.

To address this concern, Werden (1996) pioneered FOAM by arguing that the \compensating marginal cost reductions" necessary to o set the anticompetitive e ects of a merger could be calculated from rst-order properties of the demand system.[8] In particular, such e ciencies would have to o set the change in rst-order conditions created by the new opportunity cost of a sale due to the diversion from a product of a merger partner. This approach is computationally simple and transparent. Additionally, Shapiro (1996) observed that, regardless of functional form, merger e ects appeared to be increasing in this \value of diverted sales" that has come be known as \Upward Pricing Pressure" (UPP). Building on this work, antitrust o cials in the United Kingdom, led by Peter Davis and Chris Walters, began to use UPP to evaluate mergers (Walters, 2007 nor matrices.

A The general model

Consider a market with N rms denoted $i = (1, \ldots, n)$. Firm i produces m_i goods, and chooses a strategy vector $_i = (_{i1}, _{i2}, \ldots, _{im_i})$ from a m_i-dimensional strategy space. Thus { following Werden and Froeb (2008) { w written as:

$$f_i(\quad)\qquad \underbrace{\left(\frac{dQ_i}{d\ _i}\right)^{T}{}^{1}\left(\frac{dP_i}{d\ _i}\right)^{T} Q_i}_{\text{Generalized inverse hazard rate/Cournot distortion}}\qquad \underbrace{(P_i\quad \mathrm{mc}_i)}_{\text{Mark-up}} = 0;$$

Thus GeSP is the change in the first-order condition at the pre-merger strategies. It *holds fixed the firms' strategy space and conjectures about other firms' reactions*, thus capturing only the unilateral effects of a merger. The value of GeSP is given in the following proposition.

Proposition 1. *The GeSP on firm i's strategy generated by a merger between firms i and j is*

$$g_i(\sigma) = D_{ij}(P_j - mc_j)\left(\left(\frac{d^M Q_i}{d\sigma_i}\right)^{-1}\right)^T \left(\frac{d^M P_j}{d\sigma_i}\right)^T Q_j - \Delta\left(\left(\frac{dQ_i}{d\sigma_i}\right)^{-1 T}\left(\frac{dP_i}{d\sigma_i}\right)^T\right) Q_i. \qquad (3)$$

Here $\Delta(\cdot)$ denotes the change from pre- to post-merger value of its argument; the change is due to the merger partner's strategy no longer reacting.

Proof. See Appendix A.

The first and second terms of equation (3) are the changes in firm j's profits induced by a sale by firm i (caused by changing firm i's strategy). Post-merger firm i takes into account the effect of a change in it's strategies on the quantities (first term) and the prices (second term) of its merging partner's products. The last term is the change in firm i's marginal profit due to the end of accommodating reactions: once the firms have merged, the firm no longer anticipates an accommodating reaction from its merger partner.

B Prices as Strategies

In the previous two subsections, we have taken the firms' strategies and conjectures as given exogenously. However, if firms are using a strategy other than prices, then we can still think of the two merging firms as setting prices as long as the merging firms' strategies generate unique prices – no two strategy combinations generate the same set of prices. This, of course, requires that the map from strategies to prices be invertible.[13] Assuming this is true, we can always re-conceptualize the firm's problem as a choice of prices. A firm's conjectures as well as other firms' non-price choosing behavior can be viewed as jointly forming a conjecture on how other firms will adjust price. For example, if firms are actually choosing quantities, we can think of them as choosing prices and expecting the other firms to adjust their prices so as to keep their quantities fixed.[14] The advantage of this approach is that it has a clearer concordance with UPP and the quantitative changes in price that impact welfare. In this subsection we pursue this dual strategy.

If strategies are prices then the second term on the right hand side of equation (3) vanishes because firm i's prices do not change firm j's prices. GeSP simplifies to *Generalized Pricing*

[13] A standard condition to guarantee this is that $\sigma \in \mathbb{R}^{\sum_i m_i}$ and $\frac{\partial P}{\partial \sigma}$ is either globally a P-matrix (a matrix will all positive principal minors, see Hicks (1939)) or globally the negative of a P-matrix. While this may seem a strong condition, it is trivially satisfied in many contexts; for example, if the equilibrium is Cournot (Nash-in-quantities) and consumers have quasi-linear utility then this follows directly from the fact that the Slutsky conditions imply that the Slutsky matrix $\frac{\partial Q}{\partial P}$ and thus its inverse $\frac{\partial P}{\partial Q}$ is negative definite globally, as all negative definite matrices are the negative of P-matrices. Any other sufficient condition for invertibility would be equally suitable.

