Efficient Inter-carrier Compensation for Competing Networks When Customers Share the Value of a Call

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With competition in telecommunications markets a carrier relies on competing networks to complete inter-network calls originated by its custom

1. Introduction

The Telecommunications Act of 1996 (henceforth the 1996 Act) established a framework under which local telephone markets in the United States would be opened to competition. This necessitated the development of an inter-carrier compensation regime for inter-network traffic.[1] This regime affects networks' retail prices, which affects network usage and therefore efficiency.

Under the 1996 Act all local exchange carriers (LECs) are required "… to establish reciprocal compensation arrangements for the transport and termination of telecommunications."[2] The reciprocal compensation guidelines developed by the FCC established a "calling party's network pays" (CPNP) regime, in which a calling party's (or originating) network pays the called party's (or terminating) network a termination fee equal to the Total Element Long Run Incremental Cost (TELRIC) of the traffic-sensitive facilities of the terminating network used to complete the call.[3] The rationale for this policy, which effectively imposes the entire cost of the call on the originating network,[4] is that the originating network imposes a cost on the terminating network, and therefore the originating network must fully compensate the terminating network for this cost.[5]

The main purpose of this paper is to point out that both parties to a call often share the value of the minutes of conversation consumed,[6] and in such cases it is efficient for them to share

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[1] The 1996 Act imposes on *all*

the costs.[7] I show that parties to a call should bear the incremental cost of a minute in the same proportion that they share the value. For example, if the calling party receives 65% of the value of

I concentrate on bill and keep, not because a zero inter-carrier compensation rate is likely to give rise to theoretically optimal usage levels, but because the optimal rate may be very close to zero. Thus, a zero inter-carrier compensation rate may generate very small distortions in usage, while the alternative, which is likely to be a CPNP regime, may create more significant inefficiencies and distortions.[12] Thus, as a policy matter, society may be better off accepting the small usage distortions of bill and keep, rather than the distortions arising from CPNP. Therefore, my results regarding MBAK should be viewed as benchmarks.

There are several reasons to study the efficiency property of inter-carrier compensation rates. First, though the 1996 Act requires carriers to negotiate interconnection agreements, virtually all agreements between incumbent local exchange carriers (ILECs) and competitive local exchange carriers (CLECs) have been settled by arbitration. So these results can aid regulators in setting rates.[13] Second, even if carriers were to successfully negotiate rates, there is no guarantee that negotiated rates would be efficient. For example Katz, Rosston and Anspacher (1996) and Brennan (1997) explain that carriers might agree on high compensation rates to facilitate collusion. It has also been argued that, because each network is a monopolist with respect to terminating calls to its customers, each has an incentive to set the monopoly rate for that service. While determining the conditions (if any) under which negotiation leads to efficient compensation rates is beyond the scope of this paper,[14] this work helps to characterize efficient rates.

My analysis assumes competing networks provide homogeneous services, eliminating the need to model competing networks as offering differentiated products.[15] This is important because

[12] See DeGraba (2002) for a list of problems created by cost-based inter-carrier compensation. These include: a) regulators have less information about network costs than carriers who may over- report costs to obtain higher regulated prices; b) allowing carriers to recover only traffic sensitive costs may cause carriers to invest in technologies with high traffic-sensitive costs, even if these are not the cost minimizing technologies; c) termination charges endow customers that are net terminators of minutes with rents, creating incentives for carriers to engage in rent-seeking behavior.
[13] ILECs have little incentive to reach agreements with entrants, since this would facilitate competitive entry.
[14] See for example Brennan (1997)
[15] See for example Kim and Lim (2001) and Laffont, Rey and Tirole (1998).

one issue in analyzing inter-carrier compensation rates is whether they create tipping equilibria. Differentiated products models create a natural tendency for different customers to gravitate toward different networks, which will tend to mask the tipping effects that can be caused by inappropriate termination charges.

There are two recent works that considers the idea that called parties receive benefits from a call. Kim and Lim (2001) looks at the case where two networks agree on an termination rate, and analyze how this rate will differ between the case in which customers can be charged a usage fee for receiving calls and the case in which they cannot. They do not focus on finding an optimal termination charge. Doyle and Smith (1998) examines how the "receiver pays" principle affects the prices of mobile customers, but does not focus on the optimal termination charge.

