Contests with Rank-Order Spillovers

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- Results apply/extend a variety of models/results in IO, behavioral economics, and game theory
 - Cover all standard contests and auctions, war of attrition, & all-pay auction
 - More exotic auctions and contests
 - Innovation contests with spillovers
 - Pricing games
 - Price matching policies
 - Behavioral economics (inequality aversion, loss aversion, regret, reference pricing)
 - Evolutionary equilibria (ESS)

Model and Notation

- Players: $\in \{1, 2\}$
- Actions (bids, prices, e^{x} ort, etc.): \in 0,
- Payo¤s (coin-‡ip tie-breaking rule suppressed):

- = 0
- : An arbitrary game with this structure.
- = -
- *: Symmetric pure-strategy (Nash) equilibrium
 - * : Symmetric (non-degenerate) mixed-strategy equilibrium

Proposition 1: Characterization of Symmetric Pure-Strategy Equilibria

 $\begin{array}{ll} (i) & \geq 0 \\ (ii) & \leq 0 \end{array}$

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Proposition 2: Characterization of Symmetric Mixed-Strategy Equilibria

(i) 0 0;
(ii) 0, 0 0
(iii) 0, 0 and either 0 or 0.

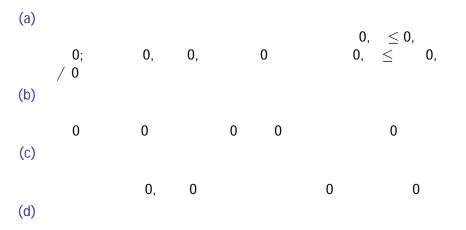
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Proposition 2: Characterization of Symmetric Mixed-Strategy Equilibria (Continued)

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Proposition 3: Summary Characterization



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Example: Partnership Dissolution

- Two partners wish to dissolve a partne
- Submit bids simultaneously; high bidde gain ownership:

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Self-Auction)

ach values at 0. other partner her bi

Example: An Innovation Contest with Spillovers

- Extend Dasgupta's (1986) all-pay auction innovation contest model
 - Each ...rm's expenditure on R&D has bene...cial spillover on rival

- Greater bene...t to winner than loser (0)
 This is a with 0, 0, and 1.
 Since 0 and 0, Propositions 2 and 3 imply that the
- Since 0 and 0, Propositions 2 and 3 imply that the unique symmetric equilibrium is

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Example: Varian/Rosenthal Sales Models

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Example: Inequality Aversion in a Job Tournament

• Two workers compete in a winner-take-all fashion for a promotion

Example: Inequality Aversion in a Job Tournament (Continued)

 Propositions 1, 2 and 3 imply that the unique symmetric equilibrium is in mixed strategies and given by

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$$1 - \int_{\exp} \int_{\mu} \int_{\mu$$

Example: Loss Aversion in a Job Tournament

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Example: Winner Regret in Auctions

 First price auction with regret (Engelbrecht-Wiggans (1989), Engelbrecht-Wiggans and Katok (2007), and Filiz-Ozbay and Ozbay (2007)):

$$- - \mu - \text{if}$$

1, 2 0 if

- is player 's bid, 0 the value of the item, and μ 0 a "regret" parameter
- Winner regret refers to the fact that the high bidder derives disutility from leaving money on the table (the di¤erence between the winning and losing bid). The payo¤s may be rewritten as

$$\begin{array}{ccccc} & -\mu & 1 & \mu & \text{Ir} \\ 1, & 2 & 0 & \text{if} \\ , & 0, & 1 & \mu & 0, & -\mu, \text{ and } -1. \\ \text{Propositions 1 and 3 imply the unique symmetric equilibrium is} \\ * & \cdot \end{array}$$

Example: Auctions with Winner and Loser Regret

- When $/\mu$, Propositions 2 and 3 imply the unique symmetric equilibrium is

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$$\int \frac{1}{-\mu} \int \int_{1-\exp} \int_{-\frac{-\mu}{1}} \int \int_{1-\exp} \int_{1-\frac{-\mu}{1}} \int \int_{1-\exp} \int_{1-\frac{-\mu}{1}} \int \int_{1-\exp} \int_{1-\frac{-\mu}{1}} \int_{1-$$

on $0, \frac{1-}{-\mu} \ln \left\{ \begin{array}{c} 1 \\ 1 \\ \mu \end{array} \right\}$ • When μ , Proposition 2 yields the standard all-pay auction form: * /, but * - /2.

Example: Evolutionary Stationary Strategies (ESS) in the All-Pay Auction

- One can also use Proposition 2 to ...nd the unique symmetric ESS equilibrium in the standard two-player all-pay auction
- Finite agent ESS equilibrium of Scha¤er (1988) requires each player maximize di¤erence in payo¤s:

• This is a with payo^xs

1, 2

Concluding Remarks

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