



Competition, Contracts, and Innovation

Christopher J. Metcalf

John D. Simpson

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Christopher J. Metcalf and John D. Simpson[|]

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Abstract

Our paper contributes to the literature on the relationship between innovation and market power by considering how changes in the intensity of product market competition a¤ect innovation when managerial compensation is a linear function of ...rm pro...ts. Changes in the intensity of product market competition a pect both the return from innovation and the cost of inducing managers to innovate. Several recent papers account for both the returns-to-investment exect and the agency-cost exect in analyzing the exect of additional product market competition on incentives to innovate (see e.g., Schmidt (1997), Raith (2003), and Piccolo, DAmato, and Martina (2008)). Our model dipers from these papers in the type of contract that we assume ... rms can use to induce innovation. With linear pro...tsharing contracts, the cost of a non-drastic innovation declines as product market competition increases because the increment gained from innovation becomes a larger fraction of the total pro...t. We argue that this decline in the cost of attaining innovation as competition increases means that competition will often lead to more innovation even in models where the returns to innovation otherwise would fall as competition increases.

1 Introduction and Literature Review

Does market power facilitate innovation? If so, under what circumstances should competition policy tolerate the short-term allocative ine¢ ciency associated with market power in order to obtain higher levels of innovation? These two questions have been a

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^IBureau of Economics, Federal Trade Commission, 600 Pennsylvania Ave NW, Washington, DC 20580. E-mail: cmetcalf@ftc.gov and jsimpson@ftc.gov. Contact author: Metcalf.

central issue in Industrial Organization for 50 years. While simple answers have proved elusive, numerous papers have made signi...cant progress toward addressing these questions. Our paper seeks to add to this stock of knowledge by considering how product market competition a¤ects innovation when managerial compensation is a linear function of ...rm pro...ts.

Changes in the intensity of product market competition a^{pect} both the return from innovation and the cost of inducing managers to innovate. An extensive literature examines the exect of product market competition on the return from innovation and ...nds that greater product market innovation can lead to either increased innovation or decreased innovation depending on the assumptions made about factors such as the nature of competition before and after the innovation, whether the innovation can be readily copied, and whether rivals can also innovate. (see e.g., Arrow (1962); Tirole (1988); Schumpeter (1947); Qiu (1997); Vives (2008)). Another set of paper examines the exect of product market competition on the cost of inducing managerial exort and ...nds that increased competition, measured as the number of entrepreneurial ...rms (i.e., without agency problems), either reduces agency problems or increases agency problems depending on the agents utility function (e.g., Oliver Hart (1983) and Scharfstein (1988)). Finally, several recent papers account for both the returns-to-investment exect and agency-cost exect in analyzing the exect of additional product market competition on incentives to innovate (see e.g., Schmidt (1997), Raith (2003), and Piccolo, D'Amato, and Martina (2008)).

2 Model

We consider a three stage model: In stage one, each of two symmetric ...rms simul-

Speci...cally, we assume that each ...rrhwith competitor m

$$d_{V|p} Ef >> \{ > f + \{ E 2 \ f^2 \} \{ 2 \ f^2 \ n \ 2 \ E \bullet f \ 2 \ f \ f^2 \} \}$$

Let us ...rst focus only on the case where it is the unique pure strategy equilibrium

Figure 1: Baseline Example: $d_{v|p}$ and $v_{|p}$ for E> f > { '•>_e^*>_{2f}^*

As an aside, let us now briety consider the parameter region where it is a pure strategy equilibrium for only one owner to invest. Proposition 1 shows that an owners additional pro...t from having a lower marginal cost ...rst decreases and then increases as competition (f) increases. Combining this result with the result for the case where $d_{v|p} A = v_{v|p}$, shows that the form of the both owners invest, and recalling that equilibrium can change in a number of ways as f increases. Speci...cally, depending on the level of j (the cost of innovating), the relationship between competition and investment may result in the following: (1) no innovation for any level of competi $d_{v|p} \not = f$), (2) no innovation followed by innovation by only one ...rm tion (j A $d_{v|p} E f$? j? $d_{v|p} E f$), (3) innovation by both ...rms, followed by innovation by one ...rm, followed by no innovation, followed by innovation by only one ...rm $(4 ? _{dv|p}? j ? _{dv|p}E$, (4) innovation by both ...rms followed by innovation by only one ...rm ($_{v|p} \not\in f ? j ? 4 ? _{dv|p}$, (5) innovation by both ...rms for all levels of competition (j ? $v_{|p}$ Ef). Note that if innovation is costless, j ' f, then

it is the dominant strategy for both ...rms to invest.