The paper is organized as follows. Section 2 explains why it is efficient for customers to share costs in proportion to the value they receive from a call. Section 3 provides a formal model showing that bill and keep can be more efficient than the existing inter-carrier compensation regimes. Section 4 discusses extensions. Concluding remarks are in Section 5.

2. Optimal cost sharing between calling and called party

This section shows that if customers share the value of a minute of calling, they consume the optimal level when they jointly pay a per minute price equal to the marginal cost of that minute, and each customer pays a proportion of this price equal to the proportion of the value he receives from that minute. The intuition behind this result is similar to the intuition behind the Lindahl equilibrium for the provision of a public good.[16] In a Lindahl equilibrium the optimal output level occurs where the sum of the marginal benefit of each customer from the last unit of a public good equals the marginal cost of producing that unit. In this paper a call can be viewed as a public good jointly consumed by the calling and called parties.

I model a call as requiring the use of a switch that serves the calling party, a switch that serves the called party and a trunk that connects the two switches, each of which generates marginal per minute costs. This structure applies both to calls between parties on the same network, which I refer to as on-net calls, and to parties served by different networks, which I refer to as off-net calls.[17]

Let $V(m)$ be the *total* value of the m^{th} minute of calling consumed by the two parties. Let p_o be the price facing the calling (originating) party and p_t be the price facing the called (terminating) party. Let c represent the marginal cost of providing a minute of calling.

Proposition 1. Suppose that $V(m)$ is a decreasing function of m, and that the calling party receives λ of the value of each minute while the called party receives $(1-\lambda)$ of the value. Then the parties consume the efficient level of minutes if the calling party faces a price equal to λc and the called party faces $(1-\lambda)c$. This is the only set of prices that sum to c that yield efficient consumption.

Proof:

A minute of calling is completed if and only if both parties to the call voluntarily engage in the minute. The calling party is willing to engage in minute m' if $\lambda V(m') \geq p_o$ and the called party is willing to engage in minute m' if $(1-\lambda)V(m') \geq p_t$. Let m^* be the efficient level of calling; i.e., m^* is the minute for which $V(m^*) = c$. Setting $p_o = \lambda c$ and $p_t = (1-\lambda)c$ implies that m^* is the last minute in which each party will engage.

To see that no other prices that sum to c yield efficient consumption levels, suppose the calling party faced a price in excess of λc. Then he would consume fewer minutes than is optimal.

[17] The wireline loop connecting the customer to the switch typically involves no traffic-sensitive costs. Society's overall cost of having multiple networks can be no higher than the costs of having a monopoly network when all switches are used to capacity. To see this start with a monopoly local network and then let it divest some of its facilities to a competitor. On average off-net calls will have a slightly higher expected trunking cost than on-net calls, since some on-net calls occur between customers on the same switch and require no trunking. I abstract from this minor cost difference.

Suppose the calling party faced a price less than λc. Then the called party would face a price in excess of $(1-\lambda)c$, which would cause him to consume fewer minutes than is optimal. *QED*

This result applies to both on-net and off-net calls. Thus, if on-net and off-net calls have the same cost, efficiency requires that they have the same price for each party. This is not surprising since both must be priced at marginal cost. Proposition 1 implies two sufficient conditions for efficiency. The first is that $p_o + p_t = c$. The second is that $p_o/p_t = \lambda/(1-\lambda)$.

Proposition 1 focuses only on prices that sum to c. It can be shown that to obtain efficient consumption only one party must face the efficient price while the other must face a price no greater than the efficient price.[18] Because each party has "veto power" over continuing joint consumption, prices must be such that neither party vetoes consumption before the efficient level is reached, but at least one party vetoes consumption at any level beyond the efficient level.

I now ask what compensation rate results in customers facing efficient prices. Let c_o be the originating network's marginal cost of providing its portion of an off-net call and c_t be the terminating network's analogous costs (so that $c_o + c_t = c$). Let a be the termination rate (or "access" charge) paid by the calling party's network to the called party's network. The calling party's network's effective cost is $c_o + a$, and the called party's network's effective cost is $c_t - a$. Assume that competition causes carriers to set usage rates equal to their incremental cost.[19] Proposition 2 presents the optimal termination rate.

Proposition 2. Suppose competition results in each carrier setting usage rates equal to her network's effective marginal cost. The efficient termination rate is $a^* = (\lambda-1)c_o + \lambda c_t$.

[18] When the inequality holds strictly, prices will not sum to c.

