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MARKET-SHARE QUOTAS

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Market-Share Quotas

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Abstract: Anecdotal evidence reveals that an import quota is not always filled when the quota is specified in terms of a market-share limit instead of a quantity limit. In a simple Cournot duopoly, we provide a theoretical rationale for this outcome. Imposing a market-share quota eliminates the possibility of a pure-strategy equilibrium. Instead, a mixed-strategy equilibrium arises where only the domestic firm mixes choices. The quota constraint is binding under one of the two equilibrium domestic strategies, but it is not binding under the other. Compared to a tariff that restrains the foreign market share to an equivalent level, domestic profits are always higher and consumer surplus is always lower under the market-share quota. Social welfare is lower under the market-share quota when the domestic firm uses its "constrained" strategy, but this outcome may be reversed when the domestic firm uses its "unconstrained" strategy.

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Market-Share Quotas
1. Introduction

Successive rounds of GATT negotiations have reduced the importance of tariffs as a policy instrument for protecting domestic industry. Nontariff barriers have emerged in their place, and prominent among them are quantitative restrictions such as quotas, voluntary restraint agreements, and orderly marketing arrangements. 1 More recently, po1icymakers have experimented with a new type of protective policy which can be referred to as a "market-share quotaBo48.54 531.85

foreign firms. In markets with imperfect competition, this policy changes strategic behavior considerably. A market-share quota limits imports to a given percentage of the market -- thus, this policy merely sets a maximum level for the ratio of import sales to domestic sales. This ratio constraint must be translated to a constraint on the absolute quantity of import sales, but that can only occur after domestic firms have committed to a given quantity of sales. To the extent that a domestic firm commits to a given quantity of sales through the chosen level of a strategic variable such as output or capacity, it can use that choice to determine the severity of the market-share quota as a quantitative restriction.
To illustrate how a market-share quota affects strategic behavior,
a Cournot duopoly is examined with one domestic firm and one foreign firm. When the market-share constraint is actually binding, the foreign firm desires to sell more of its output in the home market. Any increase in the domestic firm's sales then raises the permitted level of imports, and is therefore matched by a corresponding increase in the foreign firm's sales. This reaction dampens the incentive for the domestic firm to commit to higher sales by increasing its output. However, once the domestic firm commits to a sufficiently high sales level, the foreign firm can sell all that it desires to the home market without exceeding the market-share limit. Since the foreign firm's sales are no longer constrained by the quota, the domestic firm now expects that an increase in its sales will not induce any change in foreign sales. This result implies that the domestic firm's conjecture about rival behavior changes once its sales reach a sufficiently high level.
Technically,

firm's profit function is no longer globally concave. The domestic firm thus considers two possible strategies in choosing its output: (1) commit to a low sales

to an equivalent level, domestic profits are higher and domestic consumer surplus is lower under the market-share quota. Social welfare is lower when the domestic firm uses its "constrained" strategy, but it may be higher when the domestic firm uses its "unconstrained" strategy.
These results often continue to hold when the foreign firm has the ability to shift output across markets subsequent to the domestic firm's output commitment. However, under these conditions, it is instead possible that a pure-strategy equilibrium occurs where the domestic and foreign firms split monopoly profits according to the market shares given by the quota. Social welfare in unambiguously lower in this equilibrium than in that obtained under an equally restrictive tariff.
This paper is organized as follows. Section 2 describes the followen

letters denote the foreign firm):
l£(x,X) p(x+X)x - c(x) (1)
IT(x,X) p(x+X)X - C(X), (2)

where p is the inverse demand function, c(C) is the domestic(foreign) cost function, and x(X) is domestic(foreign) output. The corresponding first-order conditions are:

p + p'x - c x o (3)
p+p'X-Cx 0, (4)

where ' denotes the derivative of the inverse demand function.
We presume that conditions are satisfied so that a unique, stable equilibrium exists. Specifically, it is assumed that:
l£xx < l£xX < 0; llxx < llXx < o. (i)

Condition (i) implies that the reaction functions are decreasing with respect to rival output. 7
Let xN(X) and XN(x) represent the Cournot reaction functions for the domestic and foreign firms, respectively. We denote the free-trade Cournot-Nash equilibrium as (x, ,X,) For future reference, k, denotes

