Contests with Rank-Order Spillovers

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Overview: Applications

- Results apply/extend a variety of models/results in IO, behavioral economics, and game theory
  - Cover all standard contests and auctions, war of attrition, & all-pay auction
  - More exotic auctions and contests
  - Innovation contests with spillovers
  - Pricing games
  - Price matching policies
  - Behavioral economics (inequality aversion, loss aversion, regret, reference pricing)
  - Evolutionary equilibria (ESS)

Model and Notation

- Players: $\in \{1, 2\}$
- Actions (bids, prices, e¤ort, etc.): $\in$ 0, ∞
- Payo¤s (coin-‡ip tie-breaking rule suppressed):

$$, \quad \begin{cases} -\beta \quad -\delta & \text{if} \\ -\gamma - \alpha \quad -\theta & \text{if} \end{cases}$$

- ≥ 0
- $\equiv \quad \gamma \quad 0$
- Γ: An arbitrary game with this structure.
- $\eta \equiv \alpha \quad \theta - \beta - \delta$
- *: Symmetric pure-strategy (Nash) equilibrium
- * : Symmetric (non-degenerate) mixed-strategy equilibrium

Proposition 1: Characterization of Symmetric Pure-Strategy Equilibria

Γ

(i) $\beta \geq 0$

(ii) $\alpha \leq 0$

Proposition 2: Characterization of Symmetric Mixed-Strategy Equilibria

Γ

(i) β 0 α 0;

(ii) β 0, α 0 $\eta\theta$ 0 η α

(iii) β 0, α 0 and either α η 0 or η θ 0.

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Proposition 2: Characterization of Symmetric Mixed-Strategy Equilibria (Continued)

$*, \quad *$

$$* \qquad ' \in \Big) - \overline{\eta}, \infty \Big\{ \begin{array}{lll} 0 & \text{if} \quad \alpha \quad 0 \\ & \text{if} \quad \alpha \quad 0 \end{array} ,$$

$$* \qquad \begin{array}{ll} -\overline{\eta} & \text{if} \quad \alpha \quad 0; \beta \quad 0; \eta \quad 0 \\ \overline{\eta} \Big) \alpha / \beta^{\frac{\eta}{\alpha - \beta}} - 1 & \text{if} \quad \alpha \quad 0; \beta \quad 0; \alpha \,/\, \beta; \eta \,/\, 0 \\ \overline{\eta} \Big) \exp \eta / \alpha - 1 & \text{if} \quad \alpha \quad \beta \quad 0; \eta \,/\, 0 \\ \overline{\alpha - \beta} \ln \frac{\alpha}{\beta} & \text{if} \quad \alpha \quad 0; \beta \quad 0; \alpha \,/\, \beta; \eta \quad 0 \\ / \alpha & \text{if} \quad \alpha \quad 0; \beta \quad 0; \alpha \quad \beta; \eta \quad 0 \\ \infty & \text{if} \quad \text{otherwise} \end{array}$$

Proposition 3: Summary Characterization

Γ

(a)

$\beta \quad 0, \ \alpha \leq 0,$

$\eta \quad 0; \quad \beta \quad 0, \ \alpha \quad 0, \quad \eta \quad 0 \quad \beta \quad 0, \ \eta \leq \alpha \quad 0,$

$\theta \ / \ 0$

(b)

$\beta \quad 0 \quad \alpha \quad 0 \quad \beta \quad 0 \ \ \alpha \quad 0 \quad \eta\theta \quad 0 \quad \eta \quad \alpha$

(c)

$\beta \quad 0, \ \alpha \quad 0 \quad \alpha \quad \eta \quad 0 \quad \eta \quad \theta \quad 0$

(d)

Γ

Example: Partnership Dissolution (… Self-Auction)

- Two partners wish to dissolve a partner … ach values at 0 …
- Submit bids simultaneously; high bidde … other partner her b … gain ownership:

— if

,

Example: An Innovation Contest with Spillovers

- Extend Dasgupta's (1986) all-pay auction innovation contest model
  - Each …rm's expenditure on R&D has bene…cial spillover on rival

$$\square \quad , \qquad - \quad -\delta \qquad - \quad -\theta \qquad , \qquad .$$

  - Greater bene…t to winner than loser (δ θ 0)
- This is a Γ with δ θ 0, 0 γ, and α β 1.
- Since $\alpha - \beta$ 0 and η 0, Propositions 2 and 3 imply that the unique symmetric equilibrium is

$$*$$

Example: Varian/Rosenthal Sales Models

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Example: Inequality Aversion in a Job Tournament

- Two workers compete in a winner-take-all fashion for a promotion

Example: Inequality Aversion in a Job Tournament (Continued)

- Propositions 1, 2 and 3 imply that the unique symmetric equilibrium is in mixed strategies and given by

$$ {}^{*} \qquad \frac{1-}{} \left(\exp\left(\frac{}{\mu} \right) - 1 \right) $$

on $\left(0, \frac{\mu}{}\right)$

Example: Loss Aversion in a Job Tournament

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Example: Winner Regret in Auctions

- First price auction with regret (Engelbrecht-Wiggans (1989), Engelbrecht-Wiggans and Katok (2007), and Filiz-Ozbay and Ozbay (2007)):

$$1, \quad 2 \qquad - \quad -\mu \quad - \quad \text{if} \qquad 0 \quad \text{if} \quad .$$

- is player 's bid, 0 the value of the item, and μ 0 a "regret" parameter
- Winner regret refers to the fact that the high bidder derives disutility from leaving money on the table (the di¤erence between the winning and losing bid). The payo¤s may be rewritten as

$$1, \quad 2 \qquad - \quad \mu \quad 1 \quad \mu \quad \text{if} \qquad 0 \quad \text{if}$$

- , α θ 0, β 1 μ 0, δ $-\mu$, and η -1.
- Propositions 1 and 3 imply the unique symmetric equilibrium is

* .

Example: Auctions with Winner and Loser Regret

- Can show both effects arise in this case

$$ {}_{1},\ {}_{2} \quad - \quad \mu \quad 1 \quad \mu \quad \text{if} \qquad -\rho \quad - \quad \text{if} \quad . $$

- This is a Γ with $1 \quad \rho$, $\alpha \quad 1 \quad \rho \quad 0$, $\beta \quad 1 \quad \mu \quad 0$, $\theta \quad -\rho$, $\delta \quad -\mu$, and $\eta \quad 0$.
- When $\rho \ / \ \mu$, Propositions 2 and 3 imply the unique symmetric equilibrium is

$$ * \quad \left(\frac{1 \quad \rho}{\rho - \mu}\right)\left(1 - \exp\left(-\frac{\rho - \mu}{1 \quad \rho}-\right)\right) $$

on $0, \frac{1-\delta}{\delta-\mu} \ln \left(\frac{1 \quad \rho}{1 \quad \mu}\right)$
- When $\rho \quad \mu$, Proposition 2 yields the standard all-pay auction form: $*\quad /$, but $* \quad -\rho \ /2$.

Example: Evolutionary Stationary Strategies (ESS) in the All-Pay Auction

- One can also use Proposition 2 to …nd the unique symmetric ESS equilibrium in the standard two-player all-pay auction
- Finite agent ESS equilibrium of Scha¤er (1988) requires each player maximize di¤erence in payo¤s:

$$1, \ 2 \qquad \begin{cases} \_ \quad \_ \quad \_ & \text{if} \\ \_ \quad \_ \quad \_ & \text{if} \end{cases}$$

- This is a Γ with payo¤s

$$1, \ 2$$

Concluding Remarks