Idertifying Demand in EBay Auctions

Christopher P. Adams[*]
Federal Trade Commission
Email: cadams@ftc.gov

June 28, 2004

Abstract

This paper presents assumptions and identi¯cation results for eBay type auctions. These results are for private value auctions covering three major issues; censoring bias, auction heterogeneity and dynamic bidding. The ¯rst section of the paper presents two identi¯cation results for second price open call auctions with private values and unobserved participation (eBay

1 Introduction

EBay and eBay type auctions are an economic phenomena. EBay is fast becoming a major distribution channel in everything from Beanie Ba and identically distributed then the value distribution is identified. While the first result is based on a fairly restrictive assumption, it gives a simple function that is straight forward to estimate. The second result shows that this assumption on the distribution of potential number of bidders is not necessary for identification. The second section presents results that generalize the first result to auctions with heterogenous bidders and heterogenous auctions (both observed and unobserved). The third section of the paper presents assumptions and identification results for auctions in which the bidding is interdependent.

The results presented in the first section build on results in Athey and Haile (2002) and Song (2003). Athey and Haile (2002) show that in open call second price auctions with independent private values and a known number of bidders, the value distribution is identified from the observation of one order statistic. For example in second price auctions the price is the $N-1:N$ order statistic. That is, the price is all that is needed to identify the value distribution.[3] While this result shows that the value distribution can be identified despite bids being censored, it does not account for the possibility that the existence of the bidder may also be censored. Song (2003) presents a solution to this second censoring problem. Her result is that the value distribution can be identified from the observation of two order statistics. This result works well in the case where the $N:N$ and $N-1:N$ order statistics are observed as is the case if eBay provides the data (Adams and Bivins (2004); Zeithammer (2004a)). However, it is unusual to observe the actual highest bids from eBay data and other bidders may have their highest proaches. The section's second result uses information on the timing of bids to identify demand. Zhang et al. (2002) assume that a Poisson distribution determines the entry probability and the timing of bids. There are two concerns with their approach. First, it is fairly obvious looking at bidding behavior on eBay that bids are congested towards the end of the auction (Adams et al. (2004)), which suggests the Poisson distribution is not a reasonable representation of bidding behavior. Second, the authors assume that each bidder only bids once at their \last opportunity" which casual observation also suggests is not true.[4] The results presented below indicate that an estimator with less restrictive assumptions can be used to estimate the demand for items on eBay.

The second section of the paper presents results for heterogenous auctions. Athey and Haile (2002) present results for \asymmetric" auctions, that is auctions in which bidders draw their values from di®erent distributions. Again, however, these results are for auctions in which the number of bidders is known. This paper presents results for asymmetric auctions where the number of bidders is unknown.[5] The section also presents results for auctions of di®erentiated goods. The paper shows that the joint value distribution over multiple items can be identi¯ed under certain conditions. The paper further shows that hedonic models are identi¯ed and provides assumptions su±cient to identify hedonic models with unobserved item heterogeneity. Bajari and Bankard (2004) presents non-parametric identi¯cation results for transactions data with unobserved characteristics. The ¯nal result of the section considers auctions with unobserved heterogeneity. Krasnokutskaya (2003) presents identi¯cation results for ¯rst price auctions and discusses other work on this issue. Athey and Haile (2002) present results for second price auctions with a known number of bidders. Froeb et al. (2001) present a parametric estimator for second price auctions with a common unobserved shock.

[4] A Poisson assumption on entry and a non-parametric assumption on the \last opportunity" may be more reasonable. Song has preliminary work on such an estimator.

[5] Froeb et al. (2001) show that power-related parametric distributions can be used to estimate value distributions for asymmetric bidders.