These results arise in large part because of the lumpiness of innovation investment. This lumpiness gives rise to asymmetric outcomes in a symmetric situation, and this asymmetry in turn leads to the increase in investment after the initial decrease. If investment was not lumpy, then as the return to investment falls with increased competition (given that the opponent is also investing) both ...rms would reduce investment leading potentially to a symmetric equilibrium. Since investment is lumpy, as the re-

$$e_{v|p} A j$$
(7)

Since $d_{v|p} A_{v|p}$, equation 7 is the binding condition in terms of de...ning the region of *f* such that innovating is a dominant strategy for managers of both ...rms. De...ne *f* $E_{j} > f > d_{s}$ the implicit function de...ned by: $v_{|p} E_{j} > f > f > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > f > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$. The implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p} E_{j} > d_{s}$ distributes the implicit function de...ned by: $v_{|p}$

Proposition 2 For any e5 E frightarrow, f Ee > j? f Ej.

Proof. From proposition 1, $v|_{p} A f$, $\frac{C v|_{p}}{Cf}$? f, and f? e?•. $\frac{C v|_{p}}{Cf}$? f implies that this implicit function is well-de...ned. If *f* is such that e $v|_{p}$ ' j, it must be that $v|_{p} A j =$

A second result is that an owner must pay a manager a larger share of ...rm pro...t in order to obtain innovation as competition increases. However, because the ...rm's overall pro...t decreases as competition increases, the total amount paid to managers decreases as competition increases. Denote_{|p} as the minimum ethat will induce the agent to innovate given j and the other ...rm investing, i.e., $e_{v|p}$ ' $\frac{j}{v|p}$.⁹

Proposition 3 For the case where both rivals invest, whenj ? v | p, the fraction of pro...ts needed to induce the manager to invest increases as competition increases, CGyá • Æ žA • a Uò'g t#†P58.957 0 m 26.735 0 I S Q 3.006A6 7.97 Tf 42.196 -18.043 92.89.3739.2479er83955 Tf 19.697 1.794 T2Td 4j /1 /TT7 A

Proof. See Appendix.

Denote $e_{d_{v|p}}$ as the minimum ethat will induce the agent to innovate given j and the other ...rm not investing, i.e., $e_{d_{v|p}}$ ' $\frac{j}{d_{v|p}}$.¹⁰ Since $d_{v|p} A = \frac{1}{v|p}$, it is the case that $e_{v|p} A = \frac{1}{v|p}$

Since the principal will not pay an excess sum to induce an action, the only pure strategy equilibria of stage 1 are $\text{Ee}_{v|p} > \sqrt{e_p}$, $\text{Ee}_{dv|p} > f$, $\text{E} \$_{de_{v|p}}$, and E \$ f (See Table In stage 2, the choices of from the stage 1 determine the equilibrium investments. If the principals choose $E_{v|p} > e_{p}$, then investing is a dominant strategy for both agents, and thus $E > \{$ is the unique stage 2 Nash Equilibrium. If the principals choose $\text{Ee}_{v \mid p} > _{d} \text{e}_{v \mid p}$ then investing remains the dominant strategy for the agent of ...rm However, since the agent for ... rm 2 will only invest when the agent for ... rm 1 1. declines to invest, the unique stage 2 Nash Equilibrium is now E xf. Therefore, an equilibrium is not formed by $E_{v|p} > e_{v|p}$ because principal 2 would unilaterally deviate to $Ee_{v|p} > f$ since $_2 E \clubsuit \{ A E \bullet e_{dv|p} \ _2 E \clubsuit \{ similarly for Ee_{dv|p} > \sqrt{e_p} \}$. If the principals choose $\mathbf{E}_{v|p} \neq \mathbf{f}$ then investing is a dominant strategy for the agent of ...rm 1 while not investing is a dominant strategy for the agent of ...rm 2. Thus, the unique stage 2 Nash Equilibrium is $E_{x/p} \neq 1$. However, an equilibrium is not formed by $E_{x/p} \neq 1$ $E \bullet e_{v|p} \bullet E \clubsuit$ { similarly for $E \clubsuit_{vp}$. If the principals choose $E_{dv|p} > e_{vp}$ then, there are two pure strategy Nash equilibria: $E \neq and E \neq \{$ Therefore, an equilibrium is not formed by $\operatorname{Ee}_{d_{V|P}} > d_{e_{V|P}}$ (with a pure strategy equilibrium in stage 2) because the principal whose agent does not invest in stage 2 would unilaterally deviate fromed v | p to 0 since $|E \$ \{ A E \bullet e_{d \vee | p} | E \$ \{ = If the principals choose E \$ f then for both$ agents not investing is a dominant strategy, thus the unique Nash equilibrium is E ff. Neither $e_{r|p}$ nor $e_{dv|p}$ is ever a dominant strategy in stage 1 because in response to