[19]

nonetheless are recovered on a traffic-sensitive basis. To the extent that termination rates represent fixed costs that are recovered on a per minute basis, my results indicate those termination rates are too high and could inefficiently reduce network usage.[21]

Finally, the optimal termination rate applies to individual calls and is completely independent of the total amount of traffic originated on one network as opposed to the other. Thus, bill and keep regimes can be efficient when there is an imbalance of traffic between networks (i.e., when one network originates more traffic than the other). This result contradicts the widely held belief that bill and keep is appropriate only when traffic is balanced.[22]

3. Equilibrium analysis when customers share the value of a call equally

I now present a model in which customers share the value of a call equally. Assuming equal call value significantly reduces the complexity of calculating equilibria. Under this condition, and assuming competing networks have access to the same technology, I show that MBAK maximizes social surplus. (I discuss asymmetries in the next section.)

There are two carriers, 1 and 2, that operate competing interconnected telephone networks. The cost of providing one minute of origination for each network is 1 cent, and the cost of a minute of termination is also 1 cent.[23] A completed minute of phone conversation between two customers

and the same cost of termination, then only one call will be made in equilibrium.)

For every off-net minute originated by its subscribers, the originating network pays the terminating network a per minute termination charge, denoted a, which is set by the regulator.

Given this structure, I present the following game, played among the carriers and the customers. The regulator sets a exogenously. Carriers observe a, and in Stage 1 announce their per minute usage prices. They can set four different usage prices; $p_{iof} \equiv$ carrier i's price for originating off-net minutes, $p_{ion} \equiv$ carrier i's price for originating on-net minutes, $p_{itf} \equiv$ carrier i's price for terminating off-net minutes, and $p_{itn} \equiv$ carrier i's price for terminating on-net minutes; $i \in \{1, 2\}$, subscripts o and t represent "origination" and "termination" respectively, and n and f represent "on-net" and "off-net," respectively. Let q_{iof}, q_{ion}, q_{itf} and q_{itn} be the corresponding quantities of these different minutes sold by carrier i.

In Stage 2 each customer observes the prices and subscribes to one network. This choice is made to maximize the customer's expected consumer surplus from consuming minutes of conversation given the prices and the subscription decision of the other customer. The existence of network externalities makes the customers' choice of networks non-trivial.[28]

Once customers have chosen their networks, the remainder of the game is played out mechanically. Nature chooses which customer initiates the call, and the customers engage in all minutes of conversation for which their private value (weakly) exceeds their private cost.

The profit for each carrier is the revenue she receives from her customers plus the access revenue she receives, less her cost of providing switching and the access payments she makes. Formally, $\Pi_i = p_{ion}\, q_{ion} + p_{itn} q_{itn} + (p_{iof} - a) q_{iof} + (p_{itf} + a) q_{itf} - (1)(q_{ion} + q_{itn} + q_{iof} + q_{itf})$.

The payoff to each customer is the consumer surplus he receives, which is the sum of the value he receives from each minute of calling less the price of that minute.

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exceeds his private cost of initiating the call. Nature continually chooses one or the other to initiate calls until no more calls will be completed regardless of which caller is chosen to initiate the call.

[28] The possibility that on-net and off-net prices could be different means that the value a customer receives from his choice of network depends on the subscription choice of the other customer.

both customers subscribe to network 2.[31] Thus, in specifying an equilibrium, one must specify equilibrium subscription rules for customers as well as prices for carriers.

I now show that if a = 1, the prices in Proposition 1 cannot be supported in an equilibrium. a = 1 is important because it is the CPNP rate, currently imposed by many regulators.

Proposition 4. If $a = 1$, no equilibrium exists with $p_{iof} = p_{ion} = p_{itf} = p_{itn} = 1$.

Proof:

Suppose that carriers set $p_{iof} = p_{ion} = p_{itf} = p_{itn} = 1$. If each customer were to subscribe to each this behavior in telecom markets. Certain entrants into local markets choose to serve primarily Internet Service Providers (ISPs) in order to collect large amounts of termination from the off-net dial-up Internet access these ISPs generate. This is known as the ISP reciprocal compensation problem.[32]

It is of course interesting to ask what kind of equilibrium occurs when $a = 1$.

Proposition 5. When $a = 1$, there are two equilibria in which $p_{iof} = 2$, $p_{itf} = 0$ and $p_{ion} = p_{itn} = 1$. In one equilibrium, both customers subscribe to network 1. In the other, both subscribe to network 2.

Proof: The formal proof is provided in Appendix A.

