Collusion and Optimal Reserve Prices in Repeated Procurement Auctions

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Abstract

Previous research on collusion in procurement markets uses static mechanism design theory to address the limitations on collusive activity imposed by asymmetric information, but in most instances it does not address how to enforce the proposed mechanisms. This paper uses repeated game theory to examine the sustainability of two commonly reported collusive schemes that have been identified as optimal static mechanisms. If a buyer does not select its reserve price strategically, or if its value is large relative to the sellers' costs, then collusion may be sustainable for a wide range of plausible discount factors. However, even mildly sophisticated reserve price selection can dramatically shrink the set of discount factors for which collusion can be sustained. These findings provide a rationale for existing arguments that buyers are vulnerable to collusion, but suggest that buyers possess tools that may profitably induce sellers to act competitively. The analysis also reveals that collusion tends to be more easily sustained if the sellers' costs have a low mean or a high variance, or, in some instances, if the number of sellers increases.

1 Introduction

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This is not a trivial concern, because sellers may have incentives either to set prices other than the ones specified by the mechanism, or to not make agreed-upon transfer payments. That is, while truthfully revealing costs is made incentive compatible, abiding by the mechanism's rules is not. For example, suppose that two firms report their costs of 3 and 4 and are told to offer prices of 6 and 7, respectively. The firm with cost 4 has incentives to set its price just below 6 in order to win the contract. As the mechanism design approach is static, it does not address the possibility or prevention of such deviations.

In this paper I use repeated game theory to evaluate the ability of sellers in first-price procurement auctions to enforce adherence to two commonly reported rigid-pricing collusive schemes identified as being optimal stsitit - t dtsdtsnsal tste to ttposit 4.8 y 0 e TD (e) Tj 8.64 0 TD -0.0298 Tc e(s) Tj or below the level that is optimal versus static Nash equilibrium price-setting, the critical discount factor for sustaining collusion is high because the short-term gain from defection is large relative to the per-period loss from rivals' retaliation to that defection. Hence, ¯rms mus have the at

while the Appendix contains all proofs.

2 The One-Shot Procurement Auction

A prospective buyer of a product solicits price o®ers simultaneously from each of N sellers. Prior to the making of o®ers to the buyer, each seller i draws its production cost, c_i, independently from the cumulative distribution $F(c)$. Assume that F has a di®erentiable density f with support $[\underline{c}; \bar{c}]$. The buyer purchases from the low-priced seller at the o®ered price. In auction terminology, this is a symmetric independent private value (IPV) ¯rst-price auction. Assume further that the buyer and sellers all are risk neutral, the number of ¯rms is exogenous, entry is blockaded, and it is costless for sellers both to learn their production cost and to participate in the procurement process. The buyer's next best supply alternative costs c_B $\bar{c}$, and the buyer's pro¯t from purchasing from one of the N sellers at price p is c_B p. Seller i's pro¯t from winning with price p_i is p_i c_i. Prior to the submission of o®ers, the buyer imposes a commonly known reserve price, r, that is less than or equal tor0 TD 0 Tc (o) Tj ETD 0.0182 Tc (e) Tj 82q and its ex ante expected profit in the static Nash equilibrium, when it has not yet drawn its cost, is

$$\pi_S^{NE}(r) = \int_{\underline{c}}^{r} F(c)\,[1 - F(c)]^{N-1}\,dc.$$

Result 1 illustrates that sellers shade their price above their cost by an amount that depends on the reserve price and the number of rivals. As the reserve price increases, the amount of shading and expected profits increase. As the number of rivals increases, the amount of shading and expected profits decrease.

Result 2 In the symmetric IPV one-shot procurement auction, i^{i}

strategies each period is a subgame perfect equilibrium, of greater interest is the characterization of subgame perfect equilibria yielding the sellers higher expected pro¯ts. In what follows, I assume that the buyer does not reveal the winning price, as is common in private sector procurement.[9] Additionally, I assume that the winner in each period is revealed, and that the buyer commits at the outset to a reserve price to be used throughout the game.[10] Subject to the reserve price, the sellers are presumed to select the most collusive outcome possible.

I examine two rigid-pricing collusive schemes in Ilulri.