¹⁰ $e_{dv|p} @ \frac{j}{\{+5, f^5, ++, f_5, +5+, f_5, +4, f_5, +5+, f_5, +4, f_7, +4, f_7, +5\}}$

and $e_{v|p}$

competition leads to greater innovation is most extensive where cost is relatively low and where innovation leads to relatively small cost reductions.

Proposition 4 There is $j_{v|p} E > f > \{ >splich that if j ? j_{v|p} then the unique (pure strategy) equilibrium is for both principals to induce the agents to invest by setting e ' <math>e_{v|p}$ and if j A $j_{v|p}$ then both principals inducing investment is not an equilibrium. There is an $\{$ and a f E_{i} such that if $\{$? $\{$ and f ? f E_{i} , then there are $f_{O} E_{i} > \{$ and $f_{X} E_{i} > \{$ such that $\frac{C_{i}}{C_{f}}$ A f if f_{O} ? f ? f_{i} otherwise $\frac{C_{i}}{C_{f}}$ f.¹² Finally, $\frac{C_{i}}{C_{i}}$ ' f, $\frac{C_{i}}{C_{i}}$ A f, $\frac{C_{i}}{C_{i}}$ A f, $\frac{C_{i}}{C_{i}}$? f, and $\frac{C_{i}}{C_{i}}$? f.

Proof. See Appendix.

¹²For instance at $f @ \frac{4}{5}$, then $\frac{C_{\frac{|f|}{2}}}{C_f} A$ 3if $\{?, \frac{4 < 3}{84} and f?, 4, \frac{65}{4 < 3}, 4\}$

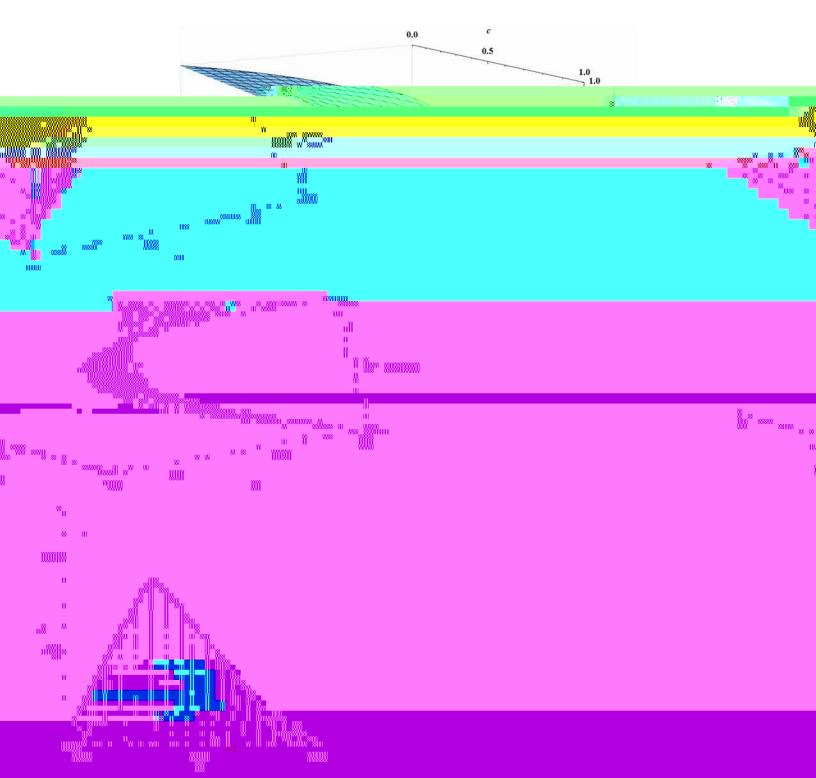
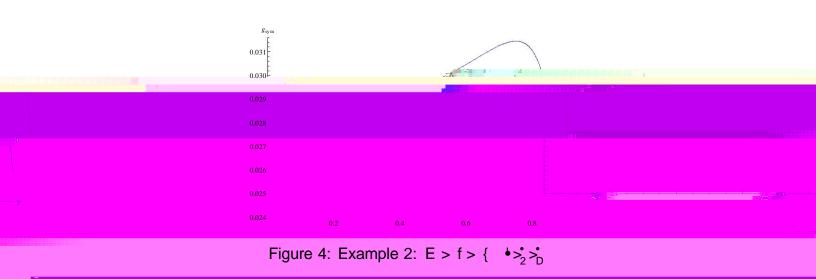


Figure 2: The Investment Increases with Competition Intensity Region: For each $\mathbb{H} > \{$ the upper bound and lower bound are shown for the region of f for which as f increases, the extent of the investment region increases.

