Moral Hazard and Renegotiation: Multi-Period Robustness

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Abstract

Is the second best outcome of static agency models renegotiation proof? In models with one period of renegotiation, Fudenberg and Tirole (1990) answer no when the principal makes the o er, while Ma (1994) and Matthews (1995) answer yes when the agent makes the o er. This paper analyzes the robustness of these two claims when there are more periods of renegotiation. With a known number of periods, if the principal makes at least one o er, even if the agent makes the o er in every other period, the equilibrium is identical to Fudenberg and Tirole equilibrium. With an uncertain number of periods, the agency problem is even more severe than in the Fudenberg and Tirole model.

1 Introduction

Consider the standard principal-agent model: a risk neutral principal contracts with a risk averse agent to induce the agent to exert unobservable e ort to increase the principal's pro fit. The second best solution to this problem occurs when the parties can commit not to renegotiate the contract (Holmstrom (1979), Shavell (1979), Grossman and Hart (1983)). In many cases, however, the realization of the principal's pro fit occurs sometime after the agent chooses his e ort level. (Throughout the paper, I will use female pronouns for the principal and male pronouns for the agent.) A product's profitability occurs long after the product development e ort of the manager; there are often many weeks between a sharecropper's farming e ort and the realization of the field's crop yields. In these, and many other similar, situations, there is ample opportunity for the principal and agent to renegotiate the original incentive contract after the agent has already chosen his e ort level. The question then arises, is the standard commitment solution to the principal-agent problem renegotiation proof?

In the current literature, there are two conflicting answers to this question. Fudenberg and Tirole (1990) show that, when the principal can make a take it or leave it o er to the agent after the agent has chosen his action, the principal cannot obtain the outcome that she could obtain if she could commit not to renegotiate. In fact, the principal cannot induce the agent to exert high e ort with probability one. To see this, imagine the agent did exert high e ort. After the action was taken, the principal would have an incentive to completely insure the agent. Knowing this would occur, the agent would have no incentive to exert high e ort in the first place. On the other hand, Ma (1994) and Matthews (1995) show that, when the agent makes the take it or leave it er in the renegotiation period, the second best outcome remains an equilibrium. $^\mathrm{1}$ The reason the agent can be induced to take the high e ort action when the agent, rather than the principal, makes the r-336(t)3(1(ter4(c)-2(r(e)-1r4(c)-2-2(a.8)]TJ-0-351(after)]TJ-0.0037Tc-33.77E33400-1(st0)-2-2(a.8)4(e)- of renegotiation. First, I analyze the case where the number of rounds of renegotiation is known with certainty. Contrary to what one might think, the equilibrium does not depend on who makes the final o er, or the first o er. I fi

even more risk than in the one period contract.

The plan of the paper is as follows. Section 2 describes the model. Section 3 analyzes the case where the number of renegotiation rounds is known with certainty. Section 4 analyzes the case where the number of rounds is uncertain. Section 5 concludes.

2 Model

Consider a simple two e ort, two outcome model (very similar to the one in Fudenberg and Tirole (1990)). The two e ort levels are $\overline{e} > \underline{e}$. The two outcomes are $g > b$. The probability of outcome g is given by $p(e)$. As shorthand, I call this p when the agent exerts e ort \bar{e} and p when the agent exertse ort $\underline{\text{e}}$. The agent's utility function for income w and \textbf{e} ort \textbf{e} is additively separable: $V(w, e) = U(w) - D(e)$ where $U' > 0$ and U'' $<\alpha$ (the agent is risk averse). Fthe wage according to the compensation scheme in place. If the outcome does not occur, then there is another period with the same structure. Notice that discounting will not a ect either party's incentive to agree given that the actual time of payment is only a function of when the outcome occurs, not when any agreement is reached. Thus, for simplicity, I assume no discounting.