[14] See the Nash-in-quantities section of VI.C for a fleshing out of this example.

Pressure (GePP):

$$g_i(P) = \bar{D}_{ij}(P_j \quad mc_j) \qquad \left(\frac{dQ_i}{dP_i}^{1}\right)^{T} Q_i.$$ [15]

Here, $\bar{D}_{ij} \quad D^P_{ij}$ is the diversion matrix *holding fixed the price of the merger partner* and allowing *all other firms' prices to adjust as they are expected to in equilibrium*. This diversion ratio is the general conjectures and matrix equivalent of the commonly used ratio of the derivatives of demand.

III Price Changes

As we show in Appendix B, the ith entry of the error vector in equation (4) takes the form "

$$E_i = \frac{1}{2} \sum_j$$

and Shapiro (2010a) argued informally that because UPP is essentially the opportunity cost of sales created by the merger, multiplying it by the *pre-merger* pass-through rates should approximate merger effects. Farrell and Shapiro (2010b) and Kominers and Shapiro (2010) prove in the symmetric case that bounds on pre-merger pass-through, in conjunction with those on UPP, over the range between pre- and post-merger prices can be used to establish bounds on merger effects. However, it is not clear whether pass-through or demand curvature is the crucial quantity since they use a constant marginal cost framework under which the two are equivalent. In the following section we reconcile this apparent conflict between pre- and post-merger pass-through rates as the crucial quantities, and resolve the ambiguity between pass-through and demand curvature.

A Pre-merger, post-merger and merger pass-through

Marginal costs (and thus Marshallian specific taxes) enter quasi-linearly into the expression for f_i for an individual firm i. That is $f_i(P) = \tilde{f}_i(P) + \text{mc}_i(P)$ and thus if we were to impose on the firms a vector of Marshallian specific (quantity) taxes t, the post-tax (but pre-merger) equilibrium would be characterized by

$$f(P) + t = 0;$$

so that by the implicit function theorem

$$\frac{\partial P}{\partial t}\frac{\partial f}{\partial P} = -I:$$

The *pre-merger pass-through matrix* is

$$\frac{\partial P}{\partial t} = -\left(\frac{\partial f}{\partial P}\right)^{-1}: \tag{5}$$

After the merger between firm i and firm j takes place, the marginal cost of producing good i enters quasi-linearly, with a coefficient of 1, into h_i, but also enters h_j quasi-linearly with a coefficient of $-D_{ji}$. This follows directly from the fact that following the merger, the GePP also enters h_j and includes the mark-up on good i which depends (negatively) on the specific tax applied to this good. Thus if we let

$$K = \begin{pmatrix} 1 & -D_{ij} \\ -D_{ji} & 1 \end{pmatrix};$$

then the post-merger and post-tax equilibrium is characterized by

$$h(P) = -Kt$$

and thus the *post-merger pass-through matrix* is[19]

$$\frac{\partial \overrightarrow{P}}{\partial t} = -\left(\frac{\partial h}{\partial P}\right)^{-1} K: \tag{6}$$

[19] The term with $\frac{\partial K}{\partial P} t$ drops out because the tax is zero to begin with.

Our result from the previous section is that $P^M \quad P^0 \quad \frac{\partial h(P)}{\partial P} \;^{1} g(P^0)$. Thus, *merger pass-through*

g) - 5 9 (h)] T J / g 0 G B T / F 1 7 6 3 d 1 4 - 2 6 8 5 . 7 1 c m [ ] 0

(Jaffe and Kominers, 2011), horizontality seems frequently to be a good approximation to demand in a discrete choice context (Gabaix et al., 2009; Quint, 2010).

In non-Nash equilibrium concepts, calculation becomes even more difficult. Consider the conjectural variation framework. The only way to avoid relying on firms' reports of what they conjecture $\frac{dP_{-i}}{dP_i}$ to be is to assume their conjectures are consistent along the lines of Bresnahan (1981) consistent conjectures. In that case, because there is no guarantee that Slutsky symmetry is satisfied by the relevant residual demand system, calculating the price changes requires a direct observation of the relevant second derivative of demand both when other prices adjust (which requires the derivative of the reaction function) and when they are held fixed. It is possible that a large number of instruments allowing for sufficient variation to identify these higher-order derivatives could be found, but it seems unlikely in practice.

