solution, the vertically integrated regulated firm has a new variable that it can adjust: the transfer price of affiliated purchases.

Therefore, the vertically integrated regulated firm selects its purchases from affiliated supplies, its purchases from nonaffiliated suppliers, and its transfer price to maximize equation (1) subject to the constraint given in (6). The Lagrangian for this constrained maximization problem is:

(7)
$$L = R(X) - c(x_a) - w(x_n) \cdot x_n$$
$$+ \lambda \cdot [\sigma + \alpha \cdot (w_a x_a + w(x_n) x_n) / (x_a + x_n) - P(f(x))]$$

Necessary conditions for profit maximization are:

(8a)
$$\frac{\partial L}{\partial x_{a}} = \frac{\partial R}{\partial x} - \frac{\partial c}{\partial x_{a}}$$
$$+ \lambda \cdot \alpha \cdot \frac{[w_{a} \cdot (x_{a} + x_{n}) - w_{a} \cdot x_{a} - w(x_{n}) \cdot x_{n}]}{(x_{a} + x_{n})^{2}}$$
$$- \lambda \cdot \alpha \cdot \frac{\partial P}{\partial f} \cdot f' = 0$$

(8b)
$$\frac{\partial L}{\partial x_n} = \frac{\partial R}{\partial x} - w(x_n) - \frac{\partial w}{\partial x_n} \cdot x_n$$
$$\begin{bmatrix} -w_a x_a + w(x_n) \cdot (x_a + x_n) + \frac{\partial w}{\partial x_n} \cdot x_n \cdot (x_a + x_n) - w(x_n) \cdot x_n \end{bmatrix}$$
$$+ \lambda \cdot \alpha \cdots]$$
$$\end{bmatrix} = \begin{bmatrix} -w_a x_a + w(x_n) \cdot (x_a + x_n) + \frac{\partial w}{\partial x_n} \cdot x_n \cdot (x_a + x_n) - w(x_n) \cdot x_n \end{bmatrix}$$