Antitrust Contests

Michael R. Baye
Federal Trade Commission & Indiana University

Paul A. Pautler
Federal Trade Commission

&

Joshua D. Wright
Federal Trade Commission & George Mason University

Roadmap

- Theoretical foundations of structural estimation
- Overview of antitrust contests in the US
- Data
- Structural estimates
- Monte Carlo results: Reliability & bias
- Conclusions

Theoretical Foundations

- Two contestants
  - FTC (player 1)
  - Defendant (player 2)
- Potentially different values to each of winning and losing
- Contest Success Function:

Model 1 (Generalized Tullock)

$$\Pr(\textbf{FTC Wins} \mid X) = p(x_1, x_2)$$

$$= \frac{\sigma x_1^r}{\sigma x_1^r + x_2^r}$$

$$= \frac{1}{1 + \frac{1}{\sigma}\left(\frac{x_2}{x_1}\right)^r}$$

$$\frac{1}{1 + \frac{1}{\sigma} z^r} \equiv \frac{}{1 + \ldots}$$

Model 2 (Logistic)

$$\Pr(\text{FTG Wins} \mid X) = F(X\beta)$$

where *F is logistic:*

$$F(\omega) = \frac{\exp(\omega)}{1 + \exp(\omega)}$$

Pros and Cons

- Advantage of Generalized Tullock
  - Well established theoretical literature (by all of you and others)
- Advantages of Logistic
  - Structural micro foundations (McFadden and others)
  - Empirically estimable using standard logit estimation rather than problematic binomial MLE methods

Key Result: Structural Equivalence

Â

Remark 2

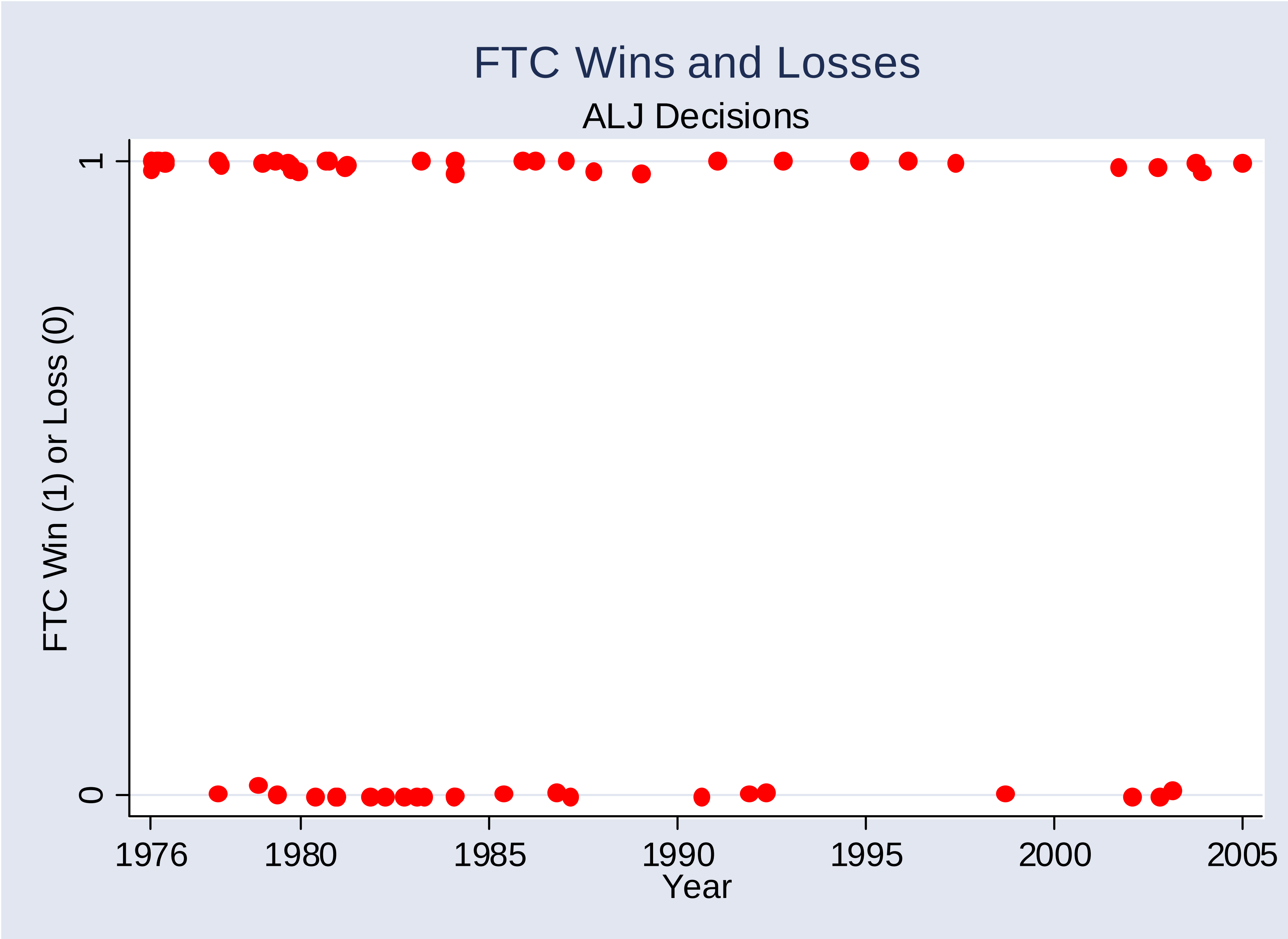
- Structural equivalence works for more general contest success functions, such as

$$p(x_1, x_2) = \frac{\alpha_1 x_1^r}{\alpha_1 x_1^r + \alpha_2 x_2^r} = \frac{1}{1 + \frac{\alpha_2}{\alpha_1}\left(\frac{x_2}{x_1}\right)^r} \equiv \frac{1}{1 + \rho z^r}$$