Approximation

However, the difference between pre-merger and merger pass-through (and post-merger pass-through) may in fact be small. For our approximation to be valid, $g(P^0)$ and the curvature of the equilibrium conditions need to be jointly sufficiently "small". If $g(P^0)$ is small, then it seems likely that $\frac{\partial g(P^0)}{\partial P}$ would also be small and thus $\left(\frac{\partial h(P^0)}{\partial P}\right)^{-1}$ would be approximately $\left(\frac{\partial f(P^0)}{\partial P}\right)^{-1}$. If this were not the case, then while $g(P^0)$ is small, if $g(P)$ were evaluated at a relatively close price in the direction of maximal gradient rather than at P^0 it would then no longer be small. To the extent that the smallness of g is "fragile" in this sense, it is unlikely to form a solid basis for using first-order approximations.

Thus, in many cases when the first-order approximation would be valid, the merger pass-through is approximately equal to pre-merger pass-through. Furthermore, if small diversion ratios, rather than other factors, cause $g(P^0)$ to be small, then post-merger pass-through will also be close to merger pass-through as K will be close to the identity matrix. If a merger is likely to have a small impact on prices, then it is likely to have a small impact on pass-through rates and thus both pre- and post-merger pass-through rates will approximate merger pass-through. Of course, using merger pass-through is very likely to be more accurate than using pre- or post-merger pass-through. An extreme example of this effect is the undifferentiated limit of N-i-q competition, where $\frac{\partial g}{\partial P}$ becomes large even though g approaches 0 at the fragile symmetric point.

Nonetheless, the interpretation which views pre-, post- and merger pass-through as close to one another has a number of benefits. First, it is consistent with the apparent coincidence (Froeb et al., 2005) that demand forms that are known to give rise to high pre-merger pass-through rates also have been found to generate high pass-through of merger efficiencies (which are driven by post-merger pass-through) and large anti-competitive effects (which are proportional to merger pass-through). Second, it shows that the Froeb et al. and the Shapiro et al. logic are on some level consistent with one another: to the extent that either is valid as a way to approximate merger effects, they are likely to give similar answers. Finally, it shows that using intuitions about pass-through rates to approximate the rate at which GePP is passed through to prices may not be overly misguided.

V Welfare Changes

The changes in prices calculated in Section III can be converted into estimates of changes in consumer or social surplus. This is useful because we generally care about price changes only in so far as they affect welfare. This normative approach based on consumer or social surplus is concordant with a large body of economic and legal scholarship on the appropriate standards for antitrust policy. While there is still strong disagreement over whether consumer or social surplus is the appropriate standard to apply, there seems to be widespread agreement that *one of these two*, or some mixture of them, should be targeted (Farrell and Katz, 2006). Additionally, focusing on surplus allows for the analysis of mergers that affect multiple products where the changes in price may vary substantially. Also, to the extent that there is substantial uncertainty in the estimates of the relevant parameters, looking at welfare combines the confidence intervals (by plugging in different estimates to the forumulas) in the appropriate way to get the corresponding bounds on the metric that we ultimately care about.[22]

Consumer Surplus

First, consider consumer surplus in the evaluated market (ignoring externalities and potential cross-market effects of the price changes). To a first-order, the change in consumer surplus is, by the classic Jevons formula, just the sum across goods of the change in price times the quantity: $\Delta CS \approx \Delta P^{\top} Q$.[23] It becomes unit-free, as with any other price index, if it is normalized by the initial value of the price index $P^{\top}Q$ yielding $I_{CS} \equiv \frac{\Delta P^{\top} Q}{P^{\top} Q}$.

Social Surplus

Estimating the change in social surplus requires an estimate for the expected change in quantity. Multiplying the Slutzky matrix $\frac{\partial Q}{\partial P}$ by the estimated price changes gives a first-order approximation for the change in quantity, $\Delta Q \approx \frac{\partial Q}{\partial P} \Delta P$.[24] Again ignoring externalities and out-of-market effects, the additional deadweight loss from the price increase is the sum of the change in quantities multiplied by the absolute mark-ups:

$$DWL \approx \Delta Q^{\top}(P - mc) \approx \left(\frac{\partial Q}{\partial P} \Delta P\right)^{\top}(P - mc).^{25}$$

The mark-ups can be pre-merger, post-merger or some combination of the two; various approaches, such as normalizing by the value of the market, construct unit-free indices. It would also be natural to include (as an additional term) an expected change in fixed (or more generally infra-marginal) costs due to the merger as in Williamson (1968).[26]

---

[22] We are grateful to Louis Kaplow for this point.

[23] Since we have calculated the first and second derivatives of Q, we could add higher order terms to this approximation, but since ΔP itself is an approximation that would be adding some second order terms and not others. Still, the formula may be evaluated at pre-merger (in the spirit of Laspeyres) or post-merger (Paasche) quantities or an arithmetic (Marshall-Edgeworth) or geometric (Fisher) average of the two.