The third section of the paper considers the issue that bidders may shave their bids to account for the option value of winning a future auction. There is a substantial literature on bidding behavior in sequential auctions.[6] However, there is an important di®erence between traditional sequential auctions and eBay auctions. In a traditional sequential auction such as an FCC spectrum auction, winning bidders leave the sequence of auctions and are not replaced, so the number of bidders and value distribution of the remaining bidders changes over time. On eBay, however, there is constant entry of new bidders into the sequence of auctions. Two recent papers analyze dynamic bidding behavior on eBay (Arora et al. (2002); Zeithammer (2003)), however both papers assume that the bidder faces a ¯nite set of future auctions. In

Let C_t be the \cut o®" price at time t. As eBay is a second price auction, $C_t = B_t^{(M-1:M)}$, where $B_t^{(M-1:M)}$ is the second highest bid as of time t. Song (2003) shows that in a Bayesian Nash equilibrium of this game, it must be that for every bidder whose value for the item is greater than C_t at their last opportunity to bid, will bid their value ($B^i_{t^i}$ =

price to have a simple functional form.[11]

Assumption 5 Let $Pr(N = n) = (1$ ¡ $¿_j p)¿_j^n p^n$, where $¿_j$ is the length of auction j, t_j 2 [¿; ¿] and p

if 5.51 0 Td[(;)97.6 f(N)TTJ 22 -1uction
75.w 11: 00 -88 pro0)]d0T121.95291340478bserve0.8 d(of) -3.1T1 -2)]TJ(te10.8 -10)]0(ie 0)]TJ 6958te10.8hrelt2 0.44 0.

Define V_{2k} similarly, such that $V_{2k} \in \{v_1; v_2; \ldots; v_K\}$. Note that $v_1 = \underline{v}$. Note further that as $K \to \infty$, $F_K(\cdot) \to F(\cdot)$. Let x_k denote the observed (large sample) probability of V_{2k}. So rewriting Equation (2) for the case of V_{2k} and noting that the marginal probability of observing v_k is $f_k(v_k)$ and the cumulative probability of observing v_k is $F_K(v_k)$ we have the following equation.

$$x_k = \frac{2(1 - \tau_j p) f_K(V_{2k})(1 - F_K(V_{2k}))}{(1 - \tau_j p F_K(V_{2k}))^3} \tag{5}$$

Step 3. First we have

$$x_1 = 2(1 - \tau_j p) f_K(\underline{v}) \tag{6}$$

So consider two sets of auctions with different lengths, τ_1 and τ_2, we have

$$x_{11} = 2(1 - \tau_1 p) f_K(\underline{v}) \tag{7}$$

and

$$x_{12} = 2(1 - \tau_2 p) f_K(\underline{v}) \tag{8}$$

Rearranging we have

$$\frac{x_{11}}{2(1 - \tau_1 p)} = \frac{x_{12}}{2(1 - \tau_2 p)} \tag{9}$$

and so

$$p = \frac{x_{12} - x_{11}}{\tau_1 x_{12} - \tau_2 x_{11}} \tag{10}$$

and

$$f_K(\underline{v}) = \frac{x_{11}}{2(1 - \tau_1 p)} \tag{11}$$

and rearranging Equation (5)

$$f_K(V_{2k}) = \frac{2(1 - \tau_j p) x_k (1 - F_K(V_{2k}))}{(1 - \tau_j p F_K(V_{2k}))^3} \tag{12}$$

Using Equation 4, by induction $f_K(\cdot)$ is identified. Letting $K \to \infty$, $F(\cdot)$ is identified. Q.E.D.

Proposition 1 states that if the potential number of bidders has a parlengths are observed, then the value distribution is identified. The proof shows that given a particular probability distribution over the number of potential bidders the value distribution unconditional on the number of bidders is also identified and has a relatively simple functional form.

$$\Pr(V_2 | N \geq 2; \tau) = \frac{2(1 - \tau p) f(V_2)(1 - F(V_2))}{(1 - \tau p F(V_2))^3} \qquad (13)$$

As one would expect the functional form is a slightly more complicated version of the standard censoring model. If one is willing to assume that a log normal distribution is parsimonious representation of the underlying distribution then the formula suggests a simple maximum likelihood estimator (Greene (2000)). Alternatively, the three steps of the proof suggest a non-parametric estimator in the tradition of Guerre et al. (2000).