3. The Effect of a Market-Share Quota on Reaction Functions
Prior to the output stage, let the policymaker commit to a market-
share quota k, where 0 < k < 1. If X- represents the quantity of foreign sales to the home market, then the quota requires that X-/(x+X-) ~ k, or

3.1 The Foreign Firm's Reaction Function

Consider the impact of the quota on the foreign firm's reaction

function. While the domestic firm sells its entire output, the quota may

potentially constrain foreign sales to the home market below the foreign

output level for that market. 9 For a given domestic output choice, the

limit on foreign sales is XO(x,k) - (k/(l-k»x. When X ~ XO(x, k), foreign

sales equal foreign output (i.e., X- - X). When X > XO(x,k), foreign sales

are instead constrained at the quota limit (i.e., X- - XO(x,k».

The foreign firm's revenue is determined by its sales X-, but its

costs are determined by its output X. Based on the prior discussion, we can

now describe foreign profits under the market-share quota:

fiO(x,X,k) - p(x+X-)X- - C(X) (5)

where X- - X for X ~ XO(x,k)

- XO(x,k) for X > XO(x,k).

9 In this section and the next section, the foreign firm commits to an output level specifically intended for the home market. Either the home market is the only source of demand, or conditions exist that make it prohibitively costly to transfer output across markets once the magnitude of the quantitative restriction becomes known. These conditions may include unique design requirements for individual markets, customer obligations (i.e., explicit and implicit guarantees to customers or distributors through contracts, orders, etc.) , or substantial transportation costs from alternative sources of supply. Even if none of these circumstances is present, the above assumption still applies when the foreign firm must commit quantities to each market prior to observing the output level chosen by the domestic firm.
Later, the analysis considers a situation where the foreign firm
sells to more than one market and output is perfectly fungible across markets. In this situation, the foreign firm commits to a total output level, and then decides at a latter stage how to optimally allocate that output across markets. Using this framework, it will be shown that the equilibrium results are often qualitatively similar to those derived below.

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Lemma 1: The optimal foreign reply under the

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it raises the quantity of foreign sales allowed by the market-share quota. Thus, the foreign firm can sell more of its available output. By contrast, when x > xQ(X,k), the quota limit exceeds the foreign output level. Within this range, any expansion in domestic output does not affect foreign sales since the foreign firm already sells its entire output. According to this reasoning, the reaction of foreign sales to an expansion in domestic output is positive for x < xQ(X,k) and zero for x > xQ(X,k). This behavior causes the marginal value of domestic output to jump upward at xQ(X,k).
The upward jump in ~Qx(x,X,k) implies that ~Q(x,X,k) must be convex
in the neighborhood of xQ(X, k); hence, domestic profits cannot attain a maximum at xQ(X,k). In other words, it is never optimal for the domestic firm to set its output so that the foreign firm's output is at the quota limit. Therefore. no pure-strategy equilibrium occurs where the foreign firm sets output at the quota limit. XQ~.
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the best constrained strategy.13 The other choice, XN(X) , satisfies the first-order condition from (lOb). We refer to

sell its entire output. It must instead sell only the quota limit.
Second, if the choice of domestic output indicated by the best constrained strategy would actually make the quota nonbinding, then the domestic firm instead maximizes profits by using its best unconstrained strategy. More precisely, at all values of X where xM(k) ~ xQ(X,k), it holds that XN(X) is the optimal domestic reply.15
Let X**(k) solve xM(k) ~ xQ(X,k) i.e., X**(k) is the foreign
output level where if the domestic firm uses its best constrained strategy, then foreign output is at the quota limit. Given that xM(k) is independent of X, and that xQ(X,k) is increasing in X, it follows that xM(k) ~ xQ(X,k) for X ~ X**(k). Thus, according to the prior discussion, xN(X) is the optimal domestic reply for X ~ X**(k). Given this domestic reply, the foreign firm can sell its entire output without exceeding the quota limit. We have just shown that xN(X) is an optimal reply for X ~ X**(k), and
that xM(k) is an optimal reply for X ~ X*(k). Define Z(X,k) - ~Q(xM(k),X,k)
- ~Q(xN(X) ,X, k) as the difference in domestic profits from using the best "constrained" and "unconstrained" strategies. It follows that Z(X, k) < 0 for X ~ X**(k), and that Z(X,k) > 0 for X ~ X*(k).
Now, assume temporarily that Z(X,k) is monotonically increasing in X
for X**(k) < X < X*(k). If this assumption holds, then it follows from the above results that a unique value of X satisfies Z(X,k) o. At this value

of X, o.