[24] In many cases, such as consistent conjectures, the full Slutsky matrix is not necessary.

[25]

Pro ts

Our approach gives a simple approximation for the expected change in pro ts post-merger. While such are not typically an object of regulatory concern, an assumption that these must be positive by the rms' revealed preference for merging may provide some information.[27] If F_i is the (presumably negative) change in rm i's xed costs and mc_i is the (uniform) change inframarginal costs then

jn6he]TJ/F47 7.9701 Tf 16 (n2iTJ/F20 11.9552 Tf 9.755 0 Td 1 Tf 1181a11

VI Extensions and Examples

A Marginal cost e ciencies

Proposition 2. *In the symmetric example, the GePP from a merger of any two rms is*

$$D \quad m\frac{1 + \tilde{}(n \quad 3) \quad D(n \quad 1)\frac{\tilde{}^2}{1 \quad \tilde{}}}{1 \quad D\tilde{} \quad (n \quad 1) + D\tilde{}} \qquad D \quad m\frac{1 + \tilde{}(n \quad 3)}{1 \quad D\tilde{} \quad (n \quad 1) + D\tilde{}}; \qquad (7)$$

where $\tilde{} = \frac{}{1+}$ *is the post-merger accommodation by the un-merged rms and the approximation is valid for small .*

Proof. See Apprendix D.

In analyzing (7), we begin by focusing on the approximate formula. Note that $\tilde{}$ is strictly increasing in . When $n = 2$, we are considering a merger to monopoly, equation (7) is proportional to 1 $\tilde{}$, which is clearly decreasing in . That is, as discussed above, if accommodation by the merger partner is the only issue, GePP declines with the degree of accommodation as Farrell and Shapiro (2010a) conjecture. However, when $n = 3$ equation (7) is proportional to $\frac{1}{2 \quad D\tilde{}}$ which is clearly increasing in . This e ect gets stronger as $n \; ! \; 1$; in the limit the expression is proportional to $\frac{\tilde{}}{1 \quad D\tilde{}}$ which increases even more quickly in . Thus, in this basic example, \somewhere between" a merger to a monopoly and a merger by two rms within a triopoly the e ect of accommodation on GePP switches from negative to positive. Using the precise rather than the approximate formula weights things further towards GePP decreasing with as it subtracts a term strictly increasing in .

Often, the two merging rms are closer competitors (and potential accommodators) with

If the strength of the within-merger interaction is small compared to that outside the merger, GePP increases with anticipated accommodation. Conversely, if the strength of

This gives us a pre-merger condition of

$$f_i(P) = (P_i - \text{mc}_i) - \frac{Q_i}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-i}}\left(\frac{\partial Q_{-i}}{\partial P_{-i}}\right)^{-1}\frac{\partial Q_{-i}}{\partial P_i}} = 0.$$

After the firms merger, firm i starts taking firm j's price as given, so, following the same logic as above, the GePP is

$$g_i(P) = \underbrace{\frac{\frac{\partial Q_j}{\partial P_i} - \frac{\partial Q_j}{\partial P_{-ij}}\left(\frac{\partial Q_{-ij}}{\partial P_{-ij}}\right)^{-1}\frac{\partial Q_{-ij}}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-ij}}\left(\frac{\partial Q_{-ij}}{\partial P_{-ij}}\right)^{-1}\frac{\partial Q_{-ij}}{\partial P_i}}}_{\text{Diversion Ratio}}(P_j - \text{mc}_j) - \underbrace{Q_i\left(\frac{1}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-ij}}\left(\frac{\partial Q_{-ij}}{\partial P_{-ij}}\right)^{-1}\frac{\partial Q_{-ij}}{\partial P_i}} - \frac{1}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-i}}\left(\frac{\partial Q_{-i}}{\partial P_{-i}}\right)^{-1}\frac{\partial Q_{-i}}{\partial P_i}}\right)}_{\text{End of Accomodating Reactions}}.$$

The limit as one approaches undifferentiated N-i-q competition demonstrates the importance of the assumption of the invertibility of the map from strategies to prices. In the undifferentiated case, the GePP is 0, but this is meaningless, because it is impossible to change the price of one firm holding fixed the other firm's price. Under no differentiation, one would need to apply the GeSP formula above when strategies are quantities and use the merger quantity pass-through (Weyl and Fabinger, 2009), but we do not pursue this further here.[28]

