Moral Hazard, Mergers, and Market Power

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[1]L wkdqn Mdh Qdkp dqg vhplqdu sduwlflsdqwv dw wkh Ihghud0 Wudgh Frpplvvlrq dqg wkh Dphulfdq Odz dqg Hfrqrplfv Dvvrfldwlrq phhwlqjv iru kh0six0 frpphqwv1 Wklv sdshu grhv qrw qhfhvvdul0| uhsuhvhqw wkh ylhzv ri wkh Ihghud0 Wudgh Frpplvvlrq ru dq| lqglylgxd0 Frpplvvlrqhu1

Prvw dqd0|vlv ri pdunhw srzhu dvvxphv wkdw pdqdjhuv dfw dv shuihfw djhqwv iru wkh vkduhkroghuv1 Wklv sdshu uhod{hv wklv dvvxpswlrq1 Zkhq pdqdjhuv ri d pxowlsurgxfw up pxvw h{huw xqre0

— ⁻ d uc i

Ghvslwh wkh idfw wkdw wkh odvw ihz ghfdghv ri uhvhdufk rq wkh wkhru| ri wkh up kdv vkrzq wkdw pdqdjhuv zloo riwhq dfw dv d shuihfw djhqw iru wkh up*v vkduhkroghuv/ prvw dqdo|vhv ri phujhuv dqg pdunhw srzhu frqwlqxh wr dvvxph wkdw dq| pxowl0surgxfw up zloo fkrrvh lwv ghflvlrq ydulde ohv wr pd{lpl}h wkh mrlqw sur wv ri wkh surgxfwv zlwklq wkh up1 Wklv sdshu lv rqh ri wkh uvw wr uhod{ +dw ohdvw lq vrph uhvshfwv, wkdw dvvxpswlrq dqg h{dplqh wkh lpsdfw rq wkh sulflqj/ lqqrydwlrq/ dqg lqfhqwlyh vwudwhjlhv ri pxowl0surgxfw upv1[1] Wkhvh lpsdfwv duh sduwlfxoduo| uhohydqw zkhq irupxodwlqj phujhu srolf| vlqfh/ lq gl huhqwldwhg surgxfwv lqgxvwulhv/ phujhuv riwhq uhvxow lq pxowl0 surgxfw upv1

Frqvlghu wkh iroorzlqj k|srwkhwlfdo1 Wkhuh duh wzr rzqhu pdqdjhg upv/ D dqg E/ wkdw pdnh vriwzduh iru vrph lqgxvwu|1 Zkloh wkhvh surgxfwv erwk phhw wkh vdph qhhg iru wkh vdph jurxs ri upv/ wkh| duh qrw lghqwlfdo1 Pd|eh surgxfw D lv vrphzkdw ehwwhu iru vpdoohu upv zkloh E lv ehwwhu iru odujhu rqhv1 Ru pd|eh D*v surgxfw grhv vrph vhw ri wdvnv ehwwhu dqg rwkhuv qrw dv zhoo/ dqg wkh lpsruwdqfh ri wkhvh wdvnv gl huv dprqj upv lq wklv lqgxvwu|1 Qrz/ lpdjlqh wkdw D dqg E phujh1 Vlqfh wkhvh surgxfwv duh idu iurp shuihfw +wkh| duh vriwzduh surgxfwv/ diwhu doo,/ dqg hdfk pdqdjhu nqrzv khu surgxfw idu ehwwhu wkdq grhv wkh rwkhu pdqdjhu/ hdfk pxvw uhpdlq dv sduw ri wkh frpelqhg up wr lpsuryh khu surgxfw1 Mxvw ehfdxvh wkh| duh sduw ri rqh up/ krzhyhu/ grhv qrw phdq wkh| zloo qrz rqo| dfw lq wkhlu mrlqw/ dv rssrvhg wr wkhlu lqglylgxdo/ lqwhuhvw1 Pruhryhu/ ehfdxvh D dqg E vhuyh vo ljkwo| gl huhqw +wkrxjk ryhuodsslqj, pdunhwv/ dqg hdfk pdqdjhu nqrzv idu pruh derxw khu surgxfw dqg krz srwhqwldo fxvwrphuv shufhlyh lw/ wkh pdqdjhu ri hdfk surgxfw kdv sulydwh lqirupdwlrq derxw wkh ghpdqg fxuyh iru khu surgxfw1 Dv d uhvxow/ hdfk pdqdjhu pxvw kdyh wkh vroh dxwkrulw| wr vhw wkh sulfh iru khu surgxfw1[2] +Wklv lv frqvlvwhqw zlwk hpslulfdo hylghqfh wkdw vkrzv wkdw phujhg upv duh iuhtxhqwo| rujdql}hg lq d pxowlvxevlgldu| irup dqg wdujhw pdqdjhphqw lv riwhq uhwdlqhg +Suhfkho/ Erlhv/ dqg Zrrgv 4<<<>]h| dqg Vzhqvrq 4<<<,1 Wlph Zduqhu/ iru h{dpsoh/ rshudwhg hdfk ri lwv gl huhqw glylvlrqv dv lqghshqghqw sur w fhqwhuv zlwk d juhdw ghdo ri lqwudO up frpshwlwlrq1,