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) TD 0.0182 Tc (e) TD 0.0182 Tc (e) TD 0.0182 Tc (e) TD 0.0182 50.0182 Tc (b) Tj 6.12 0 (o) Tj 5.52 0 TD 0.0509 Tc prior to the buyer's ¯rst-price auction. They refer to a cartel using this scheme as a **strong cartel**. Strong cartels require more frequent contact than do weak cartels, which increases the likelihood that the antitrust authorities may discover and be able to prove the existence of the agreement. However, this concern is balanced against the fact that the strong cartel ¯nds collusion easier to sustain than does the weak cartel.[11]

Four potential stumbling blocks to successful collusion are the selection of the winning ¯rm, the selection of a price, the detection of deviations, and the punishment of deviations. The schemes used by the weak cartel (WC) and the strong cartel (SC) clearly overcome the ¯rst three. The winner and the price are predetermined, and deviation can be recognized after the current round of o®ers and dealt with in the next period. To punish deviations, I employ perpetual reversion to the static Nash equilibrium. Though there exist more severe punishments, imposing them su®ers from di±culties similar to those preventing e±cient collusion. For example, suppose all ¯rms $i = j$ are supposed to punish ¯rm j in each period by setting $p_i = c_i$. Because no prices are revealed and the sellers' costs are private information, a punishing ¯rm could deviate to a price $\tilde{p}_i > c_i$ without being detected if it lost when it should have won, and earn a positive expected pro¯t. Thus, it seems such a severe punishment could not be implemented credibly. In contrast, by de¯nition Nash reversion can be implemented credibly.

Collusion can be sustained provided that the pro¯t from colluding exceeds the pro¯t from defecting today and facing punishment in the future, for any current cost realization. It is important that the incentive constraints be satis¯ed for all possible cost draws. If they were not, then the positive probability of the designated seller's being undercut by sellers with particular cost realizations will prompt the designated seller to undercut as well. Such a response precludes the existence of an equilibrium in which the selected seller o®ers a price of r. I de¯ne the critical di wqsnsnew deoawqwr ce a public randomization device to select a winner for each contract as it is offered. The designated winner competes unopposed and offers a price equal to the reserve price. The seller that is selected through the randomization process is able to costlessly decline the invitation to be the winning firm, which it will do if its cost exceeds the reserve price. The next seller on the list is then designated to be the winning firm, it also has the option of declining, and so on. The remaining sellers either do not submit price offers, or submit price offers that exceed the reserve price. This is a form of bid rotation, though the designated seller is determined randomly each period rather than selected sequentially from a predetermined ordering of the firms.

Lemma 1 in the Appendix proves that the outcomes using bid rotation are identical in expectation to those using the optimal mechanism, without transfer payments, determined by McAfee and McMillan [1992]. Their mechanism specifies that each seller with a cost less than the reserve price submits a price equal to the reserve price. Other sellers either do not bid or set a price equal to their cost.[12] However, for several reasons the bid rotation scheme I have constructed may be easier to implement than the optimal mechanism. First, while bid rotation does not require that the winning price be revealed, enforcing the optimal mechanism is difficult unless the buyer announces the winning price; otherwise, sellers can deviate by slightly undercutting the reserve price. Such a deviation cannot be detected immediately and dealt with in the next period, which impedes collusion. Second, bid rotation can be helpful in avoiding investigation by the antitrust authorities, as opposed to other schemes, such as the optimal mechanism, that have several firms submitting the same price offer.[13] Third, bid rotation is immune to the buyer's disrupting the scheme, say by using a non-random tie-breaking procedure.[14]

For a weak cartel to sustain supracompetitiveaimm r mm prnon p m edutt 82 uo c(e) se Tj 8.4 0 TD -0.0e

¼$_S^{WC}$(r) denotes a seller's ex ante expected pro¯t per-period from participating in the weak cartel when the reserve price is r.[15] The left hand side of (1) is the net present value of future expected collusive pro¯t, while the right hand side is the short-term gain from defecting plus the net present value of future expected sta e ec¯e.0077 Tc (r) Tj 4.32 0 TD 0.0254 Tc (t) Tj2D 0.043