Consistent Conjectures

Bresnahan (1981) proposed a method for empirically tying down firms' beliefs about other firms' reaction to changes in their strategy (for example prices). He argued that firms' beliefs should be *consistent* with what actually occurs when they are induced, say by a cost shock, to change their price. More formally, if we consider the case of prices as strategies, firms' conjectures are said to be consistent if $\frac{dP^k}{dP^i} = \frac{dP^k}{dt^i}\left(\frac{dP^i}{dt^i}\right)^{-1}$ where $k \neq i$ is any other firm and t^i is a vector of specific quantity taxes on the m_i goods of firm i or, equivalently, any other *while the matrix formed by* $\begin{matrix} \frac{df_1}{dx} \\ \frac{df_2}{dx} \end{matrix}$ *is non-singular. Then observing* $\frac{dP_1}{dx}$, $\frac{dP_2}{dx}$, $\frac{dQ_1}{dx}$ *and* $\frac{dQ_2}{dx}$ *identifies* $\frac{dQ_1}{dP_1}$, $\frac{dQ_1}{dP_2}$, $\frac{dQ_2}{dP_1}$ *and* $\frac{dQ_2}{dP_2}$ *and thus* $\bar{D}_{12}$, $\frac{d^M Q_1}{dP_1}$, $\frac{d^M Q_2}{dP_2}$ *and finally the Generalized*

A Simplifying the formula

While it seems that UPP is, in some sense, a simpler calculation than those we suggest, this is simply because a UPP-based calculation imposes simplifying assumptions. For example, if we were to assume all firms produced a single product, that conduct were Bertrand, that all cross-product pass-through rates were zero, then our formula would simplify to $\sum_i Q_i \rho_i UPP_i$, where ρ_i is the own-pass-through rate of each product.

Of course this is a very extreme example, but the general point is that beginning with our formula there are numerous simplifying assumptions one might make to reduce the complexity of the analysis. A few categories of assumptions one might consider are:

1. Pass-through: one could assume all cross pass-through rates (across firms and/or within particular products of a given firm) are zero so that we can ignore the impact of change in one merging firm's (opportunity) cost on the price of the other's product. One could impose symmetry on own- and cross- pass-through rates or, through an assumption akin to the horizontality assumption discussed in Subsection IV.B, assume some general relationship between pass-through rates and elasticities. Any of the assumptions discussed in Section IV above would aid in the identification of pass-through rates.

2. Heterogeneity: imposing some form of symmetry, either between the two merging firms, among all non-merging firms, between the merging and non-merging firms or all of the above would simplify the equations. Or one could summarize all non-merging firms into a single firm, as in Subsection VI.B above. Any of these would greatly reduce

would generate all the higher order terms for the Taylor expansion and yield the same precise result as MS. In practice, these assumptions typically go further than tying down higher-order effects and actually restrict quantities, such as pass-through rates and elasticities (Crooke et al., 1999; Weyl and Fabinger, 2009).[29]

Thus MS is accurate in cases when the local information available prior to the merger is determinative of the predicted effect *and* the misspecification of functional forms does not overly restrict the implications of this local information. Our approximation is likely to be precise whenever the first of these conditions is satisfied. Furthermore, in any case where the local information *is* determinative (the effects are small), our results guarantee that our formula will closely agree with the predictions of MS, so long as the functional form assumptions used in MS are not misspecified. Furthermore, the robustness of conclusions derived from MS to differing functional form, cost-side, conduct and other assumptions can first stages (using extensions of our formula as described in V), initially in a highly restricted way and then, again, these restrictions may gradually be relaxed as the analysis progresses. Thus our approach aims to incorporate all of the standard stages of an analysis continuously into a unified framework.

VIII Conclusion

Our work provides a general modeling framework for the quantitative analysis of oligopoly behavior. It shows how such a framework may be analyzed using inversion techniques to study general conduct and illustrates how first-order approximations may be applied to ap-

Appendix

A Deriving GeSP

Proof of Proposition 1. Writing P_i for $P_i(\ )$ and Q_i for $Q_i(P(\ ))$ for conciseness, the rm's rst order conditions are

$$\left(\frac{\partial P_i}{\partial\ _i} + \frac{\partial P_i}{\partial\ _{i}}^{T}\frac{\partial\ _{i}}{\partial\ _i}\right)^{T} Q_i + \left(\frac{\partial Q_i}{\partial\ _i} + \frac{\partial Q_i}{\partial\ _{i}}^{T}\frac{\partial\ _{i}}{\partial\ _i}\right)(P_i \quad mc_i(Q_i)) = 0.$$