' • > f' $\frac{1}{2}$ > {' $\frac{1}{D}$ thus, an innovation would reduce marginal cost by 40 percent. In this example, $j_{v|p}$ declines modestly when the substitution parameter is low, increases substantially when the substitution parameter has moderate values, and then

)1(l)6(y)-24320)**f@l(s)}T**arj>B/462ven(a)`gh(h}vel(ana(t)ByeT934).21(s))B8(T6(20),27&a&d))@(6)7((s))O(()p7(t1)9(e)B)(1)2(l320tr).eh}s-:



[®]w^{ll}w^{lln} n=^ww^{ll}wⁿⁿ

In our model, the relationship between product market competition and innovation changes dramatically when we add the assumption that ...rms can only use pro...t-sharing contracts to motivate managers to innovate: Absent agency problems, the level of innovation declines monotonically with greater product market competition; when ...rms must use pro...t-sharing contracts to solve agency problems, the level of innovation initially declines slightly, then rises over a broad region, and then falls sharply.

The intuition for this result can be seen by combining the equality formed by condition 8 and e incremental pro...t to the total pro...t because the owner will pay a fraction of the total but the size of the fraction depends on the size of the increment relative toj.¹³

The change of the boundary of the investment region, $\frac{C_{ij|p}}{C_f}$, can then be written as in equation 12 (since $\frac{C_{ij|p}}{C_f}$? f and $\frac{C_{ij}}{C_f}$ A f from Proposition 3).¹⁴

$$\frac{C_{\downarrow|p}}{C_f} - \frac{C_{\vee|p}}{C_f} - \frac{C_{\vee|p}}{C_f} - \frac{C_{\vee|p}}{C_f}$$
(12)

The maximum cost where both ...rms innovate decreases with competition if and only if $\frac{C}{Cf} = U$ is greater than $\frac{C}{Cf} = V|_{P}$. In other words, if the expect of the decrease in the incremental pro...t is greater than the expect of the decrease in the cost of investing multiplier (relative to the levels) then investment decreases with competition. This can be understood as the decrease in the return to innovation dominating the decrease in the cost of inducing innovation. The maximum cost where both invest increases with competition if $\frac{C}{Cf} = V|_{P} A \frac{C}{Cf} = V|_{P} U$. This can be understood as the decrease in the cost of inducing innovation dominating the decrease in the return to innovation. Therefore, the non-monotonic relationship between product market competition and innovation results from one exect being dominant over the other for some ranges of competitive intensity but not all.

¹³Formally, the cost can be divided into two parts, a fraction of the incremental pro...t and a fraction of the baseline pro...t. The fraction of the incremental pro...t must equal to induce innovation, which determines $e_{r|p}$. This same fraction of the baseline pro...t also becomes part of the cost. Therefore for a given level of incremental pro...t, if the baseline pro...t decreases then the cost decreases because the necessary fraction stayed the same but the total payment decreased. Alternatively, for a given level of total pro...t, as more of the pro...t is shifted into the increment from the baseline, the necessary fraction of the total decreases sincej stayed the same but decreases (because the total pro...t stayed the same).ded [(1)-69(3)-36<0070>]TJ /233.8649r

5 Generalization for Pro...t Functions

This section considers the generality of our results by deriving conditions for general (stage 2) pro...t functions under which the incentive exects driving our results hold. First, the cost of innovation, $e_{r|p} \in R > \{$ can be divided into two parts: (1) the sat-

$$\frac{C \operatorname{GE} \{> \binom{C \operatorname{RE} \{>}{C f} n \frac{C \operatorname{GE} \{>}{C f} @ \operatorname{GE} \{>\}}{C f} @ \operatorname{GE} \{> \{A \frac{C \operatorname{GE} \{>}{C \operatorname{R}} \frac{C \operatorname{RE} \{>}{C f} n \frac{C \operatorname{GE} \{>}{C f} @ \operatorname{GE} \}}{C f} @ \operatorname{GE} \} \{$$
(14)

If the cost of innovation is decreasing, then overall innovation increases with competition if either the return to innovation increases with competition or if the cost of innovation falls more quickly than the return. In a model such as Singh-Vives (1984), where the return to innovation increases with competition, the cost reducing exect of pro...t sharing contracts simply ampli...es the increasing return to innovation. In a Schumpeterian model where the return to innovation falls with competition (such as the one we consider), the rates of change of the return and the cost must be compared.