The second result of this section shows that there is a third way to identify demand from eBay auctions, and that this third way doesn't rely on some arbitrary distributional assumption. However, additional assumptions are still used to prove the result. In particular, the following assumption states that every bidder's last opportunity to bid is independently and identically distributed.

Assumption 6 Let $G^i(.) = G(.)$ for all i.

Assumption 7 If given the opportunity to do so, all bidders make a bid at their "last opportunity" to do so.

Assumption 7 states that we are going to restrict the set of BNEs to those in which all bidders bid at their last opportunity to do so, if they have that opportunity. Song (2003) shows that such equilibria exists although this is a more restrictive structural assumption than what is presented in Song (2003). As discussed above, there is a very large tendency for eBay bidders to bid at the end of the auction (see Adams et al. (2004) for example), and so I don't believe it is an overly restrictive assumption. Below, I present a lemma which suggests that bidders will always bid late in equilibrium (Lemma 2).

Prop osition 2 Given that Assumptions (1 - 4) and Assumptions 6 and 7 hold, and fM g

Assumptions	Variable	Actual	Reps	Mean	SD	Min	Max
Song							
drop if							

ariable

Assumptions	Variable	Actual	Reps	Mean	SD	Min	Max
Song							
drop if	μ	2.00	1000	2.01	.06	1.78	2.18
$t_2 < .4$	σ	.50	1000	.50	.04	.43	.55
No drop	μ	2.00	1000	2.08	.03	2.00	2.16
	σ	.50	1000	.49	.02	.43	.55
Adams							
	μ	2.00	1000	2.00	.07	1.79	2.26
	σ	.50	1000	.47	.01	.45	.50
	p	.02	1000	.59	.10	.03	.81

Table 2: Monte Carlo Estimates with $p_n = (1 - p)^{100-n} p^n$

set by between 60% and 80%. The results show that Song's estimator gives a slightly biased estimate of μ, 2.02 rather than 2, however the estimator is relatively inefficient. The second set of results is based results in which no restriction is made about which auctions can be used. We see that in this case the estimator is biased upwards but the estimate of μ is made with a lot more precision. Comparing the results to the estimator based on Proposition 1, the estimates for μ are not biased while being a little more efficient than Song's less biased estimator.

Table 2 presents results from Monte Carlos under a different assumption on the number of potential bidders in each auction. It is assumed that $p_n = .98^{100-n}.02^n$. Note that while this distribution is different from the one presented above, it still places most of its weight on there being a small number of bidders in each auction. The results show that even though the distributional assumption does not hold, the estimator presented above still gives an unbiased estimate for μ. However that estimate is slightly less efficient than the less biased estimator based on Song (2003). Note also that the Adams estimator gives a biased estimate for σ and this estimate is more biased than the two Song estimates for σ. Finally, note that the estimate for p from the Adams model is nonsense, which is not surprising given that the distributional assumption is incorrect.

3 Heterogenous Auctions

This section presents results which generalize Proposition 1 to situations where the auctions heterogenous, including heterogenous bidders (asymmetric auctions), heterogenous items, and unobserved auction heterogeneity.

3.1 Heterogenous Bidders

The next two results generalize the first result to the case where bidders have observable characteristics. The major issue here is that the level of observation is an auction, not a bidder. Thus it is necessary to infer information about the population of bidders from observing just the identities of the winning and second highest bidders.

Assumption 8 Let V_A^i be distributed $F_A(\cdot)$ for all bidders such that i has observable characteristic A. Let V_B^j be distributed $F_B(\cdot)$ for all bidders such that j has observable characteristic B.

Assumption 8 states that bidders can be one of two types and conditional on their type their valuations are independently and identically distributed. The following proposition states that the identification result presented above can be generalized to this case.