3 Known Number of Periods

In this section, I assume that $\mathfrak{q}_\mathfrak{t}$ = 0 for all $\mathfrak{t}<$ T and \mathfrak{q}_T = 1. That is, both the principal and the agent know that the contract in e ect in period T will be the contract that determines the agent's compensation. In this model, one might expect that the final contract would only depend on who makes the final o er. In other words, one might think that if the principal makes the final o er, then this model will generate results identical to that of Fudenberg and Tirole (1990), while if the agent makes the final o er the model's results will be identical to that of Ma (1994). As the first proposition shows, however, this conjecture is incorrect. In fact, so long as the principal makes an er in at least one period $\tilde{\text{t}}-\text{T}$, both the final contract and the maximum probability that the agent exerts high $\rm e$ ort are identical to what they are in Fudenberg and Tirole (1990). 4

Before proceeding to show that the Ma (1994) and Matthews (1995) result about the feasibility of the second best with renegotiation is not robust to any o ers by the principal, it is useful to analyze why this result holds. If the agent proposes any contract other than the second best e cient contract under commitment, then the principal believes he exerted low e ort. Thus, the high e ort agent cannot get more insurance through renegotiation, making it optimal for him to exert high e ort. Even though more insurance is ex post Pareto improving, asymmetric information prevents the high e ort agent fr, 302(c5s)-26c4.302(ing)-388339335(5407(u(u)110Tc5ru(u)11er)-387028Tc9(s)]TJ-0.06(long) (c) $pU_g(\underline{e}) + (1 - p)U_b(\underline{e}) = pU_g(\overline{e}) + (1 - p)U_b(\overline{e})$ (d)

larger when the principal believes the agent chose \bar{e} with probability x^L . If there are two di erent sequences of o ers and responses by the agent that induce the principal to have beliefs x^H and x^L , then if the agent chooses \bar{e} he will choose the sequence that produces x^H , and if the agent chooses e then he will choose the sequence that produces x^L . Thus, one must have $x^H = 1$ and $x^L = 0$. In this case, there must be a flat wage contract that the principal can o er in period t that the agent would accept and that would not be renegotiated. Knowing that the final contract would be a flat wage contract, the agent would not choose high e ort, which contradicts $x > 0$.

So, there are two cases to consider: either \tilde{c} is o ered by the agent or by the principal. If the principal $\mathsf o\,$ ers $\widetilde{\mathsf c},$ then doing $\mathbf s$ must reduce the principal's expected compensation costs below what they would be if c were in e ect at t and must give each type of agent at least as much expected utility as he would receive if c were in e ect at t. In this case, the principal would have ered $\tilde{\text{c}}$ in period t when the initial contract is $\widehat{\text{c}}$. If $\widetilde{\text{c}}$ is $\text{o}\,$ ered by the agent, then doing so must give the agent more expected utility than he would receive if c were in e ect at t and must not increase the principal's expected compensation costs above what they would be if c were in e ect at t. In this case, the agent would have o ered \tilde{c} prior to t when the initial contract was \hat{c} and the principal would have accepted. Moreover, the agent would never accept a later o er by the principal of c . So, c could not be in e ect at t when the initial contract is \widehat{c} , a contradiction.

Since the agent's e ort depends only on the final contract, which will be identical so long as the contract in e ect at t and the principal's beliefs at t are unchanged, $(x, 1 - x)$ remains an equilibrium e ort distribution. Q.E.D.

Next, I show that the principal will not o er a contract that the agent will renegotiate after the principal's final renegotiation o er.

Lemma 2 Let t be the period where the principal makes her final renegotiation o er. If there is a Nash equilibrium where the distribution over e ort levels is $(x, 1 - x)$ with $x > 0$, the initial contract is \hat{h}

of the renegotiation proof constraint is to ensure that the original contract must be a F-T contract, despite the fact that the agent can make up to $T - 1$ of the renegotiation o ers.

Lemma 3 Say there is at least one period \hat{t} T when the principal makes an o er. For any given distribution (x, 1 – x) with $x > 0$, if a contract c is renegotiation proof, then c is a F-T contract.