  - But cannot separately identify α's, only the ratio

Remark 3

- Not generally feasible to exploit additional structure
  - For $\sigma > 0$, $r > 0$ equilibrium in mixed-strategies guaranteed, but structure of strategies generally unknown except for specific values of σ and r
  - For some parameter configurations, equilibrium is in pure strategies, but these regions depend on r, σ, as well as the *(unknown)* values of winning and losing


FTC Wins and Losses
ALJ Decisions
FTC Win (1) or Loss (0)
1
0
1976
1980
1985
1990
1995
2000
2005
Year

Results: The Good

Table 2: Implied Structural Parameters

	Baseline		Random Effects			
	(1) [illegible]	(2) [illegible] of Cases	(3) [illegible] Cases Heterogeneity	(4) [illegible] Cases Heterogeneity	(5) [illegible] All Heterogeneity	(6) [illegible] All Heterogeneity
r	0.26	0.31	0.27	0.33	0.26	0.31
Sigma (Pooled)	0.74		0.75		0.78	
Sigma (Merger Case)		0.49		0.51		0.51
Sigma (Other Case)		0.78		0.79		0.79

Results: The Good

Table 2: Implied Structural Parameters						
	Baseline		Random Effects			
	(1) Baseline	(2) Controls for Type of Case	(3) Unobserved Case Heterogeneity	(4) Unobserved Case Heterogeneity	(5) Unobserved ALJ Heterogeneity	(6) Unobserved ALJ Heterogeneity
r	0.26	0.31	0.27	0.33	0.26	0.31
Sigma (Pooled)	0.76		0.74		0.75	
Sigma (Merger Case)		0.51		0.49		0.51
Sigma (Other Case)		0.79		0.78		0.79

Results: The Good

Table 2: Implied Structural Parameters						
	Baseline		Random Effects			
	(1) Baseline	(2) Controls for Type of Case	(3) Unobserved Case Heterogeneity	(4) Unobserved Case Heterogeneity	(5) Unobserved ALJ Heterogeneity	(6) Unobserved ALJ Heterogeneity
r	0.26	0.31	0.27	0.33	0.26	0.31
Sigma (Pooled)	0.76		0.74		0.75	
Sigma (Merger Case)		0.51		0.49		0.51
Sigma (Other Case)		0.79		0.78		0.79

Results: The Good

Table 2: Implied Structural Parameters

	Baseline		Random Effects			
	(1) Baseline	**(2) Controls for Type of Case**	**(3) Unobserved Case Heterogeneity**	**(4) Unobserved Case Heterogeneity**	**(5) Unobserved ALJ Heterogeneity**	**(6) Unobserved ALJ Heterogeneity**
r	0.26	0.31	0.27	0.33	0.26	0.31
Sigma (Pooled)	0.76		0.74		0.75	
Sigma (Merger Case)		0.51		0.49		0.51
Sigma (Other Case)		0.79		0.78		0.79

Results: The Bad

Table 1: Results of Logistic Structural Estimation						
	Baseline		Random Effects			
	(1) Baseline	(2) Controls for Type of Case	(3) Unobserved Case Heterogeneity	(4) Unobserved Case Heterogeneity	(5) Unobserved ALJ Heterogeneity	(6) Unobserved ALJ Heterogeneity
LN(z)	-0.256	-0.309	-0.269	-0.327	-0.258	-0.311
	(0.72)	(0.84)	(0.65)	(0.78)	(0.68)	(0.80)
MERGER DUMMY		-0.435		-0.458		-0.432
		(0.79)		(0.77)		(0.79)
CONSTANT	-0.281	-0.231	-0.297	-0.247	-0.287	-0.238
	(0.26)	(0.21)	(0.25)	(0.20)	(0.25)	(0.20)
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

Results: The Bad

Table 1: Results of Logistic Structural Estimation

	Baseline		Random Effects			
	(1) Baseline	(2) Controls for Type	(3) Unobserved Case	(4) Unobserved Case	(5) Unobserved ALJ	(6) Unobserved ALJ
[illegible]	-0.256	-0.309	-0.269	-0.327	-0.258	-0.311
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Results: The Bad

Table 1: Results of Logistic Structural Estimation

	(1)	(2)	(3)	(4)	(5)	(6)
LN(z)	-0.327	-0.258	-0.311	-0.256	-0.309	-0.269
	(0.79)	(0.68)	(0.80)	(0.72)	(0.84)	(0.65)
[illegible]	-0.458		-0.432		-0.435	
	[illegible]		[illegible]		[illegible]	
CONSTANT	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Note: Robust z statistics in parentheses

Results: The Bad

Table 1: Results of Logistic Structural Estimation

Random Effects					Baseline	
	-0.435		-0.458		-0.432	MERGER DUMMY
	(0.77)		-0.79			(0.79)
-0.297	-0.247	-0.287	-0.238	CONSTANT	-0.281	-0.231
(0.25)	(0.20)	(0.25)	(0.20)		(0.26)	(0.21)
60	60	60	60	Observations	60	60
		17	17	Number of ALJ's		
60	60			Number of Cases		