When customers share equally in the value of a call, it is efficient for them to share the cost equally as well. A positive access charge creates an inefficiency for off-net calls. In particular, it induces carriers to set prices that impose all incremental costs on the calling party. This causes too few minutes of inter-network calling to be consumed. This inefficiency can be eliminated if all customers subscribe to the same network, because the access charge does not affect the carrier's cost of providing on-net calls. Each carrier sets the usage rate for on-net calls at the efficient level, equal to the cost each customer imposes on the network. Thus, because the access charge creates an inefficiency only with respect to calls between customers on different networks, both customers have an incentive to subscribe to the same network. In this model, the positive access charge leads to a tipping equilibrium.[33]

It is interesting to ask what would happen if carriers were not allowed to distinguish between on-net and off-net calls when setting usage rates to customers. Such a constraint will

[32] RBOCs report that they paid out $2 billion in termination fees to CLECs serving ISPs in year 2000.
[33] An alternative explanation is to note that if the government were to impose a tax on transactions between vertically related firms, the two firms would vertically integrate to avoid the inefficient tax. In telecommunications an inefficient access charge can cause all customers to join the same network to avoid inefficiently priced inter-carrier services.

have no effect on prices if the regulator sets the efficient termination rate. When *a* is set efficiently, the unconstrained carriers will charge the same rate for on-net calls as off-net calls. Thus, the constraint has no effect on equilibrium pricing. So if $c_t = c_o = c/2$ then $a^* = (2\lambda - 1)c/2$ yields efficient prices for any λ under the constraint.

If *a* is not set at the efficient level, then, as in Proposition 4, the efficient prices cannot be sustained in equilibrium. To see this, suppose $a = c_1 = c_2 = c/2$, but the calling party receives only ¾ of the benefit of any minute. Then the efficient price for origination is $3c/4$ and the efficient price for termination is $c/4$. However, when a carrier's customer originates an off-net minute, the carrier incurs a cost (including the termination charge) of c, but collects only $3c/4$ in usage fees, and thus loses money. Similarly, when a customer terminates an off-net minute the carrier incurs a cost of $c/2$, but receives revenue (including the termination charge) of $3c/4$, and thus earns a profit. Therefore, at the efficient prices, each carrier would prefer to have *B* on her network and *A* on the other network. If carrier 1 were setting the efficient rates, carrier 2 could set $p_{of2} = 3c/4 + \varepsilon$ and $p_{tf2} = c/4 - \varepsilon$. Under these rates *B* would subscribe to Carrier 2 and *A* would subscribe to Carrier 1. For ε close to zero, Carrier 2 would earn a strictly positive profit.

The results of this section suggest that when customers share equally in the value of a call and competing networks have the same production costs, 3all

4.1. More than two customers

The model of the previous section assumed that there are only two customers. DeGraba (2000) presents a model that (among other things[34]) replaces *A* and *B* with a continuum of customers of type *A* and a continuum of customers of type *B*. All results from section 3 hold under this generalization. In particular, when the access charge is zero, carriers set all usage rates equal to 1, as in Proposition 3. Both customer types subscribe randomly to each network. The subscribe to network 1 and half of whom subscribe to network 2. Each customer talks to the same number of customers on his own network as on the other network. There is a single joint valuation curve that applies to calling between all pairs of customers. The joint valuations of a minute can fall in the interval [0, ***v***]. For any given valuation $v' \in [0, \boldsymbol{v}]$, there is a symmetric distribution of the allocation of *v*' between the calling party and the called party. That is, the proportion of the value the calling party receives is symmetrically distributed over the range [0, 1].[37] Finally, every customer initiates each minute with probability ½.

Suppose that the cost of origination as well as the cost of termination equals 1, and that carriers set usage rates equal to this cost plus access. (When $a = 0$ all usage rates equal 1. When $a = 1$, on-net usage rates equal 1, off-net origination rates equal 2, and off-net termination rates = 0.)

Proposition 6. If for every $v' \in [0, \boldsymbol{v}]$ the distribution of the allocation of the value of a minute between the parties is symmetric and single peaked,[38] then $a = 0$ leads to a more efficient then bill and keep causes only the small fraction of minutes, ones for which one party receives most of the benefit, (and for which total benefit is slightly larger than total incremental cost) to not be consumed. Imposing all of the cost on the calling party however, cause a larger fraction of minutes, ones for which the benefit is relatively evenly divided between the parties (and for which total benefit is slightly larger than total incremental cost) not to be consumed. Conversely, if primarily one party or the other enjoys most of the value of each minute, imposing all of the cost on the calling party will be relatively more efficient.