quota is completely described.
In fact, Z(X,k) is monotonically increasing for X**(k) < X < X*(k). Within this range, the foreign firm is unable to sell its entire output if the domestic output response is xM(k). Hence. domestic profits under the constrained strategy are unaffected by increases in foreign output. By contrast, the foreign firm is able to sell its entire output if the domestic

output response is xN (X) .16 Hence. domestic profits under the unconstrained strategy decline as foreign output increases. Since 1fQ(xM(k) ,X, k) is independent of X and 1fQ(xN(X) ,X, k) is decreasing in X, it

follows that Z(X,k) (i.e., 1fQ(xM(k),X,k) - 1fQ(xN(X),X,k» is increasing in X i

of foreign output, the domestic firm uses a constrained strategy where it uses the quota to constrain foreign sales below the foreign output level. As foreign output falls, the domestic firm's profits from the constrained strategy are unaffected, but its profits from the unconstrained strategy increase. When foreign output reaches a sufficiently low level, both strategies result in the same level of profits. Any further decline in foreign output then causes the domestic firm to use the unconstrained strategy, which renders the quota nonbinding.

4. Results

Based on our prior discussion, the following proposition can be established:

Proposition 1: No equilibrium exists iTm uil 11.4705 0 0 10

does not preclude the existence of a mixed-strategy equilibrium: 17

Proposition 2: A unique mixed-strategy equilibrium exists whenever the quota is set at or below the free-trade foreign market share. In this equilibrium, the foreign firm sets output at XE(k) as a pure strategy while the domestic firm mixes output choices between xM(k) and XN(XE(k». When the domestic firm chooses xM(k), foreign output is above the quota limit (i.e., the foreign firm cannot sell its entire output). When the domestic firm chooses XN(XE(k», foreign output is below the quota limit.

Proof: Refer to Figure 1. It has been previously shown that at (xM(k),XE(k», the foreign firm is unable to sell its entire output. Thus, XE(k) >

and that ~xx < 0, it follows that ~X(XN(XE(k» ,XE(k) ,k) > 0. 19
Let Y(p,X,k) - p~(xM(k),X,k) + (l-p)~(xN(XE(k»,X,k), where p(l-p)
is the probability that the domestic firm chooses xM(k)(XN(XE(k»). From our prior assumptions, Y(p,X,k) is concave in X.20 Thus, Y(p,X,k) attains a maximum if X satisfies Yx(p,X,k) - O.
Since it has been previously shown that ~x(xM(k) ,XE(k), k) < 0 and ~X(XN(XE(k»,XE(k),k) > 0, there exists p-p* such that Yx(p*,XE(k),k) - o. Thus, if the domestic firm chooses xM(k) with probability p* and xN(XE(k» with probability l-p*, the foreign firm maximizes its profits by choosing XE(k). Since, from Lemma 2, the domestic firm earns the same maximum level of profits by choosing either xM(k) or XN(XE(k» in reply to XE(k) , a mixed-strategy equilibrium arises. QED

The intuition behind Proposition 2 is as follows. Cons ider two

identical firms with constant marginal costs. Let a quota be imposed at the

free-trade foreign market share, i.e., k - kl - 1/2. If the foreign firm

continues to select its free-trade output XI' then the domestic firm's best

strategy is to reduce its output to xM(k/), which represents one half of the monopoly output level. Since the quota constrains foreign sales to the same

level as domestic output, this strategy implies that the domestic firm

claims one half of monopoly profits instead of one half of Cournot profits.