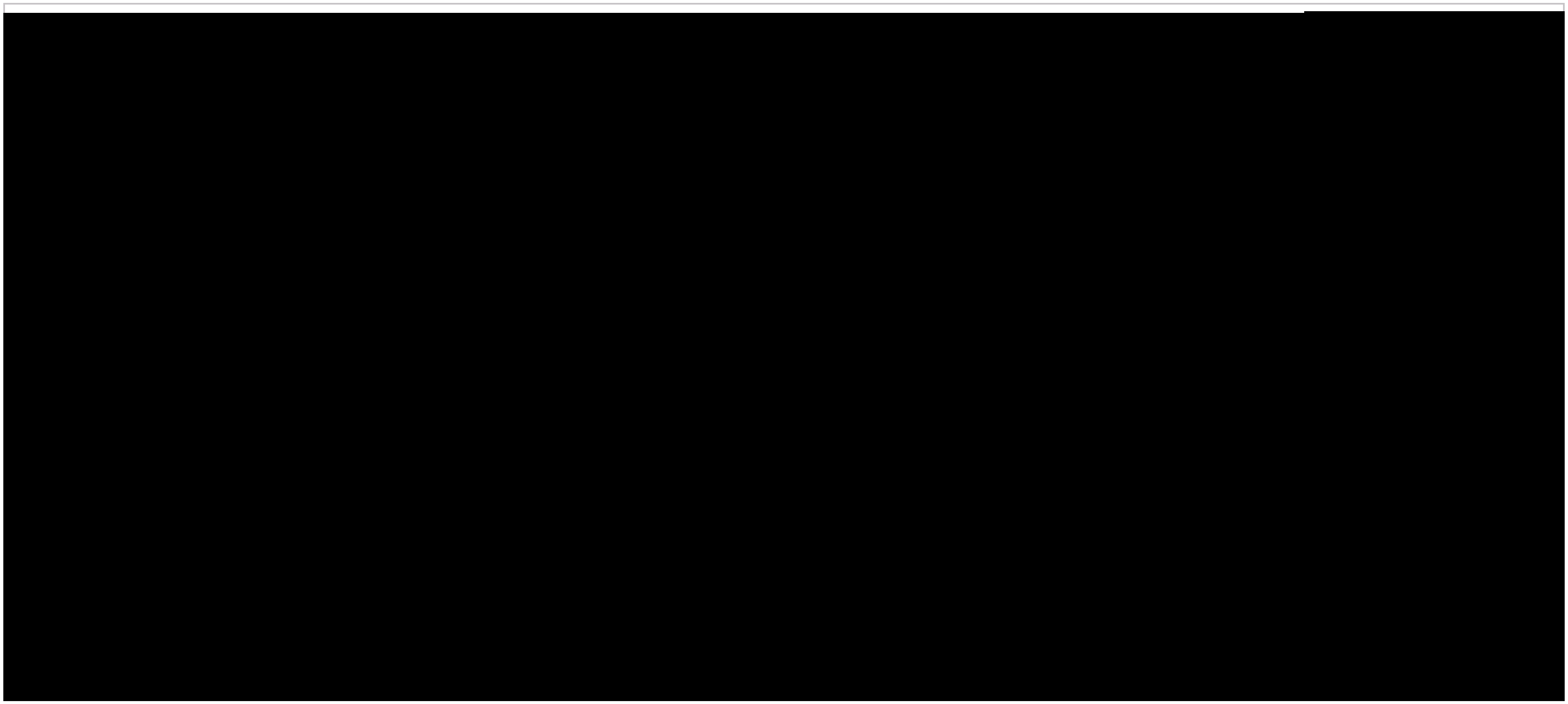
Results: The Bad

Table 1: Results of Logistic Structural Estimation

		Baseline		Random Effects		
	Controls for Type of Case	(1) Unobserved Case Heterogeneity	(2) Unobserved Case Heterogeneity	(3) Unobserved Deal Heterogeneity	(4) Unobserved Deal Heterogeneity	(1) Baseline
LN(z)	-0.309	-0.269	-0.327	-0.258	-0.311	-0.25
	(0.84)	(0.65)	(0.78)	(0.68)	(0.80)	(0.72
MERGER DUMMY	-0.435		-0.458		-0.432	
	-0.79		-0.77		-0.79	
Observations	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Number of [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Number of Cases	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Note: [illegible]