Remembering that $\frac{dA}{dB_i} = \frac{\partial A}{\partial B_i} + \frac{\partial A}{\partial B\ _i}^{T}\frac{\partial B\ _i}{\partial B_i}$, the matrix of full derivatives including the e ects of other rms adjusting their strategies as expected, and then multiplying by $\frac{dQ_i}{d\ _i}^{T\ 1}$ the rm's rst-order conditions can be rewritten as:

$$f_i(\ ) \qquad \frac{dQ_i}{d\ _i}^{T\quad 1}\frac{dP_i}{d\ _i}^{T} Q_i \quad (P_i \quad mc_i(Q_i)) = 0.$$

After a merger of rms i and j, the newly formed rm takes into account the e ect of $_i$ on $_j$ and no longer expects $_j$ to react to $_i$ since the two are chosen jointly. The merged rm's rst-order derivatives with respect to $_i$ can be written:

$$(P_i \quad mc_i(Q_i)) \quad \left[\frac{dQ_i}{d\ _i}^{T} \quad \frac{\partial Q_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right]^{1} \left(\frac{dP_i}{d\ _i} \quad \frac{\partial P_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right)^{T} Q_i$$
$$\left[\frac{dQ_i}{d\ _i}^{T} \quad \frac{\partial Q_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right]^{1}\left(\frac{dP_j}{d\ _i} \quad \frac{\partial P_j}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right)^{T} Q_j + \left(\frac{dQ_j}{d\ _i} \quad \frac{\partial Q_j}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right)^{T}(P_j \quad mc_j(Q_j))\ \#$$

Subtracting $f_i(\ )$ from these rst-order conditions gives the Generalized Pricing Pressure, $g(\ )$, so that post merger $f(\ _{ij}) + g(\ _{ij}) = 0$. This is given by:

$$g_i(\ ) = \left(\left[\frac{dQ_i}{d\ _i}^{T} \quad \frac{\partial Q_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right]^{1}\left(\frac{dP_i}{d\ _i} \quad \frac{\partial P_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right)^{T} \quad \frac{dQ_i}{d\ _i}^{T\ 1}\frac{dP_i}{d\ _i}^{T}\right) Q_i +$$
$$\left(\left[\frac{dQ_i}{d\ _i}^{T} \quad \frac{\partial Q_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right]^{1}\left(\frac{dP_j}{d\ _i} \quad \frac{\partial P_j}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right)^{T} Q_j + \left(\frac{dQ_j}{d\ _i} \quad \frac{\partial Q_j}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}\right)^{T}(P_j \quad mc_j(Q_j))\right).$$

Using the convention $\frac{d^M Q_i}{d\ _i} = \frac{dQ_i}{d\ _i}^{T} \quad \frac{\partial Q_i}{\partial\ _j}\frac{\partial\ _j}{\partial\ _i}$ and similarly for price, we get the formulation in Proposition 1.

B Taylor Series Error Term

For notational convenience let $x = h^{-1}$. The error term is

$$\frac{1}{2}\begin{pmatrix} \sum_i \sum_j \frac{\partial^2 x_1}{\partial h_i \partial h_j} g_i(P') g_j(P') \\ \vdots \\ \sum_i \sum_j \frac{\partial^2 x_n}{\partial h_i \partial h_j} g_i(P') g_j(P') \end{pmatrix} = \frac{1}{2}\left[\sum_i \begin{pmatrix} \frac{\partial^2 x_1}{\partial h_i \partial h_1} & \cdots & \frac{\partial^2 x_1}{\partial h_i \partial h_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 x_n}{\partial h_i \partial h_1} & \cdots & \frac{\partial^2 x_n}{\partial h_i \partial h_n} \end{pmatrix} g_i(P') \right] g(P')$$

$$= \frac{1}{2}\left[\sum_i \frac{\partial^2 x}{\partial h_i \partial h} g_i(P') \right] g(P'). \qquad (9)$$

We know $\frac{\partial x}{\partial h}\frac{\partial h}{\partial x} = I$. Differentiating with respect to h_i gives:

$$\frac{\partial^2 x}{\partial h_i \partial h}\frac{\partial h}{\partial x} + \frac{\partial x}{\partial h}\begin{pmatrix} \sum_k \frac{\partial^2 h_1}{\partial x_i \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_1}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \\ \vdots & \ddots & \vdots \\ \sum_k \frac{\partial^2 h_n}{\partial x_i \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_n}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \end{pmatrix} = 0.$$