Proof. Say c is not a F-T contract because either (a), (b), or (c) does not hold. Then, by lemma 2.1 in Fudenberg and Tirole (1990), either it the interim utility of an agent who chooses low-e ort is greater than that of the agent who chooses high e ort, in which case the agent would not choose to work hard $(x = 0)$, or the principal can o er di erent contract when it is her turn to make a renegotiation o er that gives each type of agent as much utility as the original contract and lower the principal's expected payment. The agent will only reject this new o er if, by doing so, it can expect the principal to accept an o er (call this contract \tilde{c}) that he will make in a later bargaining round. If this is the case, then c is not renegotiation-proof. Now say c violates (d). Then, by lemma 2.2 in Fudenberg and Tirole (1990), either (a), (b), or (c) does not hold, or c does not minimize the principal's expected payments subject to each type of agent receiving at least as much expected utility as he receives from c. Thus, there is another contract, c , that lowers the principal's expected payments while leaving each type of agent's expected utility unchanged. The principal with o er c when she can make a renegotiation o

the probability of facing a high e ort agent is large enough that principal can reduce her expected compensation costs when it is her turn to o er by decreasing the riskiness of the high e ort contract even though this requires paying the low e ort agent more.

If the original contract must be a F-T contract, then the constraints facing the principal in her period zero contracting problem are identical as in the game where the principal makes the only renegotiation o er. Thus, as the next proposition establishes, the unique equilibrium is identical also.

Proposition 1 Say there is at least one period \hat{t} T when the principal makes an o er. The optimal final contract that induces $x > 0$ and the agent's e ort distribution $(x, 1 - x)$ is identical to the optimal final contract and the agent's e ort distribution (x, $1-x$) when there is only one period of renegotiation and the principal makes the take it or leave it renegotiation o er.

Proof. By Lemmas 1 and 2, the optimal final contract can be implemented as an initial contract that is renegotiation-proof. By Lemma 3, a renegotiation-proof initial contract must be a F-T contract. Fudenberg and Tirole (1990) show that the optimal contract that induces $x > 0$ with one period of renegotiation where the principal makes a take it or leave it renegotiation o er must also be an F-T contract. Thus, the principal in this game faces the same problem as the principal in the game of Fudenberg and Tirole (1990), hence the final contract and e ort distribution is identical. Q.E.D.

Proposition 1 demonstrates that a more general renegotiation process does not alter the results in Fudenberg and Tirole (1990) so long as the principal gets to make at least one o er and there is no uncertainty about when the outcome will occur (and, thus, about the number of periods of renegotiation). On the other hand, it shows that the Ma (1994) and Matthews (1995) results that renegotiation need not undermine the optimal commitment contract is much less robust. The following corollary gives the most important implication.

Corollary 1

given period when the principal makes o ers is the sum of the probability of the game ending in any of the periods in between the last o er by the principal and her next o er.

First, consider the case with two periods of renegotiation. That is, the principal o ers a contract $c^0 = \{U_g^0(e), U_b^0(e)\}_{e=\overline{e},\underline{e}}$ in period 0. If the agent accepts, then the agent chooses an action in period 1/2. The principal makes a renegotiation o er, $c^1 = \{U_g^1(e), U_b^1(e)\}_{e=\overline{e},\underline{e}},$ in period 1, which the agent accepts or rejects. After this period, the outcome occurs with probability \mathfrak{q}_1 = \mathfrak{q} Since the game could end after this period, the agent must also choose a compensation scheme from the menu of the contract in e ect (either c^0 or c^1). With probability 1 – q, the outcome does not occur, and there is a period 2 where the principal makes another renegotiation o er, $c^2 = {U_0^2(e), U_b^2(e)}_{e=\overline{e},\underline{e}}.$ After the agent accepts or rejects this o er and chooses a compensation scheme, the outcome occurs with probability \mathfrak{q}_2

high e ort that the principal can implement, x

$$
x\{q[\bar{p} (U_g^1(\bar{e})) + (1-\bar{p}) (U_b^1(\bar{e}))] + (1-q)[\bar{p} (U_g^{2\bar{e}}(\bar{e})) + (1-\bar{p}) (U_b^{2\bar{e}}(\bar{e}))\}
$$

+
$$
(1-x)\{r (\underline{U}) + (1-r)[q(p
$$
 (1-*r*)] $q(p$ (1-*r*)] $q(p$

principal wants to implement, she must give the agent at least as much rent as he would receive in the one-period game. The following constraints, then, represent the best case for the principal: she gives each agent type exactly the same expected utility as they would receive in the one period game.