.1 TD -0.52 0 TD -080 0.0254 2 Tc (e) Tj 1 Tc (d) D -0.0298 Tc (s) Tj fraction of the cartel's surplus, $\frac{r-T(c)}{N-1}$, and receives $T(c)$, where

$$T(c) = r - \frac{\int_c^r (r-s)(N-1)[1-F(s)]^{N-1} f(s)ds}{[1-F(c)]^N}:$$

Thus, a firm can receive a payment even if its cost exceeds the reserve price. The side payment scheme can be implemented by the sellers' holding their own first-price auction prior to the buyer's first-price auction, which Graham and Marshall [1987] refer to as a pre-auction knockout (PAKT). I assume that the strong cartel attempts to collude in this fashion, with the seller winning the PAKT making transfer payments to the losing sellers before the buyer's procuremen [illegible]

Note that the side payment does not come into play in (3), beet ce

that its cost is less than the reserve price. Thus, the optimal reserve price versus such collusive price-setting is the same regardless of the collusive scheme.

Result 3 Facing the collusive scheme X in which a single seller offers the reserve price, the buyer pays the reserve price if at least one seller's cost is below the reserve price. Thus, the buyer's expected profit as a function of the reserve price, r, is

$$\pi_B^X(r) = (c_B - r)\left(1 - [1 - F(r)]^N\right):$$

The buyer's optimal reserve price, r^X, solves

$$c_B = r^X + \frac{1 - \left[1 - F(r^X)\right]^N}{N[1 - F(r^X)]^{N-1} f(r^X)};$$

provided that $r^X \leq \bar{c}$. Otherwise, $r^X = \bar{c}$ and the reserve price is non-binding. If $N \geq 2$, then the reserve price is always binding. Moreover, the optimal reserve price weakly decreases as N increases.

When the N sellers collude by having the firm selected to win set its price at the reserve price, the buyer essentially is facing a single seller whose costs are drawn from the distribution $G(r) = 1 - [1 - F(r)]^N$.

that would be set if the buyer were unsophisticated, could not credibly impose a lower reserve price, or simply had a large value relative to the sellers' costs.[20] Each of these reserve prices determines a critical discount factor that the sellers' discount factor must exceed in order for collusion to be sustainable.

Third, the minimal deterrent reserve price is the lowest reserve price for which the buyer's expected pro¯t versus static Nash equilibrium price-setting, using that reserve price, is equal toerpec rc fpltc e correspond to discount factors that ¯rms might actually use to value future pro¯ts. Second, it is useful to know how the critical discount factor changes as a function of the primitives of the strategic environment. For example, if there exist conditions under which collusion is more easily sustained as the number of sellers increases, then under those conditions the buyer may wish to restrict the number of sellers from which it accepts price o®ers. Similarly, if collusion is more easily sustained as the variance of the sellers' costs falls, then there may exist incentives for the sellers to adopt production technologies that reduce that variance.

Researchers have turned to computation to obtain bothic

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distributions. The mean and the variance uniquely determine the two Beta parameters, and hence uniquely determine the distribution. One can show that in order for the Beta parameters to be strictly positive, the mean and the variance must satisfy $\mu(1 - \mu) - \sigma^2 > 0$. Moreover, I further restrict the mean and variance so that the Beta distribution is lo in the critical discount factors with the Nash reserve price strategy than with the naive reserve price stra

reserve price, which is unchanged as μ changes, indicate that the result is largely due to the increase in the mean cost causing a decrease in the difference between the per-period cooperative and noncooperative payoffs. With the exception of a weak cartel facing the naive reserve price strategy, the results in Table 2 provide even stronger evidence that collusion typically becomes easier to sustain as the variance of the sellers' costs increases. This result is caused by increasing the difference between the per-period cooperative and noncooperative payoffs at σ^2 increases. With no a priori reasoning about how the difference between the per-period cooperative and noncooperative payoffs changes as the mean or the variance change, the ivt tbeac e g tbta TD (c) TD 0.0182 Tc (c) Tj 4.92 0 rate, then the one period discount factor is $\pm = \frac{1}{(1+R)^t}$. If contracts are o®ered more than once per year, then t is less than one.[25]

Table 3 presents the per-period discount factors associated with four hurdle rates over six di®erent time periods, as measured by the number of contracts o®ered per year. For hurdle rates of 15 percent and higher, the resulting discount factors are below those reported in Table 4 as being necessary for successful collusion by a weak cartel versus a sophisticated buyer, even with a small number of sellers. With a 10 percent hurdle rate, the s (m) Tj 5.76 0 TD j ot2 0h TD 0.0254 Tc (1) Tj (4)