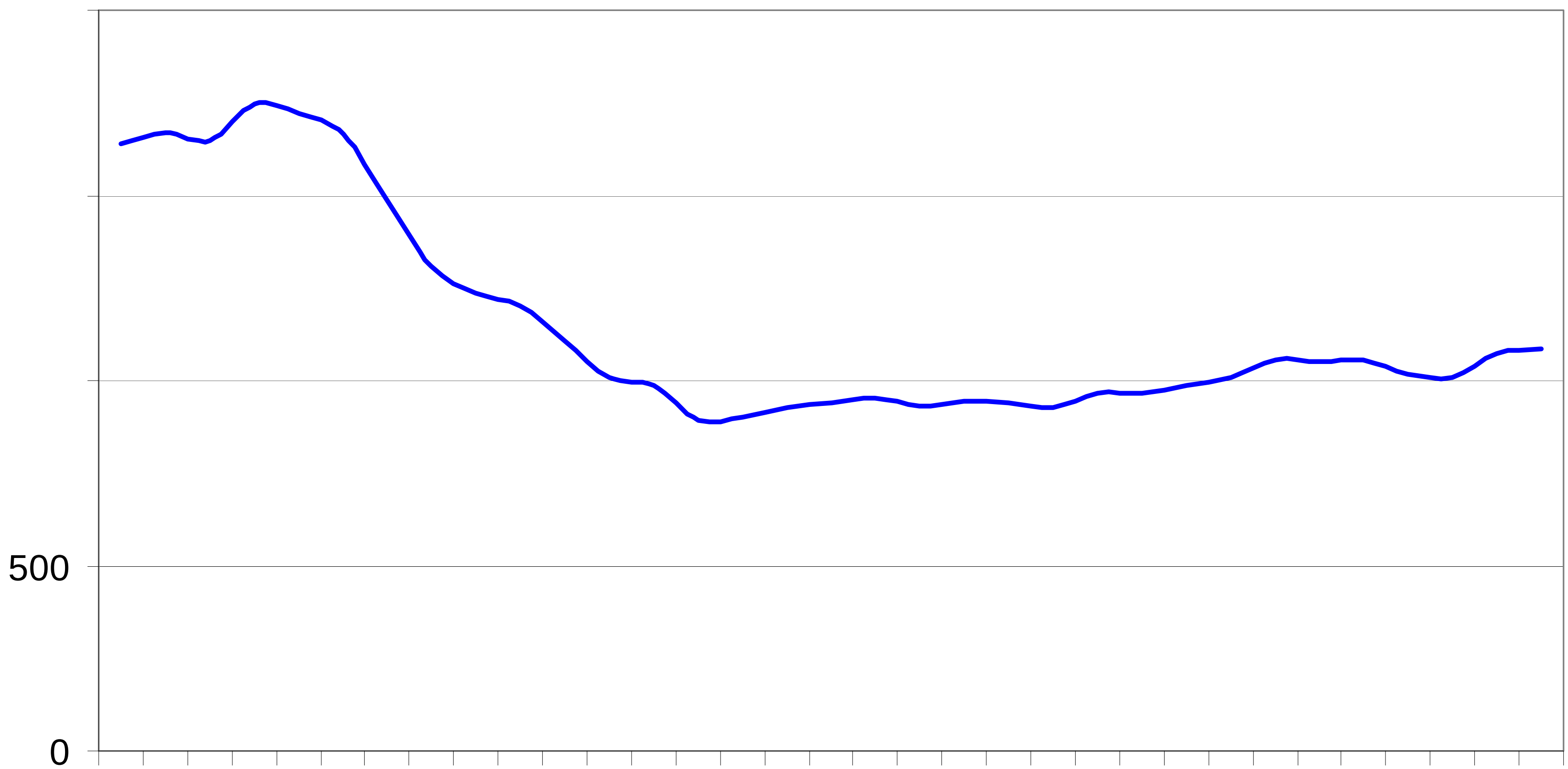
Results: The Bad