$$
q[pU_0^1(\bar{e}) + (1-\bar{p})U_b^1(\bar{e})] + (1-q)[pU_0^{2\bar{e}}(\bar{e}) + (1-\bar{p})U_b^{2\bar{e}}(\bar{e})] = pU_g(\bar{e}) + (1-\bar{p})U_b(\bar{e})
$$
(9)

$$
q[\underline{p}U_{g}^{1}(\overline{e}) + (1-\underline{p})U_{b}^{1}(\overline{e})] + (1-q)[\underline{p}U_{g}^{2\overline{e}}(\overline{e}) + (1-\underline{p})U_{b}^{2\overline{e}}(\overline{e})] = \underline{p}U_{g}(\overline{e}) + (1-\underline{p})U_{b}(\overline{e}) = \underline{U}
$$
\n(10)

If the principal cannot implement a given x at the same or lower costs as in the one period game subject to these constraints, then her costs of implementing x are necessarily higher than in the one period game. The following proposition shows that this is the case.

Proposition 3 If q $(0, 1)$, then the cost of implementing any given probability of high e ort x is strictly greater when there are (potentially) two periods of renegotiation than it is in the one period game.

Proof. The di erence in the cost of implementing any given x is given by:

$$
x\{q[p (U_g^1(\overline{e})) + (1-p) (U_b^1(\overline{e}))] + (1-q)[p (U_g^{2\overline{e}}(\overline{e})) + (1-p) (U_b^{2\overline{e}}(\overline{e}))]
$$

\n
$$
-[p (U_g(\overline{e})) + (1-p) (U_b(\overline{e}))]\}
$$

\n
$$
+(1-x)(1-r)\{[q(\underline{p} (U_g^1(\overline{e})) + (1-\underline{p}) (U_b^1(\overline{e}))) + (1-q) ([\underline{p}U_g^{2\overline{e}}(\overline{e}) + (1-\underline{p})U_b^{2\overline{e}}(\overline{e}))]
$$

\n
$$
-([\underline{p}U_g(\overline{e}) + (1-\underline{p})U_b(\overline{e}))\}
$$
 (11)

Since is convex, this implies that:

 $(\underline{p}U_{g}(\overline{e}) + (1-\underline{p})U_{b}(\overline{e})) <$ 1 $_{g}^{1}(\overline{e})) + (1-\underline{p}) \quad (U_{b}^{1})$ $\binom{1}{b}(\overline{e})) + (1-q)$ (p

$$
\frac{dU_g^{2\overline{e}}(\overline{e})}{dr} = -\frac{(\overline{p} - \underline{p})(1 - x) \cdot \langle \underline{U} \rangle}{\overline{p} \times [\overline{p} \quad \langle \langle U_g^{2\overline{e}}(\overline{e}) \rangle + (1 - \overline{p}) \quad \langle \langle U_g^{2\overline{e}}(\overline{e}) \rangle]}
$$
(17)

$$
\frac{dU_b^{2\overline{e}}(\overline{e})}{dr} = \frac{(\overline{p} - \underline{p})(1 - x) \quad '(\underline{U})}{(1 - \overline{p})x[\overline{p} \quad ''(\underline{U}_b^{2\overline{e}}(\overline{e})) + (1 - \overline{p}) \quad ''(\underline{U}_g^{2\overline{e}}(\overline{e}))]}
$$
(18)

Using these, di erentiating the di erence in expected payment to the high types with respect to r gives:

$$
\frac{(\overline{p} - \underline{p})(1 - x)(1 - q) \sqrt{(\underline{U})\{[(\sqrt{(\underline{u})}(\overline{e})) - \sqrt{(\underline{U}_0^1(\overline{e}))}] - [\sqrt{(\underline{U}_g^{2\overline{e}}(\overline{e})) - \sqrt{(\underline{U}_b^{2\overline{e}}(\overline{e}))}]\}}{\overline{p}qx[\overline{p} \sqrt{(\underline{U}_b^{2\overline{e}}(\overline{e})) + (1 - \overline{p}) \sqrt{(\underline{U}_g^{2\overline{e}}(\overline{e}))}]}}
$$
(19)