This paper uses tools from repeated game theory to examine the ability of sellers in repeated ¯rst-price procurement auctions to sustain two collusive bidding schemes that are optimal from a static mechanism design perspective. Speci¯cally, for di®erent numbers of ¯rms, di®erent reserve price strategies, and a large number of cost distributions, I numerically determine the value that ¯rms must place on future pro¯ts such that abiding by the collusive scheme is more pro¯table than is defecting from it and inciting retaliation. Computing the critical discount factors necessary for collusion to be sustainable illuminates the analytically complex e®ects of the various parameters of the strategic environment. Moreover, relating the computed discount factors to plausible real-world discount factors helps one to assess the practical relevance of the static mechanism design approach to collusion in auctions.

The computational results reveal that the buyer's choice of the reserve price has a large impact on the sellers' ability to collude. If the buyer is sophisticated in its choice of the reserve price, then collusion tends be sustainable only for extremely high discount factors that correspond to what appear to be unreasonably low hurdle rates within the ¯rm. Successful collusion requires the sellers to place such high value on future pro¯ts because the short-term gain from cheating on the collusive agreement is large relative to the per-period loss of collusive pro¯ts. Thus, sellers must value those foregone pro¯ts highly in order to resist their temptation to cheat. The necessity of such extremely high discount factors suggests it may be unlikely that tacit collusion using the previously identi¯ed bidding schemes can be supported as a subgame perfect equilibrium of the repeated game. This conclusion is consistent with the assertion by Graham and Marshall [1987] that cartels in ¯rst-price auctions in the UniteumTD -0.0182 Tc (n) Tj 6 0 TD 0.0182 Tc4.8 0 TD -0.0182 Tc (TD -0.0182 Tc (n) Tj 6

The strong conclusion that the probability of rigid-price collusion is low versus strategic buyers in an independent private values environment raises the issue of how the many reported instances of collusion in auction and procurement markets were supported. Relaxing three assumptions used in the present analysis generates three possible explanations of how collusion might more readily be sustained. First, buyers may not be very sophisticated in their reserve price selection, they may not be able to credibly commit to the reserve prices necessary to severely limit collusion, or they may have large values relative to the sellerso ooleleer n oleeiololarrteoulaolr h8f0.0 TD 0.0 Tc (o) Tj 5.52 0 TD 0.

conditional on having cost less than r

As shown

[3] Comanor, W., and Schankerman, M., \Identical Bids and Cartel Behavior." Bell Journal of Economics, Vol. 7 (1976), pp. 281-286.

[4] Cramton, P., and Palfrey, T., \Cartel Enforcement with Uncertainty About Costs." InteJouEcnuc ppJ,n 4Pna

Table 1
Regression Results by Cartel Type and Reserve Price Strategy for

$$\delta = \alpha_0 + \alpha_1 N + \alpha_2 \mu + \alpha_3 \sigma^2 + \alpha_4 N^2 + \alpha_5 N\mu + \alpha_6 N\sigma^2 + \alpha_7 \mu^2 + \alpha_8 \mu\sigma^2 + \alpha_9 \sigma^4 + \alpha_{10} N^3 + \alpha_{11} N^4$$