If the foreign firm tries to avoid "overproduction" by reducing its

own output to xM(k/), then the domestic firm earns even greater profits by

19 It is possible that XQ(xN(XE(k», k), not XN(xN(XE(k»), is the optimal reply to xN(XE(k». Nonetheless, this result still holds. Since a reply of XQ(XN(XE(k», k) puts foreign output at the quota limit, while a reply of XE(k) puts foreign output below the quota limit, it follows that XO(XN(XE(k»,k) > XE(k). Moreover, if XO(xN(XE(k»,k) is an optimal response to xN(XE(k». then ~X(xN(XE(k»,X,k) ~ 0 as X~XQ(xN(XE(k»,k)-. Since XE(k) < XQ(xN(XE(k», k), and since ~~ < 0 for X < XQ(XN(XE(k», k), it follows from the prior result that ~x(x (XE(k»,XE(k) ,k) > O.

20 Within the relevant range of foreign output choices, this condition holds unless marginal costs decrease rapidly. Moreover, this condition holds globally whenever marginal costs are nondecreasing.
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boosting output above the original free-trade level and rendering the quota nonbinding. Thus, the domestic firm will not let the foreign firm produce just enough output to reach the quota limit. This behavior eliminates any possibility for a pure-strategy equilibrium.
An equilibrium can occur only if the foreign firm produces more output than xM(k/), but less output than XI' As the foreign firm raises its output above xM(k/), it becomes less profitable for the domestic firm to produce sufficient output to render the to An not

use the quota to constrain foreign sales below the foreign output level. With this in mind, the foreign firm reduces output, but only to the point where the domestic firm is indifferent between using quota protection and abandoning quota protection.
Based on the results from Proposition 2, we compare output,

only if (xr,Xr) - (xN(X*(k»,X*(k». Since XE(k) < X*(k) by Lemma 2, foreign output is lower under the quota.
Consider the domestic reply, xN(XE(k». Given that XE(k) < X*(k) ,
and that XN(X) is declining in X, it follows that xN(XE(k» > xN(X*(k». Since -1 < dxN(X)/dX < 0 by condition (i), it also holds that xN(XE(k» + XE(k) < xN(X*(k» + X*(k). These results show that domestic output is higher, but total output is lower under the quota. Hence, price is higher (i.e., consumer surplus is lower). Domestic profits are higher under the quota because foreign output is relatively lower.
Consider the domestic reply, xM(k). We have shown that this reply is optimal only if it causes foreign sales to be constrained at the quota limit
- - i.e., xM(k) < xQ(XE(k) ,k). Given that xQ(X,k) is increasing in X, and that XE(k) < X*(k) , it follows that xM(k) < xQ(XE(k), k) < xQ(X*(k), k) - xN(X*(k». Since xM(k) < xN(X*(k», domestic output is lower under the quota. Given that the foreign market share equals k under both the quota and the tariff, total consumption must also be lower under the quota. Thus, price is higher (i. e., consume

equally restrictive tariff. The reduction in the home country's welfare due to a higher price for imports is now counterbalanced by an efficiency gain due to increased domestic output. In this situation, a qualitative welfare comparison between the quota and the tariff requires more specific cost and demand information. Based on the above discussion, we conclude:

Proposition 4: Consider a market-share quota, k S k,. Relative to a cum-tariff (or free-trade) equilibrium where the foreign market share equals k, the home country's welfare is lower under the quota when the domestic firm chooses xM(k). Home welfare may be higher, equal, or lower under the quota when the domestic firm chooses xN(XE(k».

Finally, consider a quota that lies above the free-trade foreign market share, i.e., k > k,. When the quota is set at a sufficiently large market share, the domestic firm foresakes using the quota to constrain foreign sales. The domestic firm instead reverts to an "unconstrained" strategy, and a unique pure-strategy equilibrium occurs at the free-trade equilibrium, (x"X,) - (xN(X,),XN(x,». Of course, the quota is not binding in this equilibrium.
While the above situation arises when the quota is substantially
above the free-trade market share, a far different outcome occurs when the quota is reduced to a level that is closer to the free-trade market share. In fact, there exists a range of quota levels above the free-trade foreign market share where no pure-strategy equilibrium occurs. Only a mixed-strategy equilibrium occurs in which the quota is binding with positive probability. Thus. an "apparently nonbinding" quota may alter strategic