Ri frxuvh/ vlqfh erwk pdqdjhuv qrz mrlqwo| rzq wkh wzr surgxfwv/ wkh| zloo j T41(kfw)1(kfw)2(kfw)2(kfw)48TD

holplqdwh sulfh frpshwlwlrq ehwzhhq wkhp1 Rqh zd| wr dffrpsolvk wklv zrxog eh wr iru wkhp wr djuhh wr htxdoo| vkduh wkh sur wv iurp wkh vdohv ri erwk surgxfwv +L fdoo wklv frpsohwh sur wvkdulqj,1 Wkh glylvlrq ri sur wv/ krzhyhu/ qrw rqo| d hfwv sulflqj lqfhqwlyhv/ exw lw dovr d hfwv d pdqdjhu*v lqfhqwlyh wr lpsuryh khu surgxfw1 Li surgxfw txdolw| lv xqyhul deoh/ wkhq surgxfw sur wlv wkh rqo| dydlodeoh lqvwuxphqw iru lqgxflqj wkh pdqdjhuv wr hqjdjh lq frvwo| surgxfw lpsurylqj h ruw1 Li wkh pdqdjhuv hohfw wr vkduh wkh sur wv iurp wkh wzr surgxfwv htxdoo|/ wkhq hdfk pdqdjhu zloo ehdu doo ri wkh frvw ri surgxfw lpsuryhphqw +wkh h ruw uhtxluhg wr {exjv ru dgg qhz ihdwxuhv wr wkh vriwzduh, exw uhfhlyh rqo| kdoi wkh ehqh w +wkh lqfuhdvhg sur whduqhg e| wkh wzr surgxfwv wrjhwkhu zkhq rqh ri wkhp lv ehwwhu,1 Wr lpsuryh h ruw ru lqqrydwlrq lqfhqwlyhv/ wkhq/ wkh pdqdjhuv zloo suhihu wkdw hdfk rqh uhwdlqv juhdwhu wkdq kdoi ri wkh sur w iurp lwv rzq surgxfw hyhq wkrxjk wklv jhqhudwhv vrph lqfhqwlyh iru sulfh frpshwlwlrq ehwzhhq wkh wzr surgxfwv1

Iru wkh vdph uhdvrq wkdw wkh pdqdjhuv gr qrw zdqw wr frpsohwho| vdful fh lqqrydwlrq lqfhqwlyhv wr holplqdwh sulfh frpshwlwlrq/ rqh pljkw dovr wklqn wkdw wkh| zloo qrw zdqw wr frpsohwho| vdful fh sulflqj frruglqdwlrq wr pd{lpl}h lqqrydwlrq lqfhqwlyhv1 Li lqqrydwlyh h ruw lv ri sxuho| sulydwh ydoxh +wkdw lv/ lw kdv qr h hfw rq wkh txdolw| ri wkh rwkhu surgxfw,/ wkhq wklv lqwxlwlrq lv fruuhfw1 Li wkhuh duh lqqrydwlrq vsloryhuv +zkhq rqh pdqdjhu jxuhv vrphwklqj rxw derxw krz wr {dexj ru dgg d ihdwxuh lw pdnhv lw hdvlhu iru wkh rwkhu pdqdjhu pdnh khu surgxfw ehwwhu,/ krzhyhu/ wkhq wkhuh fdq dfwxdoo| eh d uhgxfhg lqfhqwlyh wr vkduh sur w/ vr pxfk vr wkdw lw fdq eh wkh fdvh wkdw qr sur wvkdulqj lv rswlpdo1 Wkh uhdvrq lv wkdw vsloryhuv fdq fuhdwh d srvlwlyh h{whuqdolw| dvvrfldwhg zlwk surgxfw lpsurylqj h ruw/ hyhq li hdfk pdqdjhu nhhsv wkh hqwluh sur w iurp khu surgxfw1[4] Vr orqj dv wkh sulydwh ydoxh iurp wklv h ruw lv juhdwhu wkdq wkh h{whuqdo ydoxh/ h ruw lqfhqwlyhv zloo eh vwurqjhu wkh ohvv sur w vkdulqj wkhuh lv1 Wkh forvhu lq pdjqlwxgh wkh vsloryhu ehqh wv duh wr wkh sulydwh ehqh wv/ krzhyhu/ wkh zhdnhu wkh uhodwlrqvkls ehwzhhq h ruw lqfhqwlyhv dqg sur wvkdulqj ehfrphv1 Vr/ zkhq wkh vsloryhu h hfw lv vr vwurqj wkdw surgxfw E lv lpsuryhg dopruv dv pxfk dv surgxfw D lv e| pdqdjhu D*v surgxfw lpsurylqj h ruw/ wkhuh zloo eh yhu| olwwoh ydoxh/ lq whupv ri lpsuryhg h ruw lqfhqwlyhv/ wr lqwudo up sulfh frpshwlwlrq/ pdnlqj d odujh ghjuhh ri sur wvkdulqj rswlpdo1