The sign of this is the sign of:

$$
[' (Ug1(\overline{e})) - ' (Ub1(\overline{e}))] - [' (Ug2\overline{e}(\overline{e})) - ' (Ub2\overline{e}(\overline{e}))]
$$
 (20)

By (14), this is negative only if $\underline{U^{2\bar{e}}}<\underline{U}$. But $\underline{U^{2\bar{e}}}<\underline{U}$ and $r>0$ and (8) imply that (20) is positive, a contradiction. This proves the result. Q.E.D.

Unlike the e ect of adding one period of certain renegotiation, adding the possibility of a second period not only reduces the maximum probability of high e ort, it also makes any given probability of high e ort more expensive. Because the optimal renegotiation proof contract with one period of renegotiation is not renegotiation proof when another period is added, the principal has to use the first period contract to screen. This means that the first period contract is riskier than in the one period game and that a low e ort type does not choose the full insura.32(o)-22niTj π 7tTT31Tfeen. $\;$ Th Proposition 4 Let T 2. If q_t (0, 1) for $t < T$ and $q_T = 1$, then the cost of implementing any given probability of high e ort x is strictly greater than it is in the one period game.

Proof. The proof is by induction. Proposition 3 showed that the result holds for $T = 2$. Assume it is true for T − 1 periods. Any T period setting is like a T − 1 period setting with the last period split into two. So, if costs could be lower with T periods, then consider the T −1 setting where q_{T-1} = 1 but everything else is identical to the T period setting. Now consider the same contract sequence for the first T - 2 periods as the T period contract sequence that gives lower costs than the one period game. For period T - 1, o er the optimal F-T contract that gives each type the same expected utility he would get from the last two periods in the T period case. Then, collapsing these periods together does not change incentives to select among the menus of the prior contracts. By the argument above for the two period case, collapsing the last two periods into one is strictly cheaper for the principal. So, if the T period setting reduced compensation costs, then the T − 1 period setting must also, which contradicts the inductive hypothesis. Q.E.D.

Whenever there is any uncertainty about when the outcome will occur and multiple periods of renegotiation are possible, then it is more costly to induce any distribution of e ort than when there is only one (or any fixed number) of periods of renegotiation. In many settings, this is likely to be the case. In sharecropping contracts, the exact day that the crop will yields are realized is unlikely to be known with certainty. When the principal has hired an agent to develop or run a new project, often the profits from this project will come at an uncertain date. This is especially true when the project involves selling a good, since sales are usually realized at least somewhat stochastically. In fact, if contracts can be renegotiation fairly quickly, there need not be much uncertainty as to when the outcome will occur for the results in this section to apply.

5 Conclusion

With only one round of renegotiation, the e ect of renegotiation on agency contracts has been found to be very sensitive to which party, the principal or the agent, makes the o er. In this paper, I analyzed the robustness of these two di erent models by allowing for multiple rounds of renegotiation. With a fixed number of rounds, I showed that as long as the principal can make at least one renegotiation o er, o ers by the agent do not a ect the optimal contract. The equilibrium is identical to the one where the only o er is made by the principal, in which renegotiation does undermine the optimal contract with commitment. This suggests that the key di erence in the

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one period models is not which party makes the o er, but whether the principal gets to make an er. Unless the principal will not ever get the chance to make a renegotiation o er, renegotiation does prevent the principal from inducing the agent to work hard with probability one.

I then analyze the case where there is uncertainty as to when the verifiable outcome will occur, which generates uncertainty as to the number of available rounds for renegotiation. I found that in this setting, the renegotiation problem is even more severe than in the one period game. Allowing for an uncertain number of rounds both further limits the distributions of actions the principal can induce and raises her cost of inducing any given distribution. That is, not only is the pessimistic result in Fudenberg and Tirole (1990) about the e ect of renegotiation more robust than the more optimistic one in Ma (1994), but the negative e ects of renegotiation may often be even stronger than in the Fudenberg and Tirole model.

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