	Weak Cartel Naive RP	Weak Cartel Nash RP	Weak Cartel Deterrent RP	Strong Cartel Naive RP	Strong Cartel Nash RP	Strong Cartel Deterrent RP
Intercept	0.69379149	0.86431262	0.91699998	0.68188789	0.84017995	0.89139666
Number of Firms	0.03704046	0.00248617	0.00267516	0.03874445	-0.00017688	0.00342790
Mean	0.21043461	0.29331887	0.26129209	0.17315251	0.25770755	0.29180795
Variance	0.24967026	-0.40937132	-0.51774748	-0.16285821	-0.42003522	-0.57022403
Number of Firms^2	-0.00250669	0.00036730	-0.00000426	-0.00253982	0.00085835	0.00003237
Number of Firms*Mean	-0.00792688	-0.00569608	-0.00300236	-0.00711913	-0.00671399	-0.00378106
Number of Firms*Variance	-0.01335749	-0.00272237	-0.00146551	0.00159331	-0.00248975	-0.00237113
Mean^2	0.00772701	-0.15093804	-0.19797418	0.02759866	-0.07577537	-0.20118931
Mean*Variance	0.33454753	0.38034936	0.46419345	-0.18393295	0.03053952	0.47785869
Variance^2	-1.12080580	0.67125324	1.45184525	0.84536447	2.30518151	1.67026818
Number of Firms^3	0.00008605	-0.00002024	-0.00000176	0.00008394	-0.00004206	-0.00000427
Number of Firms^4	-0.00000109	0.00000032	0.00000004	-0.00000103	0.00000063	0.00000008
Data Points	13,862	13,862	13,862	13,862	13,862	13,862
Adjusted R^2	0.94958884	0.90662891	0.78330459	0.96408683	0.93118992	0.87697972
Adjusted R^2 (1^{st} order only)	0.67843531	0.68311733	0.48413183	0.71895946	0.75560037	0.61968881