Ghwhuplqlqj wkh rswlpdo ohyho ri sur wvkdulqj ehwzhhq wkh wzr surgxfwv/ ri frxuvh/ lv fulwlfdo wr dvvhvvlqj wkh frpshwlwlyh lpsdfw ri wkh phujhu ri D dqg E1 Li lqqrydwlrq lqfhqwlyhv duh olnho|

[3] i o i i o h o i i o o)o A o iv B ou o h o ou B vi v .

[4] " " u v wi h hou h i ov o i iv i h i o iv i o i wi h u i i h ou h iv .

dv d Vwdfnhoehuj ohdghu zlwk uhvshfw wr wkh rwkhu/ lqfuhdvlqj wkh frpelqhg sur wv ri wkh up dqg wrwdo txdqwlw| zklh uhgxflqj sulfh1 Wkxv/ nhhslqj wkh hqwlwlhv vhsdudwh pdnhv d krul}rqwdo phujhu sur wdeoh zkhq lw rwkhuzlvh zrxog qrw eh1 Lq qrqh ri wkhvh sdshuv/ krzhyhu/ lv xqrevhuydeoh pdqdjhuldo h ruw dq lvvxh1 Vr/ lq vrph vhqvh/ p| prgho shuirupv wkh rssrvlwh dqdo|vlv ri wkhvh sdshuv1 Lw orrnv dw wkh h hfw ri wkh qhhg wr prwlydwh pdqdjhuv rq surgxfw pdunhw frpshwlwlrq udwkhu wkdq wkh h hfw ri surgxfw pdunhw frpshwlwlrq rq krz pdqdjhuv duh frpshqvdwhg1[5]

Wkh olwhudwxuh rq uhvhdufk mrlqw yhqwxuhv +h1j1/ Ndplhq/ Pxoohu/ dqg]dqj 4<<5›]lvv 4<<7› Yrqruwdv 4<<7› Fdeudo 5333, dovr vkduhv vrph vlplodulwlhv zlwk wklv sdshu1 Wkhvh sdshuv/ krzhyhu/ w|slfdoo| dvvxph wkdw wkh mrlqw yhqwxuh fdq frqwudfwxdoo| { wkh ohyho ri uhvhdufk dqg ghyhorsO

rq wkh xvh ri sur w0vkdulqj ehwzhhq glylvlrqv wr plqlpl}h wkh glvwruwlrqv dvvrfldwhg zlwk whdp surgxfwlrq sure0hpv dqg sulfh frpshwlwlrq1