This intuition can be extended to compare the surplus from $a = 0$ to the surplus when a is any positive number, and show that $a = 0$ always yields greater surplus. Thus we have the following:

Corollary 1. If for every v' the distribution of the allocation of the value is symmetric and single peaked, then $a = 0$ is second best optimal among linear termination rates.

4.3. Inter-carrier compensation and regulated retail rates

U.S. regulators typically require ILECs to offer residential service on a flat-rated basis.[41] When carriers use only flat-rated charges, per minute access costs are not passed on as per minute retail rates and, thus, do not affect short run usage decisions. Rather, they simply transfer wealth between carriers.

Suppose carriers cannot distinguish A from B and so must set the same flat-rated charge to each customer.[42] Suppose that $c_1 = c_2 = c/2$, and that each customer engages in m_0 minutes of conversation when facing a zero usage rate. Competition drives the flat rate down to $m_0c/2$, the

[40] See Figure 2 for a graph of each distribution.
[41] Exceptions include Chicago and parts of New York City, which have metered residential service.
[42] Restricting one carrier to set flat-rated rates while the other to sets usage rates induces an equilibrium in which the carrier that sets usage rates serves all of the customers, because he can set rates efficiently.

Under $a = c/2$, competition would drive the retail price paid by ISPs to 0, since a carrier that serves an ISP receives access revenue that covers the full cost of serving the ISP. The ILEC, who serves all of the residential customers, pays for all these access revenues if the ISP is served by the CLEC. Because there is a single residential rate, these costs are averaged across all residential customers, whether or not they use the Internet. Thus non-Internet users subsidize those customers who use the Internet.

This problem is mitigated under $a = 0$. In this case, the cost of switching and half of the transport is imposed directly on the ISP, which would then be recovered in the ISP's subscription rates. Thus, only customers who use the ISP would pay for these costs. Here, only the other half of the transport costs would be averaged into the residential rates.

5. Conclusion

In this paper I have examined the effect of an inter-carrier compensation regime on market performance. I have shown that the compensation rate should be viewed as a way of dividing the incremental cost of the facilities needed to complete an off-net call between the parties. This is different from the current practice and literature, in which the inter-carrier compensation rate is viewed as the price of inputs the originating network purchases from the terminating network when it "sells completed calls" to the calling party.

The main result is that it is efficient for each customer to bear the proportion of the incremental cost of a call equal to the proportio

Second, CPNP can create a tipping effect. That is, because it overallocates the cost of off-net calls to the calling party, all customers have an incentive to cluster on the same network. While such a result may be efficient in a static sense, it would seem counterproductive in a market like telecommunications in which viable competitors are being introduced as a way to eliminate the need for regulation.

Appendix A

Proof of Proposition 3.

Under the proposed equilibrium customers subscribe randomly to each network. The customers engage in 3 minutes of conversation, because for each party the benefit of the 3rd minute is (1/2)(5-3) = 1. Each party receives a surplus of 2.25 (calculated as a total benefit of 5.25 less total usage payments of 3).

Define the surplus earned by customer *j* as S_{jii}, $j \in \{A, B\}$ where the first *i* subscript indicates *A* subscribes to network *i* and the second *i* subscript indicates *B*'s subscription choice. The following rules are subgame perfect responses for customers.

If for only one carrier, Carrier *i*, prices are such that, $S_{Aii} > S_{A\text{-}ii}$ and $S_{Bii} > S_{Bi\text{-}i}$. then both customers subscribe to network *i*.

If for both carriers, prices are such that $S_{Aii} > S_{A\text{-}ii}$ and $S_{Bii} > S_{Bi\text{-}i}$ then if $\Sigma_j S_{jii} > \Sigma_j S_{j\text{-}i\text{-}i}$ each customer subscribes to network *i*, and if $\Sigma_j S_{jii} < \Sigma_j S_{j\text{-}i\text{-}i}$ then each customer subscribes to network –*i*.

If for both carriers, prices are such that $S_{Aii} > S_{A\text{-}ii}$ and $S_{Bii} > S_{Bi\text{-}i}$ and $\Sigma_j S_{jii} = \Sigma_j S_{j\text{-}i\text{-}i}$ but $\Sigma_j S_{jii}$ is greater than the sum of customers' surplus if one customer subscribes to each network, then subscribe to network 1.