Proposition 3 If Assumptions (1 - 4) and Assumptions 5 and 8 hold, then if the distribution of $\{V_2\}$, the identity of the highest bidder and the second highest bidder and the length of the auctions are observed, and there are at

Assumption 5, from Proposition 1, $F(v)$ is identi¯ed from the observation of $\{V_2\}$. Therefore, rewriting Equation (13) for this case

$Pr(V_2; 1 \in A; 2 \in B | N \geq 2$

Assumption 9 states that the observable characteristics could have any general form. That is, it could be a dummy variable like gender or continuous variable like income.

Corollary 1 If Assumptions (1 - 4) and Assumption 5 and 9 hold, then if the distribution of fV_2g

mapped into characteristic space with an unobserved item characteristic and the function is allowed to vary with observed characteristics of the bidders.

Consider two simultaneous auctions, one for item A and the second for item B. Previous results show that under certain assumptions it is possible to identify $F_j(\cdot)$ where $j \in \{A, B\}$. The following corollary states that it is possible to identify the joint value distribution, $F(\cdot;\cdot)$.

Assumption 10 Let $Pr(N_j = n) = (1 - \tau_{jk} p_j)\tau_{jk}^n p_j^n$, where $j \in \{A, B\}$ and $p_j \in (0; \frac{1}{\tau})$.

Assumption 10 generalizes Assumption 5 to this case. Note that bidders may bid in both auctions without restriction but they don't have to. The following assumption is made for simplicity.[15]

Assumption 11 Let bidder i's bids across auctions be independent.

Corollary 2 Given Assumptions (1 - 4) and Assumptions 10 and 11, if the distribution of $\{V_{2A}; V_{2B}\}$ is observed, the length of the auctions for each set of auctions, and one bidder is observed to bid in both auctions such that she has neither the highest or second highest bid in either auction, and there are at least two auction lengths for each set of auctions, then $F(\cdot;\cdot)$ is identified.

Proof. Consider only the set of auctions in which one bidder is observed to bid in both auctions without being the highest or second highest bid in either, then the conditional probability is

$$
\begin{aligned}
x_1 = {} & (1 - \tau_{Aj} p_A)(1 - \tau_{Bk} p_B) f_A(V_{2A})(1 - F_A(V_{2A})) f_B(V_{2B})(1 - F(V_{2B})) \\
& \times F(V_{2A}; V_{2B}) \Big(\sum_{n=3}^{\infty} n(n-1)\tau_{Aj}^{n-3} p_A^{n-3} F_A^{n-3}(V_{2A})\Big) \\
& \times \Big(\Big(\sum_{n=3}^{\infty} n(n-1)\tau_{Bk}^{n-3} p_B^{n-3} F_B^{n-3}(V_{2B})\Big)\Big)
\end{aligned} \qquad (25)
$$

By Proposition 1, $F_A(\cdot)$, $F_B(\cdot)$, p_A and p_B are identified, so $F(\cdot;\cdot)$ is identified. Q.E.D.

[15] See the last section for a discussion of the case where bids across auctions are interdependent.

item, Z_i is an I dimensional vector of observable characteristics of the bidder and $»_j$ is a characteristic of the item observed by the bidder and unobserved by the researcher.

Proposition 5 shows that a model in the tradition of Berry et al. (1995) can be identi¯ed using eBay type auction data.

Proposition 5 If Assumptions (1 - 4) and Assumptions 13 and 15 hold, then if fV_2g, X_j, Z_i, the auction lengths are observed for all j

for identifying demand with both unobserved auction heterogeneity and unobserved number of bidders. The first result states that if the distribution has two mass points an equal distance from 0, then the value distribution is identified if at least one bidder i is observed bidding in two simultaneous auctions. The second result states that as the number of auctions that bidder i is observed to bid in gets large, then for any distribution for the unobserved heterogeneity (Γ), the value distribution is identified.