Wkh s0dq ri wkh sdshu lv dv ir00rzv1 Wkh qh{w vhfwlrq rxw0lqhv wkh jhqhud0 prgh01 Vhfwlrq 6 h{dplqhv wkh 0lqhdu ghpdqg fdvh zlwk 614 glvfxvvlqj rswlpd0 sur w vkdulqj/ 615 glvfxvvhv sulfh h hfwv/ dqg 616 glvfxvvhv krz prud0 kd}dug fkdqjhv wkh dqd0|vlv ri phujhuv1 Vhfwlrq 7 frqf0xghv1

h{huwlqj txdolw| lpsurylqj h ruw ^{i} lpsrvhv d sulydwh dqg xqrevhuydeoh frvw $+^{i}$, rq pdqdjhu / wkhq khu xwlolw| lv jlyhq e|=

$$^{i}+\alpha, @ \alpha\pi^{i} \ . \ +4-\alpha,\pi^{-i}- \ +^{i}, \qquad +6,$$

Vlqfh hdfk pdqdjhu kdv d v|pphwulfdo xwlolw| ixqfwlrq/ hdfk zdqwv wr fkrrvh α wr pd{lpl}h $+\alpha,1$ Dw wklv vwdjh wkhuh lv qr frq lfw dprqj wkh pdqdjhuv vlqfh hdfk pdqdjhu*v xwlolw| ydulhv zlwk α lq wkh vdph zd|1 Li txdolw| lpsurylqj h ruw zhuh frqwudfwleoh/ wkhq lw zrxog fohduo| eh rswlpdo wr vhw $\alpha @ 4/5$ vr wkdw qhlwkhu pdqdjhu zrxog kdyh dq| lqfhqwlyh wr orzhu sulfh wr vwhdo fxvwrphuv iurp wkh rwkhu1 Rq wkh rwkhu kdqg/ li sulfhv frxog eh vhw mrlqwo|/ wkhq α zrxog rqo| d hfw d pdqdjhu*v h ruw lqfhqwlyhv1 Erwk pdqdjhuv zrxog djuhh wr vhw sulfhv dw wkh mrlqw sur wO pd{lpl}lqj ohyho dqg vhw $\alpha > 4/5$ vr wkdw wkh| zrxog qrw h{huw wrr olwwoh h ruw1[7] Li/ krzhyhu/ wkh pdqdjhu ri hdfk surgxfw kdv sulydwh lqirupdwlrq derxw wkh ghpdqg iru wkdw surgxfw/ wkhq dooorzlqj wkh rwkhu pdqdjhu frqwuro ryhu wkh sulflqj ghflvlrq pd| qrw eh ihdvleoh1 Wkh huuru gxh wr lqdghtxdwh nqrzohgjh ri wkh ghpdqg ixqfwlrq frxog hdvlo| uhgxfh mrlqw sur wv idu pruh wkdq wkh lqfhqwlyh wr fxw sulfhv uhvxowlqj iurp $\alpha > 4/5$1

Ri frxuvh/ lq ghwhuplqlqj wkh rswlpdo ghjuhh ri sur w vkdulqj/ L dvvxph wkdw wkh ghpdqg ixqfwlrq iru hdfk surgxfw lv v|pphwulf zlwk wkh rwkhu/ vr wkhuh lv qr sulydwh lqirupdwlrq dw wklv srlqw1 Krzhyhu/ lw zloo riwhq eh wkh fdvh wkdw zkhq wkh pdqdjhuv duh ghflglqj rq wkh rswlpdo ghjuhh ri sur w vkdulqj wkdw wkh| kdyh frpprq h{shfwdwlrqv derxw ghpdqg/ exw zkhq wkh wlph frphv wr vhw sulfh/ hdfk pdqdjhu kdv dftxluhg vxevwdqwldo ixuwkhu lqirupdwlrq derxw wkh dfwxdo vwdwh ri ghpdqg iru khu surgxfw/ uhtxlulqj lqghshqghqw sulflqj ghflvlrqv1 Wr vlpsoli| wkh dqdo|vlv/ lqvwhdg ri vshfli|lqj wkh glvwulexwlrq ri ghpdqg ixqfwlrqv/ L mxvw dvvxph wkdw wkh pdqdjhuv qg wkh rswlpdo α xqghu wkh dvvxpswlrq wkdw hdfk pdqdjhu zloo fkrrvh khu h ruw dqg sulfh edvhg rq ghpdqg jlyhq e|$\bullet$1

Wkxv/ wkhuh duh wkuhh shulrgv lq wkh prgho1 Lq shulrg 4/ wkh wzr pdqdjhuv fkrrvh wkh ghjuhh ri sur wOvkdulqj/ $4-\alpha$1 Lq shulrg 5/ pdqdjhu fkrrvhv khu ohyho ri qrqOfrqwudfwleoh h ruw ^{i}1 L dvvxph wkdw/ zkloh qrw frqwudfwleoh/ wklv lv revhuydeoh wr pdqdjhu $-$ lq shulrg 61 Lq shulrg 6/

o o i iv .