Lemmas 1 and 2 now rule out all possible deviations by a carrier.

Lemma 1. Assume WOLG that Carrier 2 deviates from the proposed equilibrium. No deviation in which Carrier 2 sets either $p_{2of} \leq 1$ or $p_{2tf} \leq 1$ can be a profitable deviation.

Proof. Suppose that there is a deviation in which $p_{2tf} \leq 1$. If *A* subscribes to network 1, he earns an expected surplus of 2.25 regardless of the network to which *B* subscribes. This is because if $p_{2tf} \leq 1$, and *B* subscribes to network 2, *B* is willing to terminate (at least) 3 minutes worth of calls. Therefore if *A* subscribes to network 1, he can engage in 3 minutes of conversation with *B* regardless of the network to which *B* subscribes, and pay 1 cent per minute for each minute.[45] That is, if *B* is chosen as the initiator of the call and $p_{2of} > 1$, *B* will originate the call, but not consume 3 minutes of conversation (because the origination rate exceeds 1). However, *A* will then originate a call (since his origination rate equals 1), and *B* will terminate the call (since his termination rate is less than or equal to 1) and the parties will consume minutes to the point where the minutes from the initial call and the second call equal 3. Similarly, if Carrier 2 sets $p_{2of} \leq 1$, *B* is willing to originate (at least) 3 minutes. Thus, if *A* is chosen as the initiator of the first call and *B* faces a termination rate greater than 1, the initial call will not last 3 minutes but *B* will then initiate a call that covers the balance of the time.

Therefore, in any subgame perfect continuation in which Carrier 2 sets either her off-net origination rates or her off-net termination rate less than 1, any customer that subscribes to network 2 must earn a payoff at least as great as 2.25. Suppose such a customer subscribes to network 2 and engages in 3 minutes of conversation. Then to earn a surplus of at least 2.25, he must pay total usage fees that total no more than 3. Since he generates a cost of 3, Carrier 2 could never earn a positive profit from such a customer.

I know show that no customer can subscribe to network 2 and consume a quantity of minutes other than 3 and earn a surplus greater than 2.25 while Carrier 2 earns a 0 payoff. A customer that consumes *m* minutes receives a benefit of $[2.5 - m/4]m$. Letting *U* be total usage payments, the customer's surplus would be $[2.5 - m/4]m - U$. If Carrier 2 earned a zero payoff, then she would have to receive usage payments equal to *m*. Thus, the customer's surplus would equal $[2.5 - m/4]m - m$. The *m* that maximizes this expression is $m = 3$. Thus, a customer cannot earn a surplus higher than 2.25 while the carrier earns a zero payoff.

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[45] Thus, if the customer on network 2 is chosen to originate a call to the customer on network 1, but faces an off-net origination rate greater than the private benefit for some minutes less than 3, then the customer on

Lemma 2. No deviation in which Carrier 2 sets $p_{2of} > 1$ and $p_{2tf} > 1$ can be a subgame perfect profitable deviation.

Proof. Because $p_{2of} > 1$ and $p_{2tf} > 1$, if both customers were to subscribe to network 1, no customer could subscribe to network 2 by himself and earn a surplus in excess of 2.25. All minutes for such a customer would be off-net so they would all be priced above 1, and because of this the customer would consume fewer than 3 minutes. Thus we need only consider continuations in which all customers subscribe to network 2 in response to a deviation by carrier 2.

Both customers will subscribe to network 2 only if the sum of their surpluses is at least 4.5 = (2x2.25). Then it must be the case that any customer earning a surplus of 2.25 pays no more than the cost he imposes on Carrier 2, and any customer that earns a surplus greater than 2.25 must pay less than the cost he imposes on Carrier 2. Thus, such a deviation cannot result in a positive profit for Carrier 2.

Lemmas 1 and 2 rule out all possible deviations. Thus, the proposed equilibrium is an equilibrium. In this equilibrium customers consume minutes up to the point where the marginal benefit to society equals the marginal cost to society, thus maximizing social surplus. Thus, this equilibrium is efficient. *QED.*

Proof of Proposition 5

Suppose each carrier sets the prices proposed in the proposition.

Lemma 3. The customers' strategies described in Proposition 4 are also subgame perfect in this game.

Proof. Because these strategies are subgame perfect and independent of prices set by the carriers, they constitute subgame perfect behavior when $a = 1$.