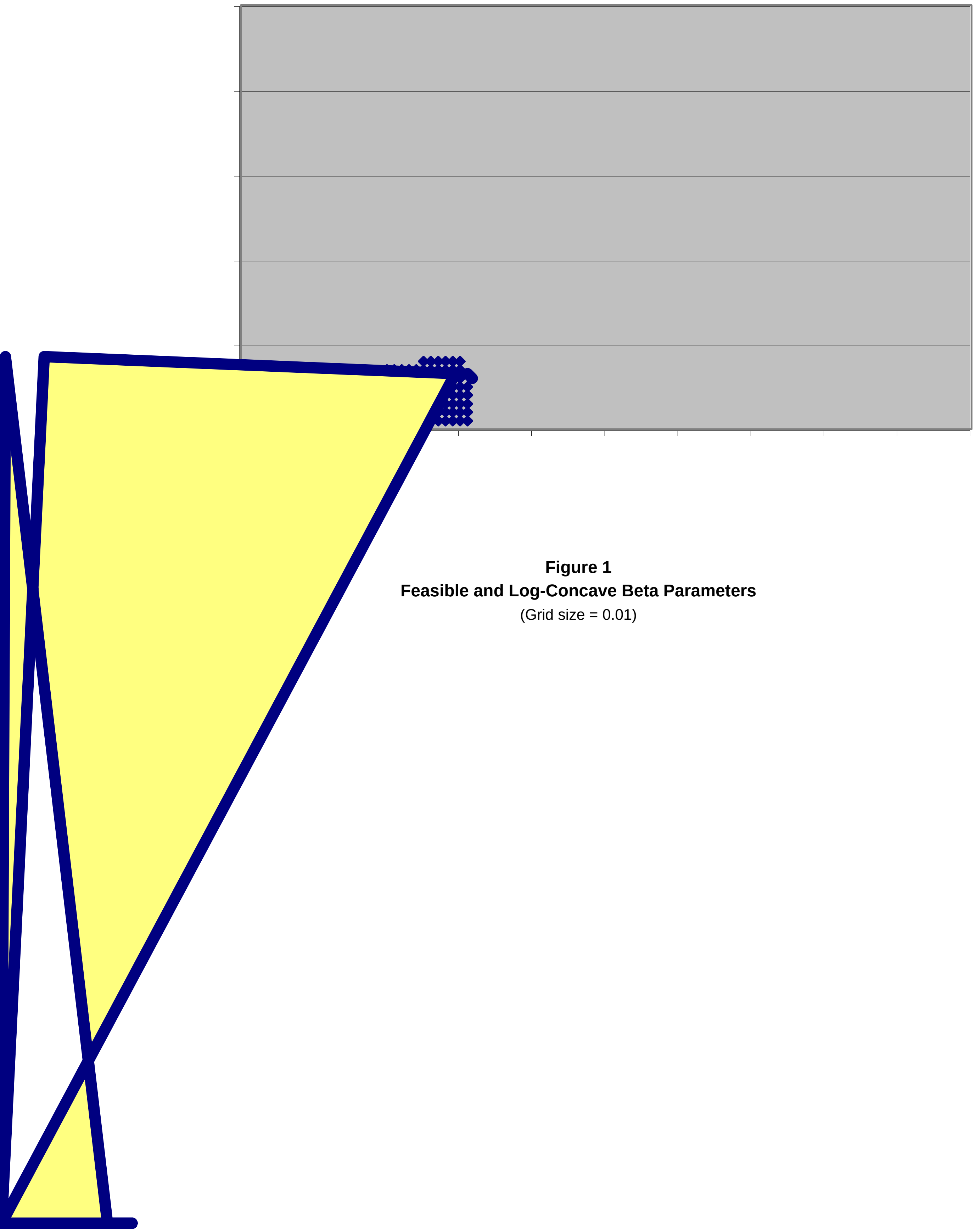

Figure 1
Feasible and Log-Concave Beta Parameters
(Grid size = 0.01)

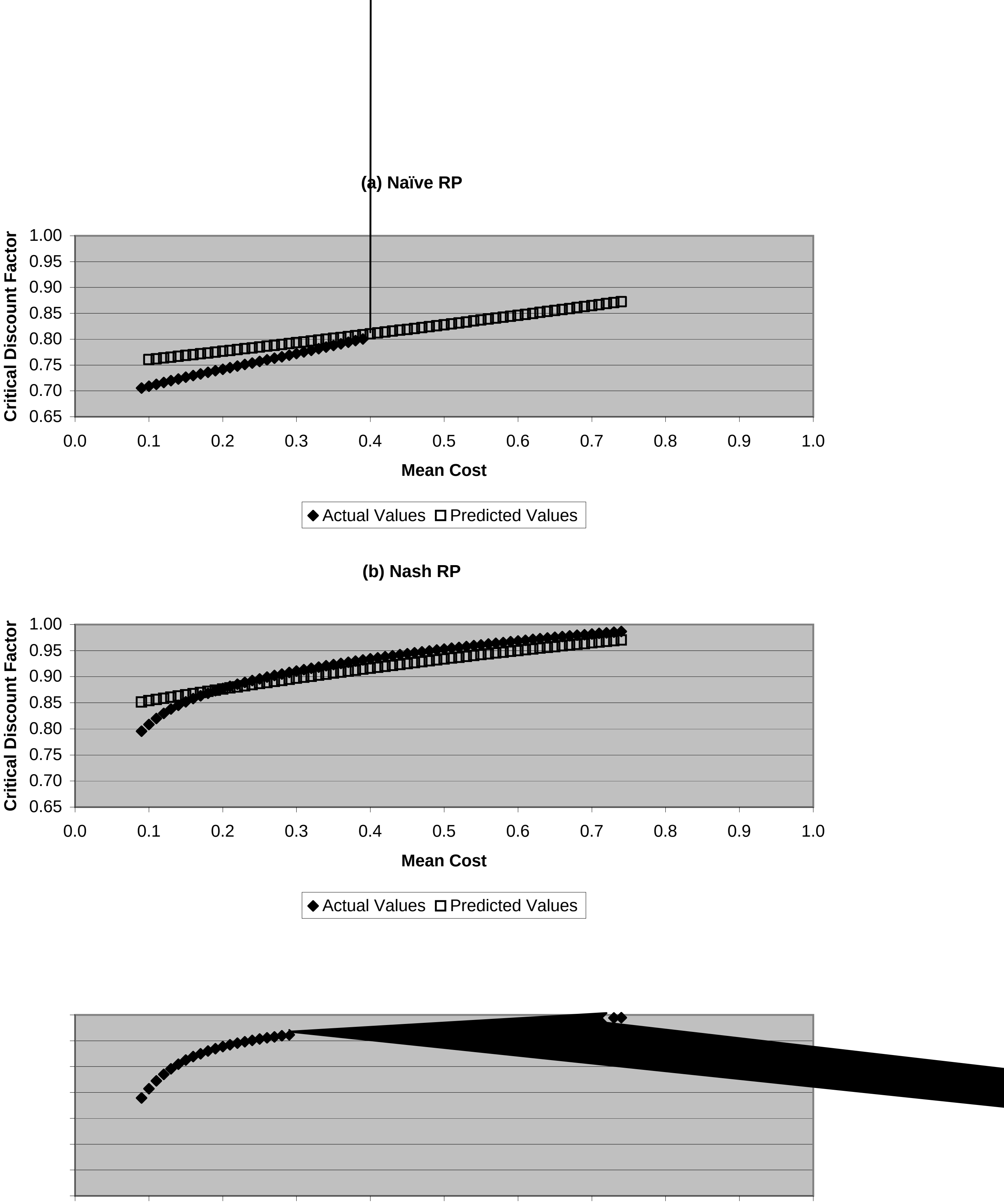

Figure 2
Critical Discount Factors for Strong Duopoly Cartel with σ^2 at Average Level