[7] wou o o i i h h wou iv i iv o o o o u o o h o h o u .

hdfk pdqdjhu vhwv wkh sulfh ri khu surgxfw dqg sur wv duh uhdol}hg dqg glvwulexwhg dffruglqj wr wkh olyhlo ri sur wlvkdulqj ghwhuplqhg lq shulrg 41

Wkh vroxwlrq frqfhsw L xvh iru wklv prgho lv vxejdph shuihfwlrq1 L vroyh wkh prgho xvlqj edfnzdug lqgxfwlrq1 Lq shulrg 6/ hdfh pdqdjhu fkrrvhv sulfh wr pd{lpl}h khu xwlow| jlyhq α, ^{i}, $^{-i}$1 E| gl huhqwldwlqj +6,/ wklv jlyhv wkh iroorzlqj uvw rughu frqglwlrq iru ^{i}=

$$\alpha+\Psi_1^{i} \ . \ + \ ^{i} - \prime,\Psi_1^{i}, \ . \ +4-\alpha,+ \ ^{-i} - \prime,\Psi_2^{-i} \ @ \ 3 \qquad +7,$$

Wkurxjkrxw wkh sdshu/ vxevfulswv zloo eh xvhg wr ghqrwh sduwldo ghulydwlyhv1 Wkxv/ Ψ_1^{i} lv wkh ghulydwlyh ri Ψ^{i} zlwk uhvshfw wr lwv uvw dujxphqw +zklfk lv ^{i}, dqg Ψ_2^{-i} lv wkh ghulydwlyh ri Ψ^{-i} zlwk uhvshfw wr lwv vhfrqg dujxphqw +zklfk lv ^{i},1 Wkh whup pxowlsolhg e| α lv wkh vwdqgdug frqglwlrq iru sur w pd{lpl}dwlrq e| d rqh surgxfw up1 Wkh vhfrqg whup lv wkh dgmxvwphqw gxh wr wkh idfw wkdw pdqdjhu dovr kdv d vwdnh lq wkh sur wv jhqhudwhg e| surgxfw $-$ / zkrvh vdolhv duh d hfwhg e| wkh sulfh ri surgxfw 1

Wkhvh wzr uvw rughu frqglwlrqv +rqh hdfh iru @ 4,5, lpsolflwo| ghwhuplqh ^{i} jlyhq α, ^{i}, $^{-i}$1 Ehfdxvh ri wkh v|ppphwu| lq wkh prgho/ rqh fdq zulwh ^{i} @ +α, ^{i}, $^{-i}$, iru @ 4,51 Wkxv/ lq shulrg 5 pdqdjhu *v uvw rughu frqglwlrq iru ^{i} fdq eh rewdlqhg e| gl huhqwldwlqj +6, zlwk uhvshfw ^{i} zkhuh ^{i} @ +α, ^{i}, $^{-i}$,1 Wklv jlyhv wkh iroorzlqj frqglwlrq=

$$\alpha+ \ ^{i} - \prime,+\Psi^{i} \ . \ \Psi_2^{i}, \ . \ +4-\alpha,+ \ \Psi^{-i} \ . \ + \ ^{-i} - \prime,+\Psi_4^{-i} \ . \ \Psi_1^{-i},, - \ '+ \ ^{i}, @ \ 3 \qquad +8,$$

Wkh whup pxowlsolhg e| α lv wkh pdujlqdo lqfuhdvh lq sur w iru surgxfw iurp surgxfw lpsurylqj h ruw1 Wkhuh lv d gluhfw h hfw gxh wr lqfuhdvhg ghpdqg dqg dq lqgluhfw h hfw gxh wr krz surgxfw lpsurylqj h ruw e| d hfwv wkh sulfh fkdujhg e| $-$ dqg krz wkdw fkdqjh lq sulfh d hfwv wkh ghpdqg iru surgxfw 1 +Qrwh/ wkhuh lv qr whup iurp wkh h hfw ri ^{i} rq wkh ^{i} vlqfh ^{i} lv vhw rswlpdoo| e| +7, douhdg|1, Wkh whup pxowlsolhg e| $4-\alpha$ lv wkh h hfw ri lqfuhdvlqj ^{i} rq wkh sur wdelolw| ri surgxfw $-$ 1 Wkh uvw whup/ Ψ^{-i}/ lv iurp wkh h hfw ri lqfuhdvlqj ^{i} rq $^{-i}$1 Wkh qh{w wzr whupv duh txdqwlw| h hfwv/ gluhfwo| iurp d odujhu ^{i} dqg lqgluhfwo| iurp wkh sulfh h hfw1 Wkh odvw whup lv wkh pdujlqdo frvw ri lqfuhdvlqj ^{i}1 Xvlqj wkh uvw rughu frqglwlrq iru $^{-i}$/ htxdwlrq +8, fdq eh uhzulwwhq dv iroorzv=