The cost decreases more quickly than the return to innovation asf increases if and only if 15 holds at a given j.

$$j = \frac{\mathbb{E} > \{ C \in E \neq \{ @ C f \in E \neq \{ C \in \mathbb{E} > \{ @ C f \in \mathbb{E} > \{ @ C f \in \mathbb{E} > \{ C \in \mathbb{E} > \{ C \in \mathbb{E} \neq \{ C \in \mathbb{E} \} \} \} \} \} \} \} \} \} \} \} \} \} \}$$

The extent of the both invest region expands with *f* if there is an increase in the highest j such that $v_{|p} \in I > \{$ The highest such j is $j_{v|p}$ de...ned earlier, and then condition 15 yields condition 16. More stringent than Condition 13, Condition 16 requires that the proportional exect on pro...t when both invest be less negative than a proportional exect on the laggard, where the laggard exect is less negative than in 13 since it is proportional to the average of the two pro...ts^{1.8} Intuitively, this condition requires that the cost of inducing innovation not only decreases as in Condition 13 but

 $G_{I} + \$$ f. $\{_{I}, \frac{CG}{C\$} @ 3, \text{ then } C_{I} + \{>f_{m}, @ C @ + s f. \}_{I}, \frac{CG}{C\$} \frac{C\$}{Cf} \cdot \frac{CG}{Cf}$. Finally, the price-cost margins cancel.

¹⁸Condition 16 is can be r1(n)16 n¤75(s)(/TT29.96)8(.)]TJ /TTn 16¤ Tf 75(s)(/TT29.96)8(d)-1(i m (king(d)-1(ih.963

with a high enough rate.¹⁹

$$\frac{C \quad E > \{@ \quad C \quad E \neq \{@ \quad C \quad f \\ \hline E \quad E > \{n \quad E \neq \{@ \quad C \quad f \\ \hline E \quad E > \{n \quad E \neq \{@ \quad 2 \end{cases}}$$
(16)

6 Conclusion

This paper analyzes the exect of greater product market competition on innovation using a model that makes two key assumptions. First, the model assumes that two symmetric ...rms, both of which can innovate, compete as dimerentiated Bertrand competitors and face a linear quadratic demand function. With this assumption, a ... rms bene...t from making a non-drastic innovation declines as product market competition increases. Second, the model assumes that a ... rm can only incentivize managers to innovate by opering them a ... xed share of pro...ts. With this assumption, a ... rms cost of investing in a non-drastic innovation declines as product market competition increases. The overall expect of increasing product market competition in this model depends on whether the reduction-in-bene...t exect or the reduction-in-cost-of-innovating exect dominates. In our paper, we ... nd that the reduction-in-bene...t exect dominates where product market competition is either very low or very high, however, the reduction-incost exect dominates for intermediate levels of product market competition. Based on these results and our general analysis, the argument that ... rms with market power are more innovative is weaker once one accounts for one plausible cost of innovating.

Of course, our ...nding that ...rms with substantial market power have a high cost of innovating is premised on the assumption that ...rms can only incentivize managers by o¤ering them linear pro...t sharing contracts. Thus, at this point, it is useful to revisit this assumption. As noted earlier, economic theory suggests that a linear pro...t-sharing

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contract may perform well in some circumstances, and empirical work suggests that the incentive contracts between ...rms and top management often have a signi...cant linear pro...t-sharing component. On the other hand, if innovation is very important, a monopoly ...rm would have an incentive to use some other type of contract because it does not want to o¤er managers a signi...cant share of pre-existing pro...ts to gain a comparatively small incremental pro...t. However, these other types of incentive contracts are likely to be costly because contracts that target one particular goal can harm a ...rm by diverting attention from other important goals. Hence, irrespective of the incentive contract a monopoly ...rm uses, such a ...rm may ...nd it costlier to induce innovation than would a competitive ...rm.