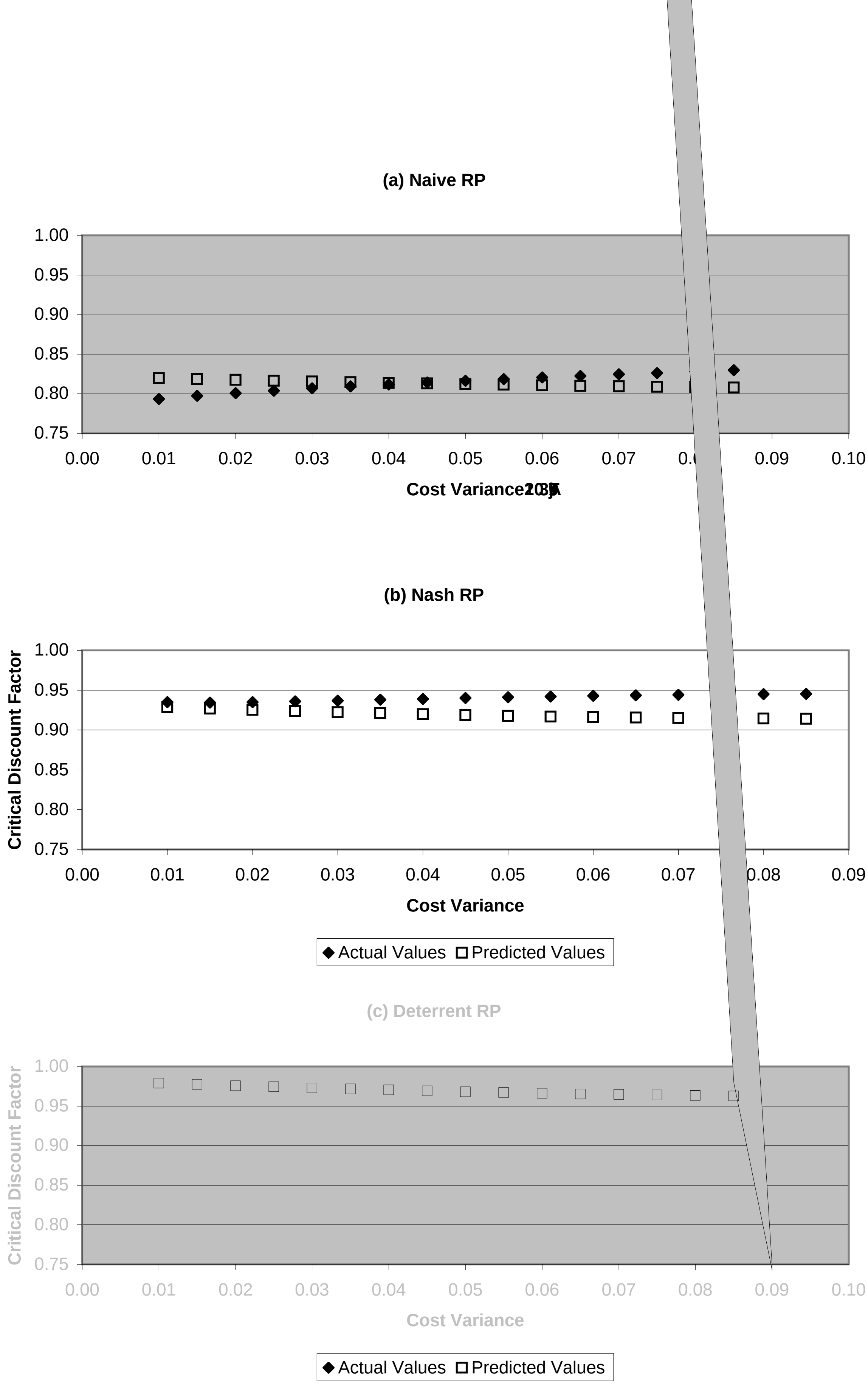

Figure 3
Critical Discount Factors for Strong Duopoly Cartel with μ at Average Level