$$\alpha\varphi^{i}+\ {}^{i}-\prime,\ .\ +4-\alpha\varphi_4^{-i}+\ {}^{-i}-\prime,\ .\ \frac{5\alpha-4}{\alpha}\ \varphi_2^{i}+\ {}^{-i}-\prime,-\ {}'+\ {}^{i},\ @\ 3\qquad +9,$$

Wkhvh wzr uvw rughu frqglwlrqv +rqh hdfk iru @4,5, lpsolflwo| ghwhuplqh ^{i} jlyhq α1 Wkxv/ rqh fdq zulwh ^{i} @ $+\alpha$, +djdlq/ wkh ixqfwlrq lv lghqwlfdo iru @4,5 ehfdxvh ri v|pphwu|,1

$$+_{1}\ .\ '_{2},\ .+\ -',\ .\ ,,\ .\ \frac{4}{5}\ '+\ -'\ \mathrm{f}$$

wkh frpsohwh sur w vkdulqj ohyho zloo lqfuhdvh h ruw1 Wkxv/ lq wklv fdvh/ wkh iroorzlqj sursrvlwlrq krogv1

[illegible] $\phi^i @ \phi^{+\,i} - {}^{+\,i} . \gamma^{-i},, {}^{-i} - {}^{+\,-i} . \gamma^{\,i},, ($ $3 \le \gamma < 4)$, fi $(\alpha @ 4/5)$ $\phi_{11} . \phi_{22} - \phi_{12} < 3.$

Lq wkh ghpdqg ixqfwlrq lq wkh Sursrvlwlrq/ γ phdvxuhv wkh pdjqlwxgh ri h ruw vsllooryhuv1 Vr/ zkhqhyhu h ruw e| rqh pdqdjhu lpsuryhv wkh txdolw| ri khu rzq surgxfw pruh wkdq lw lpsuryhv wkh txdolw| ri wkh rwkhu surgxfw/ wkhuh zloo qhyhu eh frpsohwh sur w vkdulqj1 Wkxv/ zkhq lw lv lpsruwdqw wr prwlydwh pdqdjhuv wr pdnh surgxfw lpsurylqj h ruw dqg ghpdqg ghshqgv rq| rq wkh txdolw| dgmxvwhg sulfh ri erwk surgxfwv/ d pxowl0surgxfw prqrsrolvw zloo qrw sulfh dw wkh ixoo| frooxvlyh ohyho1 Hyhq zkhq erwk surgxfwv duh xqghu frp prq rzqhuvkls/ wkhuh zloo vwloo eh vrph sulfh frpshwlwlrq ehwzhhq wkhp1

3 i $\mathbf{D}$ $\mathbf{C}$

Lq rughu wr pruh ixoo| h{soruh wkh h hfw ri xqrevhuydeoh h ruw rq wkh ghjuhh ri frpshwlwlrq ehwzhhq pdqdjhuv lq wkh vdph up/ L {d ghpdqg dqg h ruw frvw ixqfwlrq1 L dvvxph wkh h ruw frvw ixqfwlrq lv + , @ ^{2}/5 dqg wkdw wkh ghpdqg fxuyh wdnhv wkh iroorzlqj irup=

$$\phi^i @ \quad - \ {}_1{}^{+\,i} - \ {}^{i} - \gamma^{-i}, . \ {}_2{}^{+\,-i} - \ {}^{-i} - \gamma^{\,i}, \qquad +45,$$

Wkh ghpdqg iru surgxfw

$$\alpha+ \ - \ {}_1+5 \ {}^i - \ {}^i - \gamma \ {}^{-i} - \prime,, \ . \ {}_2+ \ {}^{-i} - \alpha+ \ {}^{-i} \ . \ \gamma \ {}^i, - +4 - \alpha, \prime, @ 3 \qquad +46,$$