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network 1 will originate any minutes up to 3 that were not consumed in the original call.

Lemma 4. The only subgame perfect response to the prices proposed in the proposition is for both customers to subscribe to the same network.

Proof. If both customers subscribe to network 1, each earns a surplus of 2.25, consuming 3 minutes of calling for a benefit of 5.25, and paying 3 in switching costs. Suppose *B* subscribed network 2. With probability 1/3 he will originate the call, and with probability 2/3 he will terminate the call. In either case he will consume only 1 minute of calling (which results in a benefit of 2.25). His expected surplus from subscribing to network 2 is only 1.583 = (1/3)(2.25 - 2) + (2/3)(2.25-0).

A similar calculation shows that at these prices, *A* cannot earn a higher surplus by subscribing to network 2 while *B* subscribes to network 1.

Lemmas 3 and 4 show that in response to the prices proposed in the proposition, the only subgame perfect response is for both customers to subscribe to the same network. I now show there is no deviation by a carrier that allows her to earn a strictly positive profit.

Suppose WLOG both customers subscribe to network 1. In this equilibrium both carriers earn 0.

Lemma 5. If Carrier 2 were to deviate so that there were a subgame perfect continuation in which one or both of the customers subscribed to her network, she could not earn a positive profit.

Proof. Suppose Carrier 2 deviates by setting on-net rates low enough so that $\Sigma_j S_{j22} > \Sigma_j S_{j11}$

$j \in \{A, B\}$ Then the customers must jointly receive a surplus of 4.5. But since the maximum joint customer surplus is 4.5 when a carrier earns a profit of 0 (Lemma 1), there can be no

Now, let Carrier 2 sets off-net prices so that just one customer subscribes to network 2.

The most surplus one customer could earn by subscribing to network 2 subject to Carrier 2 earning a non-negative profit would occur if $p_{2of} > 2.5$ and $p_{2tf} = 0$. In this case the customer on network 2 would never originate a call and would receive 1 minute of calling, earning a surplus of 2.25. At these prices Carrier 2 would earn a zero profit. Thus there is no deviation in which one customer would choose to subscribe to Carrier 2 that allows Carrier 2 to earn a positive profit.

QED

Proof of Proposition 6

For any valuation, *v'*, define the statistic $D_{v'}$ which is the difference between the calling party's individual valuation and the called party's individual valuation. $D_{v'}$ is distributed on the interval [-*v'*, *v'*], where $D_{v'} = -v'$ means the called party receives all of the value and $D_{v'} = v'$ means the calling party receives all the value. The conditions of the proposition imply that for each *v'* the distribution of $D_{v'}$ is symmetric about zero.

I now show graphically in Figure 1. that for each *v'* $a = 0$ results in more minutes being consumed than $a = 1$ when the distribution of $D_{v'}$ is single peaked. The graph measures the calling party's value of a minute on the horizontal axis and the called party's valuation of a minute on the vertical axis. Let v_o represent the calling party's valuation and v_t represent the called party's valuation. The locus $v_o + v_t = v'$ for $0 < v_o < v'$ and $0 < v_t < v'$ and $0 < v' < \mathbf{v}$ represents all of the possible divisions of the value of a minute that the parties value jointly at *v'*. This locus is a line with slope -1 and an intercept of *v'*.

W hen $a = 0$, and all originating and terminating minutes are priced at 1 (the cost of switching), all minutes are consumed for which $v_o \geq 1$ and $v_t \geq 1$, which is given by the locus of points "north east" of (1, 1) (Point *T* in Figure 1). When $a = 1$ and prices are symmetric in equilibrium, the usage rate for originated off-net minutes is 2, the usage rate for terminating off-net minutes is 0 and the usage rate for all on-net minutes is 1. All of the minutes northeast of (2, 1), (Point *W* in Figure 1) are consumed. In addition ½ of the minutes in trapezoid *TUVW* are consumed and ½ of the minutes in trapezoid *WXYZ* are consumed. (This is because ½ of the minutes in trapezoid *TUVW* are on-net minutes and ½ are off-net. In the $a = 1$ equilibrium, only the on-net minutes in *TUVW* are consumed. Similarly, ½ the minutes in *WXYZ* are off-net minutes and ½ are on-net minutes, and in the $a = 0$