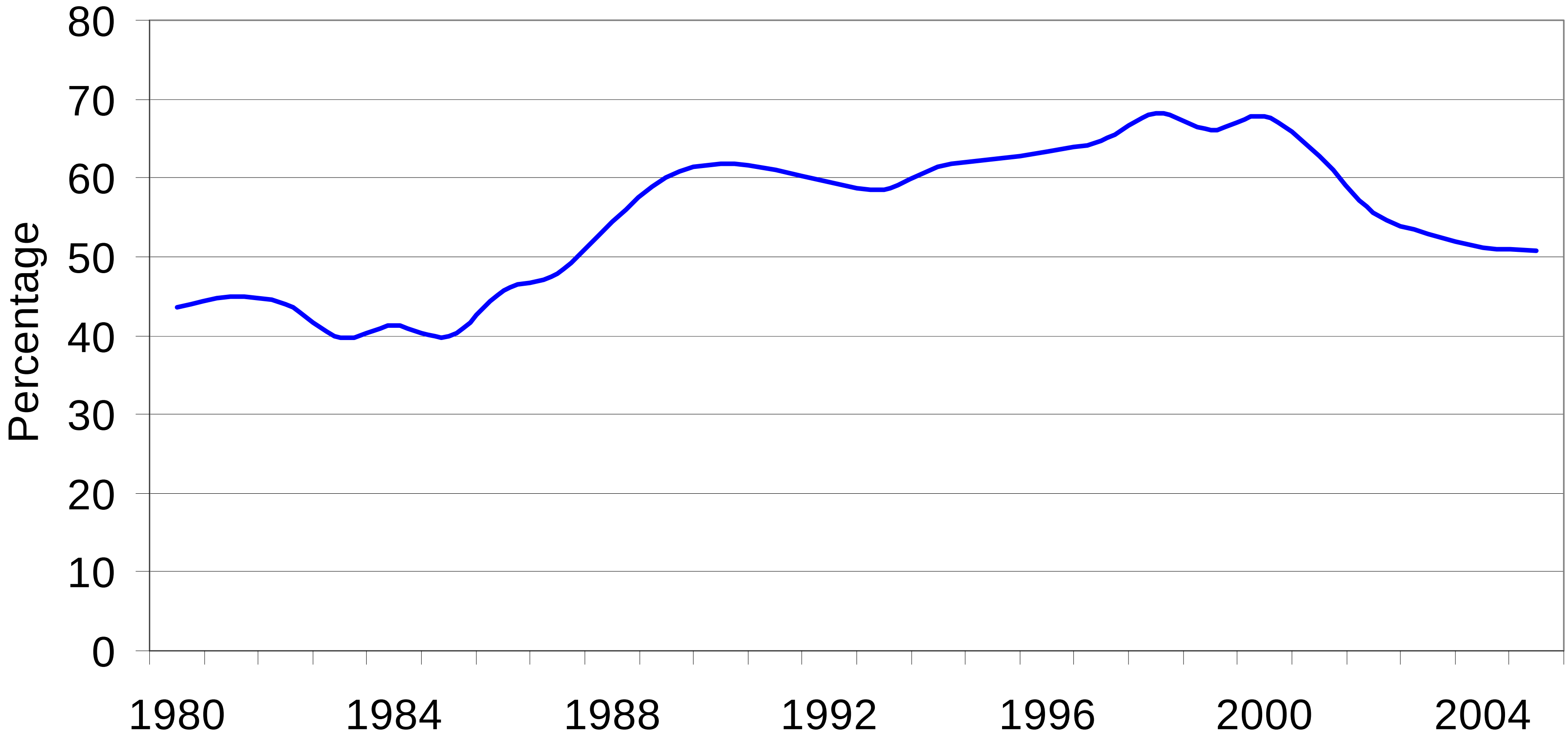
Better Data?