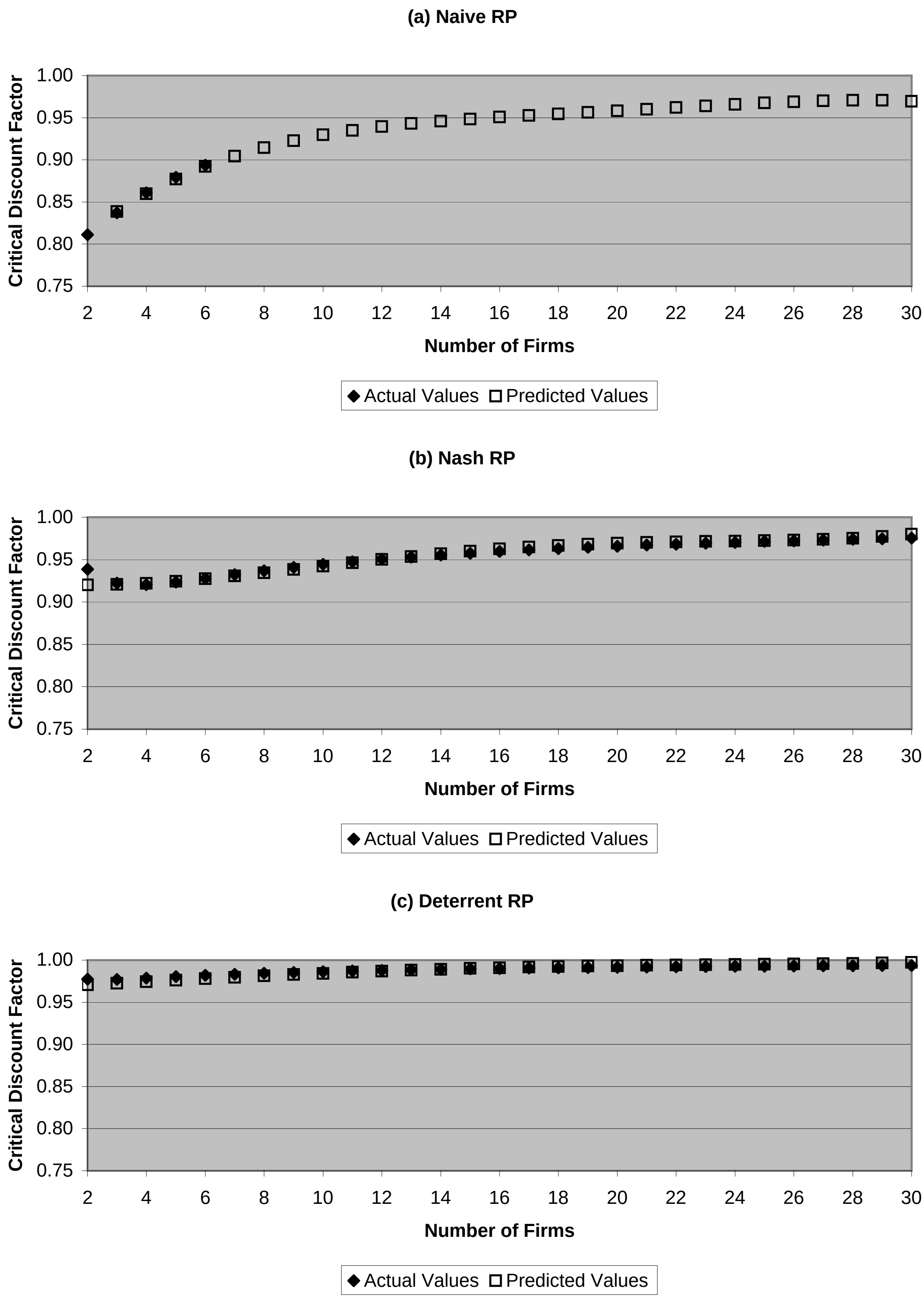

Figure 4

Critical Discount Factors for Strong Cartel with μ and σ^2 at Average Levels