Solving for $\frac{\partial^2 x}{\partial h_i \partial h}$, using $\frac{\partial x}{\partial h} = \left(\frac{\partial h}{\partial x}\right)^{-1}$ and substituting into (9) 4 9.955 T762 Td [(=)]TJ/F26 10.9091 Tf 11.51

Where $D_x^2 h_j$ denotes the Hessian. Letting $[A]_{ij}$ indicate the ij element of matrix A,

$$E_a = \frac{1}{2} \sum_j \left[\frac{\partial h}{\partial x}^{1} \right]_{aj} g^T (P_0)$$

D Conjectural variations examples

Proof of Proposition 2. The first-order condition for a single firm requires that

$$m = -\frac{q}{\frac{dQ^i}{dP^i}}.$$

Prior to the merger, by the variables we have set up and symmetry $\frac{dQ^i}{dP^i} = \frac{\partial Q^i}{\partial P^i} + (n-1)\frac{\partial Q^i}{\partial P^j}$, where partials represent Bertrand derivatives (elements of the Slutsky matrix). But the definition of aggregate diversion we gave and symmetry imply that $\frac{\partial Q^i}{\partial P^j} = -\frac{\partial Q^i}{\partial P^i}\frac{D}{n-1}$. Solving out we obtain

$$\frac{\partial Q^i}{\partial P^i} = -\frac{q}{m(1-D)}.$$

Post-merger the price of the merger partner is held fixed rather than increasing by in response to an increase in the firm's price. By symmetry, therefore, the post-merger symmetric increase in the $n-2$ remaining firms' prices in response to an increase in one of the partners' prices, ~, must satisfy

= ~ +F11 10.9091 Tf 299.213 550.146 Td 49 Td [(the)]TJ/F11 10.909

exact case than in the approximate case.

Proof of Proposition 3. Our proof here is almost entirely analogous to that of Proposition 2. The first-order condition now requires that for the merging firms

$$m = \frac{q}{\frac{\partial Q^1}{\partial P^1}(1 \quad [d^2 + \quad ^2] \quad)};$$

so

$$\frac{\partial Q^1}{\partial P^1} = \frac{q}{m(1 \quad [d^2 + \quad ^2] \quad)}.$$

On the other hand by the logic of conjectures discussed in the proof of Proposition 2, if l represents the pre-merger merging-firm-to-non-merging-firm conjecture, L represents the same between the merging firms and $\tilde{l}$ represents the post-merger version of l then

$$l = \tilde{l}(1 + L) \quad (\;) \quad \tilde{l} = \frac{l}{1 + L}.$$

Plugging in our definitions of $l = d$ and $L =$ we obtain

$$\tilde{l} = \frac{d}{1 +}.$$

Now we can compute

$$\frac{d^M Q^1}{dP^1} = \frac{\partial Q^1}{\partial P^1} \quad 1 \quad \tilde{l}d \quad = \frac{q \quad 1 \quad d^{2}\tilde{\ }}{m(1 \quad [d^2 + \quad ^2] \quad)}$$

and

$$\frac{d^M Q^2}{dP^1} =$$

E GePP with Supply Functions

To show that the analysis is not limited to prices or quantities as strategies, we brie y outline the

We wish to solve for

$$\frac{dQ}{dP_{12}} = \left(\frac{\partial Q}{\partial P} \frac{dP}{dP_{12}} \right)_{12} = \left(\frac{\partial Q}{\partial P} \frac{dP}{dx} \left(\frac{dP_{12}}{dx} \right)^{-1} \right)_{12};$$

but from the chain rule we have that $\frac{\partial Q}{\partial P} \frac{dP}{dx} = \frac{dQ}{dx}$ and thus

$$\frac{dQ}{dP_{12}} = \left(\frac{dQ}{dx} \left(\frac{dP_{12}}{dx} \right)^{-1} \right)_{12}.$$

Breaking this up by row blocks yields $\frac{dQ}{dP_1}$ and similarly $\frac{dQ}{dP_2}$; breaking these resultant matrices down by columns yields the individual effects on Q_1 and Q_2. From this the desired pre-merger quantities are extracted and the post-merger quantities calculated as in the text. For example:

$$\frac{d^M Q_1}{dP}_{\text{95Q0-mergeies}}$$

Tools for Analyzing Market Outcomes," in James J. Heckman and Edward E. Leamer, eds., *Handbook of Econometrics*, Vol. 6B, North-Holland B.V.: Amsterdam, Holland, 2007. http://www.economics.harvard.edu/faculty/pakes/files/tools8l.pdf.

Baker, Jonathan B. and Timothy F. Bresnahan, \The Gains from Merger or Collusion in Product-Di erentiated Industries," *Journal of Industrial Economics*, 1985, *33* (4), 427{444.