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assume that output can be transferred (costlessly) across markets.
Under these conditions, a mixed-strategy equilibrium may still arise
that is qualitatively similar to that obtained in the previous section. Alternatively, a pure-strategy equilibrium may arise where the domestic firm sets output at xM(k) , and foreign sales to the horne country are constrained by the quota at X0{xM(k),k). All remaining foreign output is sold to other markets. In this case, the imposition of a market-share quota leads to an outcome where the domestic and foreign firms split monopoly profits according to the market shares given by the quota. Compared to an equally restrictive tariff, welfare is unambiguously lower in the home country under the quota. Domestic and foreign output are relatively lower under the quota, which imply that prices are relatively higher in the home market. Although consumers are hurt 13.0281 021453 r5.53i8e71.93 4400ome country

abandoning quota protection. Due to this probable reaction by the domestic firm, the foreign firm could not reduce its allocation for the home market Consequently, the possibility of a pure-strategy
equilibrium was eliminated in our previous model.
In contrast. when output can be transferred across markets. the foreign firm can potentially react to any domestic output expansion by transferring output into the horne market. If the foreign firm can credibly transfer substantial output into the home market, then the domestic firm will keep its output at xM(k) and split monopoly profits even when it observes that the foreign firm has allocated XQ(xM(k),k) to the home market. In this case, a pure-strategy equilibrium occurs. Otherwise, a mixed-strategy equilibrium occurs similar to that described previously.
Thus, the equilibrium outcome depends on how much output can be credibly transferred between markets by the foreign firm. This is determined by the behavior of the foreign firm's marginal revenue function in its various markets, which, in turn, depends on demand conditions and the degree of competition in those markets. If the foreign firm's marginal revenue function is relatively flat in markets outside of the home country (possibly due to increased competition), then any expansion in domestic output above xM(k) would potentially lead to a considerable expansion in the foreign firm's sales to the home market. This behavior makes it less profitable for the domestic firm to deviate from xM(k) , and leads to a pure-strategy equilibrium. 26 If, however, the foreign firm's marginal revenue function is relatively steep in its overseas markets, then the foreign firm

26 In fact, a pure-strategy equilibrium is necessarily obtained when
the foreign firm is a prire taker in its other markets.
26

loses substantial revenue in those markets by transferring a significant amount of output into the home country. Since the foreign firm would not transfer much output under these circumstances, the domestic firm would earn higher profits by deviating from xM(k) after observing the foreign reaction to that output choice. A pure-strategy equilibrium is not attainable in this case; instead, a mixed-strategy equilibrium exists (that is similar to the equilibrium described in the previous section).
To demonstrate the above result, let the foreign firm serve one market in addition to the home country's market. The domestic firm and the foreign firm choose output simultaneously in the first stage, but the foreign firm does not divide its output between its two markets until the second stage. 27 Since the foreign firm allocates its output to each market after the domestic firm makes its output decision, the foreign firm acts essentially as a Stacke1berg follower in the second stage. In turn, the domestic firm uses its output choice in a fashion similar to a Stacke1berg leader, realizing that changes in its output may subsequently alter the foreign firm's allocation between markets. 28
Assuming that marginal costs are constant ,29 let Z, denote the quantity of foreign sales at which marginal revenue equals marginal cost in

27 If the foreign firm must make a binding output allocation to each market at the same time that the domestic firm chooses its output, then the
outcome is identical to our previous results.
28 Note that the Stacke1berg aspects of the present model share
certain similarities with the pure Stacke1berg model of Mai and Hwang
(1989). Under appropriate conditions, it will be shown that similar results
are achieved. However, the simultaneous output decisions in the first stage
of the present model are ana1agous to the struct189.13 Tm27 Tm (present) 12.56

the foreign firm's other market. 30 Under the market-share quota, the foreign output reaction function in the first stage, XR'(x,k), is described as follows:
XR'(x,k) - XR(x,k) + Z/,
where XR(x,k) - XO(x,k) for x <

domestic firm's output will cause the foreign firm to allocate a smaller share of its output to the home market whenever the quota constraint is "nonbinding" (1. e., whenever the foreign firm is able to equate marginal revenue across markets).31 Thus, similarly to our prior model, there exists a threshold level of domestic output where the domestic firm changes its conj ecture concerning the response of foreign sales to an increase in its output. The domestic firm's conjecture is positive at output levels below this threshold, but the domestic firm's conj ecture is negative at output levels above this threshold. Again, the domestic firm's conjecture declines once the threshold level is attained.

This negative change in the domestic firm's conjecture 0 21 17079 l5crke0727