Assumption 16 Let $V_j^i = V^i - y_j$, where $y_j \in [\underline{y}, \bar{y}]$ is observed by bidder i and distributed $\Gamma(.)$.

Assumption 16 states that there is some unobserved heterogeneity that is additive to the value of the item and the same for every bidder in a particular auction. Note that in this section j refers to the auction rather than the item. The following assumption states that the distribution of the unobserved heterogeneity has a simple symmetric two mass point distribution.

Assumption 17 Let $\Gamma(.)$ be such that $y \in \{-a, a\}$ where $Pr(y = a) = \gamma$ and $a > 0$ and $\gamma \in (0, 1)$

This assumption and the assumption (below) that the researcher observes at least one bidder bid in two simultaneous auctions, is enough to identify the value distribution. The assumption that the two auctions are simultaneous and the bid in each auction is independent of the bid in the other auction, is made for simplicity. The next section considers identification issues when bids are not independent.

Assumption 18 Let each bidder i bid on 2 simultaneous auctions such that her bids across auctions are independent.

Definition 1 Let A_t be the set of auctions such that at least one bidder is not censored at their last opportunity to bid after t in both auctions and is not the highest bid in the auctions.

Note that the second highest bidder is never censored. Given these assumptions the following proposition illustrates the basic result of the section.

Prop osition 6

$F(:)$ is identi¯ed. Q.E.D.

the expected price conditional on winning $E(B^{(M-1:M)}_{t+s} | B^i_{t+s} > B^{(M-1:M)}_{t+s})$ and the option value of winning that auction (O^i_{t+s}).

4.1 Homogenous Auctions

The following set of assumptions are made in order to simplify the game and turn it into a dynamic decision making problem under uncertainty.[17] Let A denote the set of all auctions and I the set of all potential bidders. The following assumption is that each bidder faces a known discrete infinite set of future auctions.

Assumption 20 Let each bidder i faces at time t a known infinite sequence of auctions, $A^i_t = \{A^i_t; A^i_{t+1}; A^i_{t+2}; \ldots\} \subset A$ where it

The assumptions contrast to the assumptions made in Zeithammer (2004b). In that paper, bidders know about future auctions and behave strategically by making bids that a®ect the expected value of winning future auctions.

Definition 2 Let $H(.)$ be the distribution of B_t^i which is on the support $[\underline{b}, \bar{b}]$.

Above and in Song (2003) it is shown that $H(.)$ can be identified from observing the auction prices and certain other data. Let δ_i define bidder i's preferences over time. Given this result and assuming δ_i is constant over time, the option value can be written in the following recursive manner

$$
\begin{aligned}
O^i(B^i) &= \delta_i \Pr(B^i > B^{(M-1:M)}) E(B^i - B^{(M-1:M)} | B^i > B^{(M-1:M)}) \\
&\quad + \delta_i (1 - \Pr(B^i > B^{(M-1:M)})) O^i(B^i)
\end{aligned}
\tag{26}
$$

By winning the auction at time t the bidder gives up the value of the winning the auction at time $t+1$ which is the probability of winning the auction ($\Pr(B^i > B^{(M-1:M)})$) by the expected value of winning the auction given that the bidder won ($E(B^i - B^{(M-1:M)} | B^i > B^{(M-1:M)})$). There is also some probability that they lose the next auction in which case their continuation value is equal to the option value. This equation can be rearranged to give

$$
O^i(B^i) = \frac{\delta_i \Pr(B^i > B^{(M-1:M)}) E(B^i - B^{(M-1:M)} | B^i > B^{(M-1:M)})}{1 - \delta_i (1 - \Pr(B^i > B^{(M-1:M)}))}
\tag{27}
$$

In this case if δ_i is known it is straight forward to determine $O^i(B_i)$. Let $H_2(.)$ denote the distribution of $B^{(M-1:M)}$ (the price), which is observed. Given this we can rewrite the option value as a function of observed variables.[20]

$$
O^i(B^i) = \frac{\delta_i H_2(B^i) \int_{\underline{b}}^{B^i} (B^i - B^{(M-1:M)}) h_2(B^{(M-1:M)}) dB^{(M-1:M)}}{(1 - \delta_i (1 - H_2(B^i)))}
\tag{28}
$$

It is often argued that real people cannot do the types of calculations that economists assume of them. In this case, eBay or some other service could provide a web based option calculator to calculate the bidder's option value and thus their optimal bid.[21] The following proposition gives the main result of the section.