_ and _ , \Estimating the Residual Demand Curve Facing a Single Firm," *International Journal of Industrial Organization*, 1988, *6* (3), 283{300.

Berry, Stephen, James Levinsohn, and Ariel Pakes, \Automobile Prices in Market Equilibrium," *Econometrica*, 1995, *63* (4), 841{890.

Berry, Steven and Ariel Pakes, \The Pure Characteristics Demand Model," *International Economic Review*, 2007, *48* (4), 1193{1225.

_ , James Levinsohn, and Ariel Pakes, \Di erentiated Products Demand Systems from a Combination of Micro and Macro Data: The New Car Market," *Journal of Political Economy*, 2004, *112* (1), 68{105.

Bowley, Arthur L., *The Mathematical Groundwork of Economics*, Oxford: Oxford University Press, 1924.

Bresnahan, Timothy F., \Duopoly Models with Consistent Conjectures," *American Economic Review*, 1981, *71* (5), 934{945.

_ , \The Oligopoly Solution Concept is Identi ed," *Economics Letters*, 1982, *10* (1{2), 87{92.

Carlton, Dennis W., \Revising the Horizontal Merger Guidelines," *Journal of Competition Law and Economics*, 2010, *6* (3), 619{652.

Chetty, Raj, \Su cient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods," *Annual Review of Economics*, 2009, *1*, 451{488.

Coate, Malcolm and Joseph Simons, \Critical Loss vs. Diversion Analysis?," *CPI Antitrust Chronicle*, 2009, *12* (1).

Commission, Competition and O ce of Fair Trading, \Merger Assessment Guidelines," 2010. http://www.oft.gov.uk/shared_oft/mergers/642749/OFT1254.pdf.

Crooke, Philip, Luke M. Froeb, Steven Tschantz, and Gregory J. Werden, \E ects of Assume Demand Form on Simulated Postmerger Equilibria," *Review of Industrial Organization*, 1999, *15* (3).

Deaton, Angus and John Muellbauer, \An Almost Ideal Demand System," *American Economic Review*, 1980, *70* (3), 312{326.

Deneckere, Raymond and Carl Davidson, \Incentives to Form Coalitions with Bertrand Competition," *RAND Journal of Economics*, 1985, *16* (4), 473{486.

Dockner, Engelbert J., \A Dynamic Theory of Conjectural Variations," *Journal of Industrial Economics*, 1992, *40* (4), 377{395.

United States Department of Justice and Federal Trade Commission, \Horizontal Merger Guidelines," http://www.justice.gov/atr/hmerger/11248.htm 1982.

— and — , \Horizontal Merger Guidelines," http://www.justice.gov/atr/public/guidelines/hmg-2010.html August 2010.

Walters, Chris, \Diversion Ahead! Approximation Diversion Ratios for Retail Chain Mergers," 2007. http://www.mmc.gov.uk/our_role/analysis/diversion_ratios.pdf.

Werden, Gregory J., \Section 7 of the Clayton Act and the Analysis of \Semihorizontal" Mergers," *Antitrust Bulletin*, 1982, *27* (1), 135{160.

— , \Horizontal Mergers: Comment," *American Economic Review*, 1991, *81* (4), 1002{1006.

— , \A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of Di erentiated Products," *Journal of Industrial Economics*, 1996, *44* (4), 409{413.

— and Luke Froeb, \Choosing Among tools for Assessing Unilateral Merger E ects," 2011. http://ssrn.com/abstract=1804243.

— and Luke M. Froeb, \The E ects of Mergers in Di erentiated Products Industries: Logit Demand and Merger Policy," *Journal of Law, Economics, and Organization*, 1994, *10* (2), 407{426.

— and — , \Unilateral Competitive E ects of Horizontal Mergers," in Pablo Buccirossi, ed., *Handbook of Antitrust Economics*, MIT Press, 2008, chapter 2, pp. 43{104.

Weyl, E. Glen, \Slutsky meets Marschak: the First-Order Identi cation of Multiproduct Monopoly," 2009. http://www.glenweyl.com.

— and Michal Fabinger, \Pass-Through as an Economic Tool," 2009. http://www.glenweyl.com.

White, Alexander and E. Glen Weyl, \Imperfect Platform Competition: A General Framework," 2011. http://ssrn.com/abstract=1694317.

Williamson, Oliver E., \Economies as an Antitrust Defense: The Welfare Tradeo s," *American Economic Review*, 1968, *58* (1), 18{36.