- Include other inputs (attorneys)
- Adjustments for time on antitrust versus other activities (consumer protection or advocacy)
- Expenditures on experts

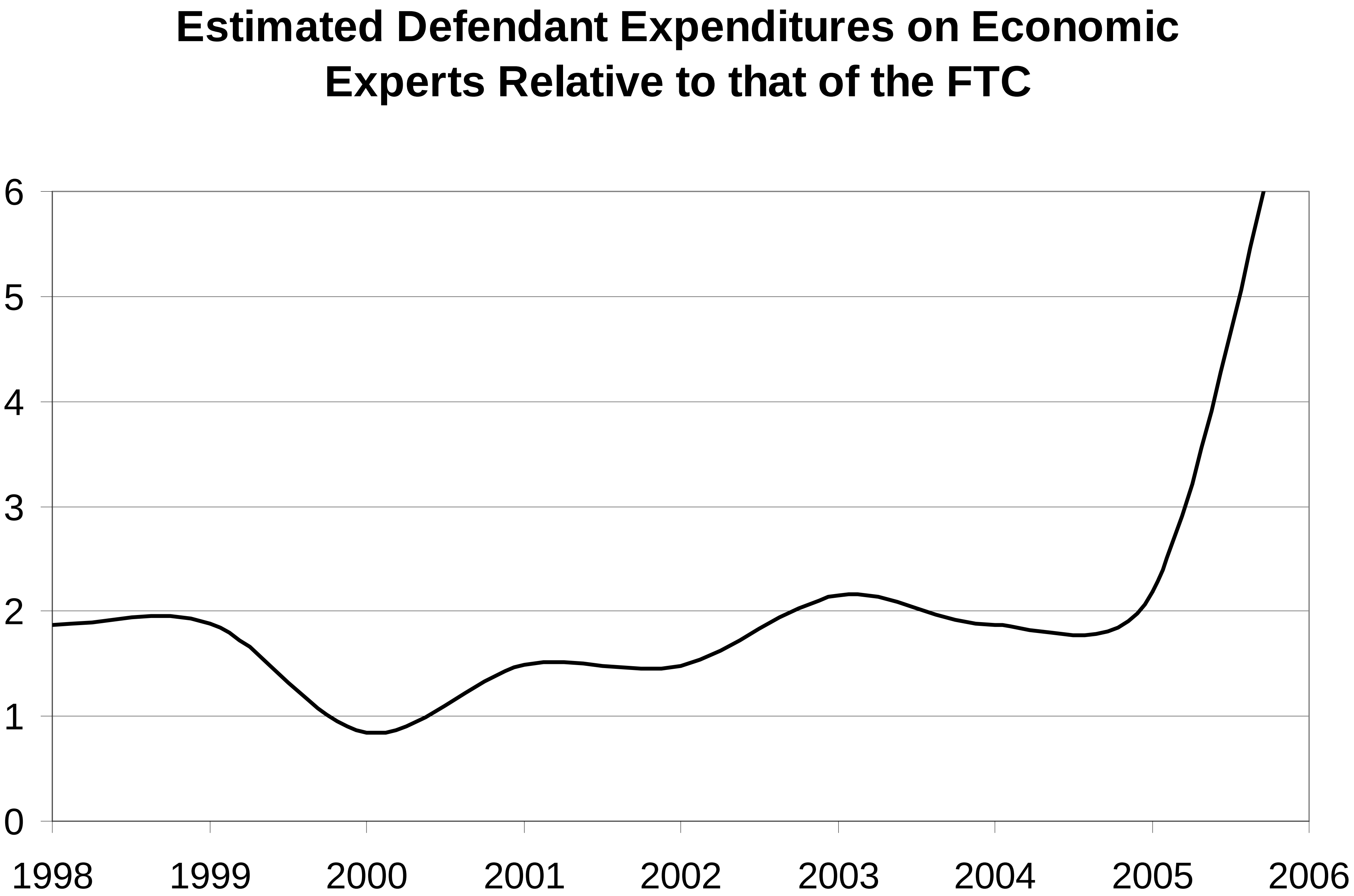
Total FTC Employees

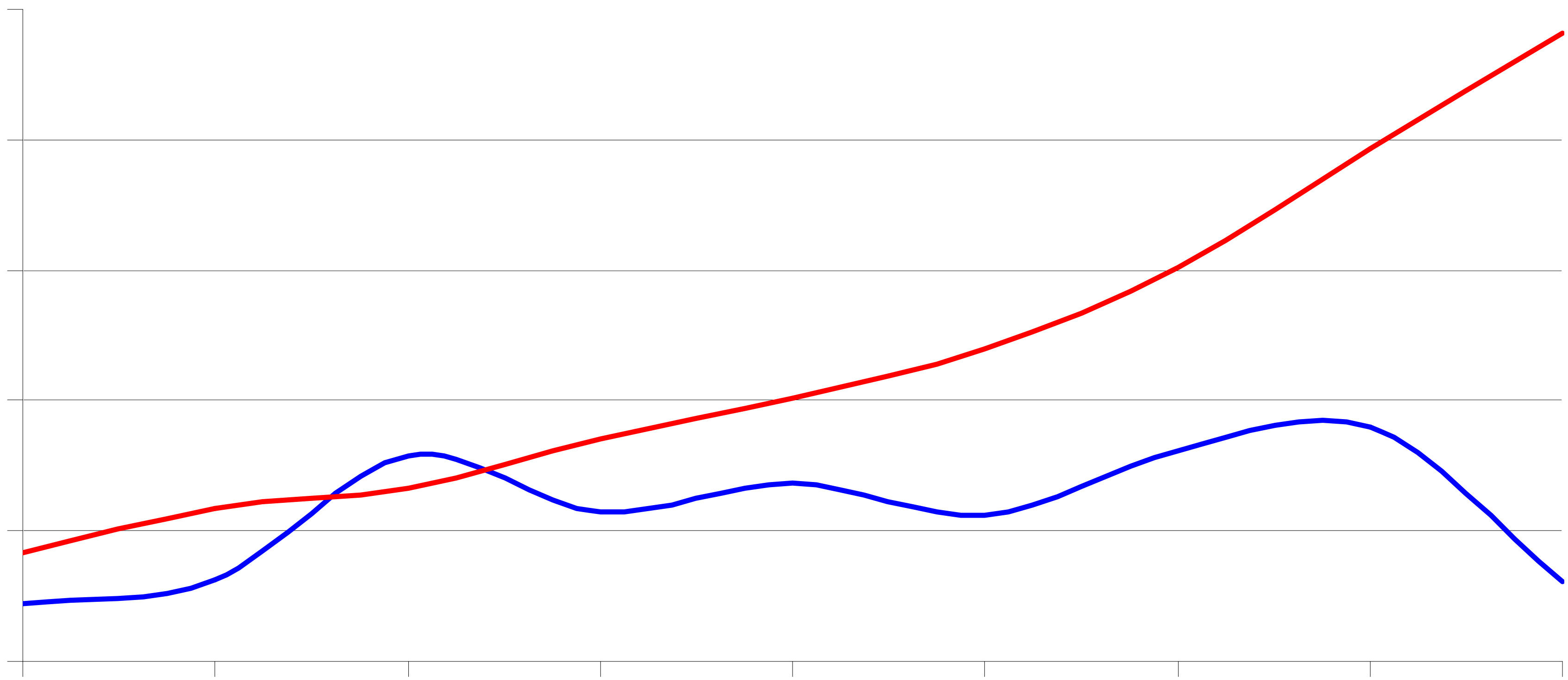


Percentage Allocation of Economist Time for Antitrust



Example of Alternative *z* Measure:





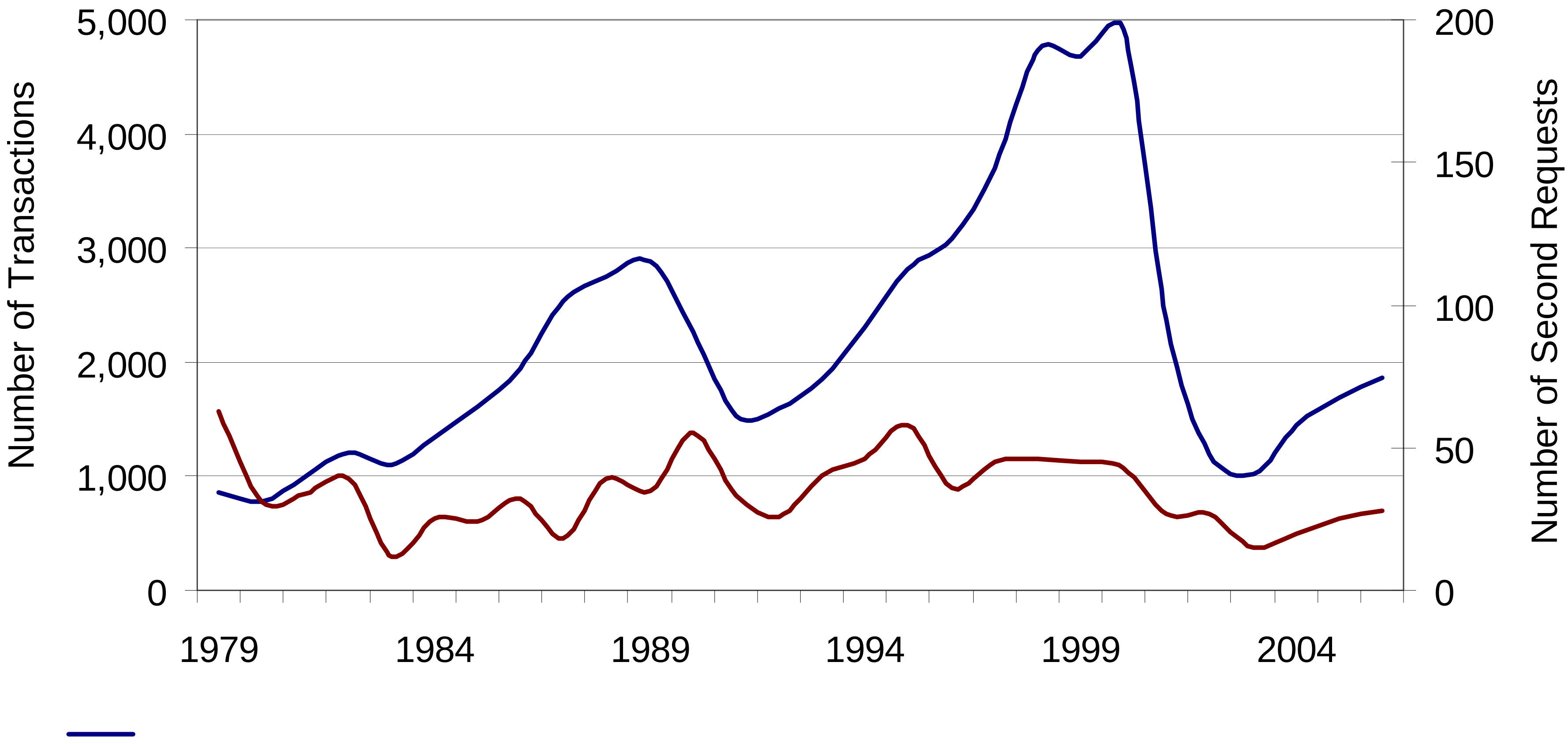
Results From These Data

- Similar sorts of estimates
- No more reliable

What About Endogeneity?

- Merger activity
- Selection issues
- Endogenous effort
  - Impose restrictions on z implied by PSNE and use proxies for values of winning

Hart-Scott-Rodino Transactions & Second Requests



Accounting for Endogeneity

- Doesn’t help!
- What’s going on?

Monte Carlo

- Generated data from a “true” model

$$r \in \{.25,\ 1,\ 1.5\}$$

$$\sigma \in \{.25,\ 1,\ 1.5\}$$

- Low, medium, high cross sectional variation in z (measured by coefficient of variation)
- 20 obs, 60 obs, 400 obs
- Replicated 10,000 times each

[illegible]

σ = 1.5			σ = .25			σ = 1			Parameter or Summary Statistic / True Parameters
1.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58	0.30	r-hat
1.99	2.00	2.01	2.05	2.03	2.55	2.57	2.65	1.97	sd(r-hat)
0.54	0.81	0.15	0.51	0.78	0.12	0.43	0.59	0.13	pseudo t-statistic
0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08	bias(σ-hat)
0.08	0.09	0.09	0.28	0.29	0.29	0.46	0.46	0.46	sd(σ-hat)
2.93	2.89	2.90	3.66	3.61	3.57	3.47	3.43	3.42	pseudo t-statistic

	Parameter or Summary Statistic	True Parameters		
$\sigma = 1.5$		$\sigma = .25$	$\sigma = 1$	

7.45	min(mat)	17.61	11.52	12.82	7.48	6.37	6.15	6.15	7.66
11.87	max(mat)	12.79	14.36	31.47	12.15	9.43	9.97	9.45	14.31

0.03	0.04	0.08	0.08	0.08	bias(σ-hat)	0.00	0.00	0.00	0.04
0.29	0.29	0.46	0.46	0.46	sd(σ-hat)	0.08	0.09	0.09	0.28
3.61	3.57	3.47	3.43	3.42	pseudo t-statistic	2.93	2.89	2.90	3.66

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60

True Parameters

Summary Statistic

σ = 1 | σ = 25

r-hat: 1.68, 0.30, 1.05, 1.68, 0.30, 1.08, 1.67, 0.25

bias(r-hat): 0.07, 0.00, 0.07, 0.11, 0.05, 0.05, 0.08, 0.05, 0.09

se(r-hat)

pseudo t-statistic: 0.78, 0.12, 0.43, 0.59, 0.13, 0.54, 0.81, 0.15, 0.51

min(r-hat): 17.61, 11.57, 12.84, 7.48, 6.37, 6.13, 8.13, 7.86

max(r-hat): 9.45, 14.31, 11.87, 12.79, 14.36, 31.47, 12.45, 9.43, 9.97

σ-hat: 1.58, 1.58, 1.58, 0.25, 0.25, 0.25, 1.04, 1.03, 1.04

bias(σ-hat): 0.08, 0.08, 0.00, 0.00, 0.00, 0.04, 0.03, 0.04, 0.08

se(σ-hat): 0.29, 0.29, 0.46, 0.46, 0.46, 0.08, 0.09, 0.09, 0.25

pseudo t-statistic: 3.61, 3.57, 3.47, 3.43, 3.42, 2.93, 2.89, 2.90, 3.66

min(σ-hat)

max(σ-hat)