[20] See Jofre-Bonet and Pesendorfer (2003) for a similar argument.

[21] I note that traders use such calculators in pricing options via the Black-Scholes formula, and that computer scientists are working on developing similar types of calculators for bidding in on-line auctions.

Noting that the LHS is observed. It is tedious but straight forward to solve for ¯ as a function of observables from $O(B(r_1))$, $O(B(r_2))$ and Equation (30). Q.E.D.

Corollary 5 states if the interest rate follows a simple Markov process and the distribution is known and observed by the bidders, then the value distribution can be identi¯ed when every bidder's discount factor is a simple linear function of the interest rate. The variation in bids caused by the changing interest rates can be used to identify the representative bidder's time preferences. It seems reasonable to expect that the more interest rate regimes there are, the more °exible the time preference function that can be identi¯ed.

The corollary shows that the richer the data the more °exible the assumptions on the approximation of the bidder's preferences over time. One useful feature of eBay data is that particular bidders can be tracked over time (see Arora et al. (2002) for an example of how this data can be used). If there is data on bidder characteristics such as their zip code or their reputation score, then it may be possible to use similar methods to identify demand when time preferences vary across observable characteristics of the bidder.

4.2 Di®erentiated Products

This section considers a bidder facing an in¯nite sequence of auctions for a single item, where the items o®ered in each auction are di®erentiated. In this case, Zeithammer (2004a) points out that knowledge of speci¯c future auction a®ects bidding behavior. In particular, if a bidder learns that her preferred item will be sold in the next auction she may not bid in the current auction as the continuation payo® may be higher than the expected payo® of winning the current auction.

Consider the following model. The following assumption states that the

the bidder will leave the sequence once she wins an auction. For example if the items are cars the bidder will leave the sequence once she wins a car, irrespective of whether it is car C or car D.

Assumptions 27 and 28 generalize the model presented in the previous section.

Assumption 27 Let each bidder i face a sequence of auctions $A_t^i = \{A_t^{ij_t}; A_{t+1}^{ij_{t+1}}; \ldots\}$ where $i \in I$ and item $j_t \in \{C; D\}$.

Assumption 28 The *ex ante* probability that the item in any auction A_t is C is p_c, with $p_d = 1 - p_c$.

	CU	CC	CD	DU	DC	DD
CU	$p_c(1 - q(t^i))$	$q(t^i)p_c^2$	$p_c q(t^i) p_d$	$p_d(1 - q(t^i))$	$p_d q(t^i) p_c$	$q(t^i)p_d^2$
CC	$(1 - q(t^i))$	$q(t^i)p_c$	$q(t^i)p_d$	0	0	0
CD	0	0	0	$(1 - q(t^i))$	$q(t^i)p_c$	$q(t^i)p_d$
DU	$p_c(1 - q(t^i))$	$q(t^i)p_c^2$	$p_c q(t^i) p_d$	$p_d(1 - q(t^i))$	$p_d q(t^i) p_c$	$q(t^i)p_d^2$
DC	$(1 - q(t^i))$	$q(t^i)p_c$	$q(t^i)p_d$	0	0	0
DD	0	0	0	$(1 - q(t^i))$	$q(t^i)p_c$	$q(t^i)p_d$