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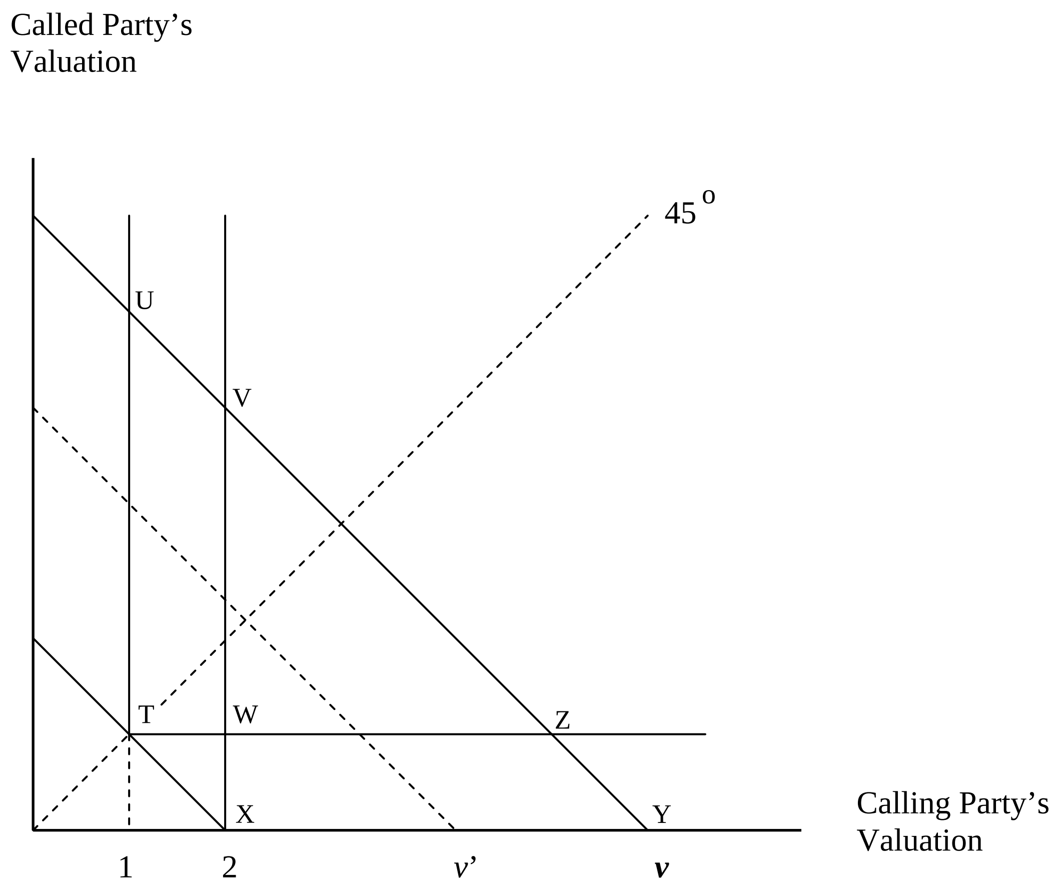

Figure 1.

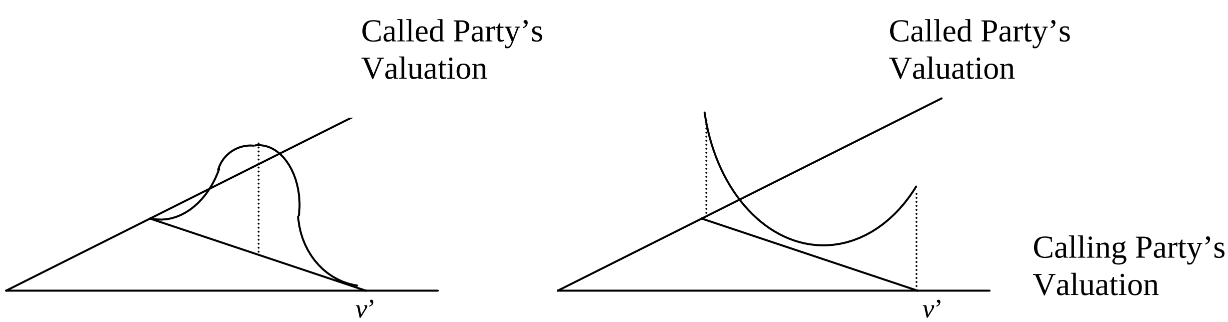

Figure 2
A single peaked distribution of the allocation of all minutes with value *v*', and a distribution of the allocation of all minutes with value *v*' with more weight in the tails.