Table 5a: Monte Carlo Results for Structural Estimation, CV(z) = 14%, Sample Size = 60

True Parameters

σ = 1.5 | σ = .25 | σ = 1

Summary Statistic									
r-hat	.09	1.57	0.25	1.07	1.61	0.30	1.05	1.58	0.30
bias(r-hat)	0.05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.05	[illegible]	[illegible]
sd(r-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
pseudo t-statistic	0.12	0.43	0.59	0.13	0.51	0.81	0.15	0.51	0.78
min(r-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
max(r-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
σ-hat	1.04	1.58	1.58	1.58	0.25	0.25	0.25	1.04	1.03
bias(σ-hat)	0.04	0.08	0.08	0.08	0.00	0.00	0.00	0.04	0.03
sd(σ-hat)	0.29	0.46	0.46	0.46	0.08	0.09	0.09	0.28	0.29
min(σ-hat)	0.33	0.35	0.61	0.51	0.55	0.01	0.02	0.01	0.35
max(σ-hat)	2.88	3.05	5.03	5.27	5.03	0.71	0.83	0.80	3.28

- Punch Line for estimating *r* with 60 obs…
  - Small bias…
  - But unreliable estimates (high variance)

What About Estimates of σ?

Table 5: … Monte Carlo Results … Structural Estimation …

Parameter or Summary Statistic	σ = .25			σ = 1			σ = 1.5		
	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5	r = .25	r = 1	r = 1.5
bias(r-hat)	0.08	[illegible]	[illegible]	0.05	0.09	0.07	0.09	0.07	0.11
pseudo t-statistic	0.81	0.15	0.51	0.78	0.12	0.43	0.59	0.13	[illegible]
max(r-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
bias(σ-hat)	0.00	0.00	0.00	0.04	0.03	0.04	0.08	0.08	0.08
pseudo t-statistic	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
min(σ-hat)	0.35	0.61	0.51	0.55	0.01	0.02	0.01	0.35	0.33
max(σ-hat)	3.28	2.88	3.05	5.03	5.27	5.03	0.74	0.83	0.80

- Punch line for estimating *σ* with 60 obs: More reliable estimates, but critically depends on the presence of "good" data on effort
  - Scaling of x_i distorts interpretation of *σ*
  - Unreliable if true effort is [illegible]

Tullock's *r*

Table 4b: Monte Carlo Results for Structural Estimation: r = .25, σ = 1

n in Effort

Variatio

=14.2%

CV(z)= 68%

Parameter or

CV(z) = 1.5%

CV(z)

man Statistic

Statistic	60	160	400	60	160	400	60	160	400	Sum
r-hat	0.25			0.38	0.74	0.21	0.32	0.25	0.25	0.31
	0.06						0.63	0.49		0.07
sd(r-hat)	0.70	0.09	0.74	0.16			43.25	19.82	7.15	4.27
max(rhat)	59.31	12.15	2.99	15.66	2.17	0.86	361.65	92.71	25.31	
σ-hat	1.16	1.04	1.00	1.16	1.04	1.00	1.15	1.04	1.01	
bias(σ-hat)	0.16	0.04	0.00	0.16	0.04	0.00	0.15	0.04	0.01	
sd(σ-hat)	0.81	0.28	0.10	0.84	0.29	0.10	0.78	0.29	0.10	
pseudo-t statistic	1.49	3.63	0.01	1.42	3.66	10.03	1.34	3.57	0.87	
	0.33	0.58	0.04	0.35	0.55	0.05	0.30	0.57	0.03	
max(σ-hat)	3.04	1.52	26.53	3.04	1.45	35.18	3.28	1.48	53.63	

Table 6b: Monte Carlo Results for Structural Estimation: r = 1.5, σ = 1									
Parameter or	Variation in Effort								
	CV(z) = 1.5%			CV(z) = 14.2%			CV(z)= 68%		
r-hat	1.62	1.51	[illegible]	1.77	1.92	1.46	1.95	1.61	1.52 / 2.02
bias(r-hat)	0.12	0.01	[illegible]	0.27	0.42	0.04	0.45	0.11	0.02 / 0.52
sd(r-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
min(r-hat)	[illegible]	1.59	[illegible]	0.11	0.80	[illegible]	[illegible]	[illegible]	[illegible]
max(r-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
σ-hat	1.17	1.03	1.00	1.26	1.04	1.00	3.48E+06	1.05	1.01
bias(σ-hat)	0.17	0.03	0.00	0.26	0.04	0.00	3.48E+06	0.05	0.01
sd(σ-hat)	0.99	0.28	0.10	9.43	0.29	0.10	3.44E+08	0.32	0.11
min(σ-hat)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
max(σ-hat)	43.79	2.39	1.44	936.14	3.05	1.47	3.44E+10	3.32	1.50

Concluding Remarks

- Structural estimation of Tullock's r problematic, unless:
  - Have **large sample,** true underlying model has **large** $\boldsymbol{r}$ and **large variation in** $\boldsymbol{z}$
- Structural estimation of σ requires exceptionally good measures of effort
- Suggests utility of developing alternative contest models more amenable to structural estimation
- Monte Carlo tests of alternative existing models
- Tullock framework still potentially useful for testing predictions via reduced form estimation

Concluding Remarks (Continued)

- Best estimate of r in antitrust contests brought by the FTC between 1976-2005:
  - $r \approx 1/4$
- Monte Carlo simulations suggest estimate is unbiased, but unreliable (high variance)