Table 3: Transition probabilities

Given this result the bidder's problem has six states with the transition probabilities given in Table 3. The six states are CU which denotes that the bidders is currently in an auction for item C and the item in the next auction unknown (the bidder has not yet observed a signal). Similarly CC denotes a current auction for item C and it is known that the next auction is an auction for item C. The transition probabilities are determined by the *ex ante* probability that the item will be C (p_c) and the probability that the bidder observes a signal of the item to be auction off in the next auction prior to her "last opportunity" ($q(t^i)$). We can write down the bidder's option value of winning a particular auction in the following recursive manner.

$$
\begin{aligned}
O^i(B^i_{CU}) = {} & \delta\Big(\sum_K p_k H_k(B^i_k) E(B^i_k - B^{(M-1:M)} \mid B^i_k > B^{(M-1:M)}) \\
& + (1 - H_k(B^i_k)) O^i(B^i_k)\Big)
\end{aligned}
\qquad (31)
$$

where $K = \{CU; CC; CD; DU; DC; DD$

Proof. Given Lemma 2 and the discussion presented above, the proof is similar to the proof of Corollary 5. Q.E.D.

As long as it is possible to observe the distribution of prices conditional on the six states of the world, we can identify the underlying value function for each state (using methods described in Song (2003) and in the previous sections). Once we have these and we know the time preference of the bidders it is just a matter of using the option value functions and some algebra to determine the underlying conditional value distributions.

5 Conclusion

There are three major issues with using eBay data to estimate the demand for an item. The ¯rst is that some bids and bidders are censored because potential bidders enter an auction after the price has risen above their willingness to pay. The second issue is that there may be observed and unobserved heterogeneity across bidders, items and auctions. The third issue is that an eBay bidder does not face a single auction for a single item, but rather faces a sequence of auctions for a single item. This paper looks at each issue in turn.

The ¯rst section develops on ideas presented in Song (2003) and Athey and Haile (2002), and suggests an alternative method for identifying demand in single eBay auctions. Athey and Haile (2002) shows that in certain auctions demand can be identi¯ed from observing the price and the number of bidders. Unfortunately, in general eBay auctions it is not possible to observe the number of bidders. Song (2003) shows that for a certain set of eBay auctions it is possible to identify demand even when the number of bidders is unknown if the distribution of two order statistics are observed. However, in many cases it is not possible to observe two order statistics in eBay auctions. This paper presents two alternative approaches. The ¯rst assumes a particular distribution on the number of potential bidders. The second makes an additional structural assumption. It is shown that under these additional assumptions,the distribution of values is identi¯ed.

The second section generalizes the results of the ¯rst section to the case where there is auction heterogeneity. The paper shows that a traditional demand model is non-parametrically identi¯ed using eBay type data. The model allows for general functional form relationships between observable characteristics of the item and the bidder and unobserved item heterogeniety. The proof of the proposition suggests a method for non-parametrically estimating the model. Athey and Haile (2002) show that when the number of bidders is known the underlying value distribution can be identi¯ed when there is unobserved auction heterogeneity. The section presents assumptions and requirements on the data for identi¯cation in this case where the number of bidders is unknown.

The third section considers an eBay bidder facing an in¯nite sequence of auctions for a single item. Following Dixit and Pindyck (1994), Zeithammer (2004a,b), Arora et al. (2002) and others, it is shown that winning an eBay auction can be thought of as \killing" an option to bid on a future auction for the same item. The implication is that the value of the item won is equal to its actual value less the value of the item's option. We can thus reinterpret the value of the item in a single auction in this way. Following Song (2003) it is still a BNE for all bidders to bid their value for the item in each auction (if they have the opportunity to bid at their last opportunity). Thus the distribution of values for the item in a particular auction is identi¯ed following Song (2003) and the results in the ¯rst section. The section shows that given certain data requirements and certain assumptions on each bidder's preferences over time, the value distribution for the item that is independent of any particular auction can be identi¯ed.

References

Adams, Christopher P. and Laura Bivins , \Demand for Cars on eBay," January 2004. Federal Trade Commission.

—, William Vogt, and Hao Xu, \Digital Demand: The demand for digital cameras on eBay," March 2004. Federal Trade Commission.