Put di¤erently, several treatments of agency problems within …rms note that agency problems are eliminated if the manager can own the …rm. Compared to a monopoly …rm, it is much less expensive for a competitive …rm to get a manager part of the way co(m)17(s33@(a)-329)-375(t)8(o8(t)-262(30314(m)17(an6(l)-359)-272(s)8(e)9333815(o)10(m(r)12(t)8(i)6(n)1t)8(o frictions related to the costliness of incentivizing employees are directly related to the ratio of the incremental pro...t from an innovation to the total pro...t-a friction that would be smaller for a smaller ...rm or an up-and-coming ...rm.

Firm 1 Pro…t		Firm 2 pro…t	
EL > L	$L > L \frac{2E \cdot f E \cdot fn}{E \cdot f E 2f^{2}}$	$L > L \frac{^{2}E \cdot f E \cdot fn\{^{2}}{E \cdot n fE 2f^{2}}$	
且 > G	2 H fn H f f^{2} F • fn $\{$	$L > G' = \frac{{}^{2} \mathbf{E} f \mathbf{E} f \mathbf{f}^{2} \mathbf{E} \cdot \mathbf{f}^{2}}{\mathbf{E} \cdot \mathbf{f}^{2} \mathbf{E} \mathbf{e} \mathbf{f}^{2}}$	
EG > L	$E \bullet f^2 = E \bullet $	$G > L' \frac{{}^{2} \underbrace{\texttt{E}_{fn} \underbrace{\texttt{E}_{f}}_{f} f^{2} \underbrace{\texttt{E}_{fn}}_{f^{2} \underbrace{\texttt{E}_{fn}}_{f^{2}}}^{2}}{E \cdot f^{2} \underbrace{\texttt{E}_{fn}}_{f^{2} \underbrace{\texttt{E}_{fn}}_{f^{2}}}^{2}}$	
EG > G	$G > G' \frac{{}^{2}E \bullet f E \bullet f^{2}}{E \bullet n f E 2 f^{2}}$	$G > G' \frac{{}^{2}E \bullet f E \bullet f^{2}}{E \bullet f E 2 f^{2}}$	

Table 1: Stage 2 Payo¤s to Investment Decision

	e₂'f	e₂' e _{dv p}	e₂'e _{v∣p}
es'f	.E≴f >	.E≸{>	.E≸{>
	2 E ≸f	E● e _{dv p 2} E{≯f	E• e _{v p 2} E{≯f
e 'e _{dv p}	E∙e _{dv p} .Ę,≯ >	E∙e _{dv p} .Ę,≴ >	E• e _{dv p} .E≸{>
	₂ E ≸ {	E• e _{dv p 2} E≸ {	E• e _{v p 2} E{≯f
e, e, e, p	E•e _{v∣p} .Ę,≾r>	E∙e _{v∣p} .Ę,≾r>	E• e _{v p} • E{ > {>
	₂ E ≸ {	E• e _{dv p 2} E≸ {	$E \bullet e_{v p} _{2} E > \{$

Table 2: Stage 1 Payo¤s, Assuming Pure Strategy (Asymmetric) Stage 3 Equilibrium in which Manager 1 Invests.

7 Appendix

Proof. De...ne: g' $E \bullet f$, i ' $E 2gn \{$, and k ' $Egn \{$.

v|p: The expression for v|p ' $\frac{2^{2} \{ E f^{2} \}}{E \cdot f^{2} E e f^{2}} E g 2k f 2k f$ is the product of two factors. The ...rst factor is always positive. The second factor has one root in the range f? f? • f' $\frac{E \cdot fn}{2 2 fn}$, such that if f? f? fthen v|p A f and if f? f? • then v|p? f. Since f? f if f? f then v|p A f. The expression for $\frac{C - v|p}{C f}$ is the product of two factors. The ...rst factor, $\frac{2x^{2}}{E \cdot f^{2} E \cdot f^{2} 2}$, is always negative. The second factor has no roots in the range in the range f? f? • and is positive in this range. Therefore $\frac{C - v|p}{C f}$?

 $d_{v|p}$ Since $d_{v|p} A_{v|p}$, it is also therefore always positive. The expression $d_{v|p}$ is continuous at f' f and f' f. The diperence, $d_{v|p} Ef_{d_{v|p}} Ef_{d_{v|p}} Ef_{d_{v|p}} Ef_{e E ef^{2}}^{2} E \cdot f^{2}}$ A f

root in E f, f, such that if f? f? f then $\frac{C d v | p}{C f}$ A f and if f? f? f then c

and f_X results in further implicitly de...ned functions, which are similarly signed by the Cylindrical Algebraic Decomposition Algorithm.

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