Arora, Ashish, Hao Xu, Rema Padman, and William Vogt, \Optimal Bidding in Sequential Online Auctions," 2002. Heinz School of Public Policy, Carnegie Mellon University.

Athey, Susan and Philip Haile, \Identi¯cation in Standard Auction Models," Econometrica, 2002, 70 (6), 2170{2140.

Bajari, Patrick and Ali Hortacsu, \Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions," RAND Journal of Economics, 2003, 34 (2).

— and —, \Economic Insights from Internet Auctions," February 2004. Duke University.

— and C. Lanier Bankard, \Demand Estimation with Heterogenous Consumer and Unobserved Product Characteristics: A Hedonic Approach," January 2004. Stanford University.

Berry, Steven, James Levinsohn, and Ariel Pakes, \Automobile Prices in Market Equilibrium," Econometrica, 1995, 60 (4), 889{917.

Cohen, Adam, The Perfect Store: Inside eBay, ¯rst ed., Little, Brown and Company, 2002.

Deltas, George, \Auction Size and Price Dynamics in Sequential Auctions," February 1999. University of Illinois.

Dixit, Avinash K. and Robert S. Pindyck, Investment Under Uncertainty, Princeton University Press, 1994.

Froeb, Luke, Steven Tschantz, and Philip Crooke, \Second-Price Auctions with Power-Related Distributions," February 2001. Vanderbilt University.

Greene, William, Econometric Analysis, fourth ed., Prentice Hall, 2000.

Guerre, E, I Perrigne, and Q Vuong, \Optimal Nonparametric Estimation of First-Price Auctions," *Econometrica*, 2000, *68*, 525{574.

Jofre-Bonet, Mireia and Martin Pesendorfer, \Estimation of a Dynamic Auction Game," *Econometrica*, September 2003, *71* (5), 1443{1489.

Krasnokutskaya, Elena, \Identi¯cation and Estimation in Highway Procurement Auctions under Unobserved Auction Heterogeniety," August 2003. University of Pennsylvania.

Lucking-Reiley, David, \Auctions on the Internet: What's Being Auctioned and How?," *The Journal of Industrial Economics*, September 2000, *68* (3), 227{252.

Mirman, Leonard, Larry Samuelson, and Amparo Urbano, \Monopoly Experimentation," *International Economic Review*, 1993, *34* (3), 549{563.

Nevo, Aviv, \A Practicioner's Guide to Estimation of Random-Coe±cients Logit Models of Demand," *Journal of Economics and Management Strategy*, 2000, *9* (4), 513{548.

Resnick, Paul, Richard Zeckhauser, John Swanson, and Kate Lockwood, \The Value of Reputation on EBay: A controlled experiment," May 2003. University of Michigan.

Rezende, Leonardo, \Auction Econometrics by Least Squares," November 2002. University of Illinois.

Roth, Alvin E. and Axel Ockenfels, \Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *American Economic Review*, September 2002, *92* (4), 1093{1103.

Rust, John, *Structural Estimation of Markov Decision Processes*, Vol. 4 of *Handbook of Econometrics*, Amsterdam: North-Holland, R.F. Engel and D.L. McFadden (eds).

Song, Unjy, \Nonparametric Estimation of an eBay Auction Model with an Unknown Number of Bidders," November 2003. University of Wisconsin.

Zeithammer, Robert , \Forward-looking Bidders in Sequential Auctions: A Theory of Bargain-hunting on eBay," September 2003. University of Chicago.

_ , \Forward-Looking Bidding in Online Auctions," April 2004. University of Chicago.

_ , \A Theory of Sequential Auctions with Information about Future Goods," April 2004. University of Chicago.

Zhang, Alex, Dirk Beyer, Julie Ward, Tongwei Liu, Alan Karp, Kemal Guler, Shailendra Jain, and Hsiu-Khuern Tang , \Modeling the Price-Demand Relationship Using Auction Bid Data," July 2002. Hewlett-Packard.