Vroylqj wkhvh uvw rughu frqglwlrqv iru ^{i} jlyhv wkh iroorzlqj htxdwlrq iru sulfh=

$$^i @ \frac{\begin{array}{c} 5\alpha^2 \ {}_1+ \ . \ {}_1+\prime \ . \ {}^i \ . \ \gamma \ {}^{-i},, \ . \ \alpha \ {}_2+ \ . \ {}_1+5\alpha - 4,+\prime - \ {}^{-i} - \gamma \ {}^i,, \\ - \ {}_2^2+4 - \alpha, \prime \ . \ \alpha+ \ {}^i \ . \ \gamma \ {}^{-i},, \end{array}}{+5\alpha \ {}_1 - \ {}_2,+5\alpha \ {}_1 \ . \ {}_2,} \qquad +47,$$

Xvlqj wklv h{suhvvlrq iru sulfh dqg +45, lq wkh uvw rughu frqglwlrq iru h ruw/ +9,/ lw lv srvvleoh wr h{soliwo| vroyh iru h ruw/ ^{i}/ dv d ixqfwlrq ri α/ γ/ dqg wkh ghpdqg dqg h ruw sdudphwhuv / $_1$/ $_2$/ dqg 1 Ehfdxvh wkh h{suhvvlrq lv hyhq pruh frpsolfdwhg wkdq wkh rqh iru ^{i}/ L uhohjdwh lw wr wkh dsshqgl{1 Qrwh/ krzhyhu/ wkdw wkh vhfrqg rughu frqglwlrq iru wkh h ruw htxdwlrq lv vdwlv hg zkhqhyhu $\geq \frac{b_1}{4}$1 Khqfhiruwk/ L zloo rqo| frqvlghu ydoxhv ri wkdw vdwlvi| wklv frqglwlrq1[8]

Xvlqj wkhvh h{soliw vroxwlrqv iru sulfh dqg h ruw/ rqh fdq jhw d uhgxfhg irup h{suhvvlrq iru xwlow| wkdw fdq eh rswlpl}hg ryhu α1 Xqiruwxqdwho|/ wkh uhvxowlqj uvw rughu frqglwlrq iru α lv dq ;0ghjuhh sro|qrpldo lq α +dovr jlyhq lq wkh dsshqgl{,/ vr lw lv qrw srvvleoh wr jhw dq h{soliw dqdo|wlfdo vroxwlrq iru α1 Hyhq dqdo|wlf frpsdudwlyh vwdwlfv duh frpsolfdwhg e| wkh idfw wkdw wkh uhgxfhg irup xwlow| ixqfwlrq lv qrw hyhu|zkhuh frqfdyh lq α1 E| h{suhvvlqj $_2$ dqg lq whupv ri wkhlu udwlr wr $_1$+l1h1/ grlqj wkh fkdqjh ri yduldeohv $_2$ @ $_2$ $_1$ dqg @ $_1$,/ krzhyhu/ wkh uvw rughu frqglwlrq iru α ehfrphv lqghshqghqw ri / $_1$ dqg $\prime$1 Dv d uhvxow/ lw lv srvvleoh wr ohduq d juhdw ghdo derxw wkh rswlpdo ghjuhh ri lqwud0 up frpshwlwlrq yld qxphulfdo vlpxodwlrq1 Wr hqvxuh wkdw wkh fkdudfwhulvwlfv ri rqh*v rzq surgxfw d hfwv lwv ghpdqg pruh wkdq wkh fkdudfwhulvwlfv ri wkh frpshwlqj surgxfw/ L dvvxph $3 < \ {}_2 < 4$1 Wr hqvxuh wkdw wkh h ruw uvw rughu frqglwlrq lv d qhfhvvdu| dqg vx flhqw frqglwlrq iru wkh rswlpdo h ruw ohyho/ L dovr uhtxluh wkdw $\geq 4/7$1

Wr rewdlq qxphulfdo vroxwlrqv iru wkh rswlpdo ohyho ri α/ L uvw qxphulfdo| vroyh wkh uvw rughu frqglwlrq1 L wkhq glvfdug doo vroxwlrqv wkdw duh qrw ehwzhhq rqh0kdoi dqg rqh1 +Lw zloo qhyhu eh rswlpdo iru d pdqdjhu wr uhwdlq ohvv wkdq kdoi ri wkh sur w iurp khu surgxfw1 L uxoh rxw d pdqdjhu uhfhlylqj pruh wkdq kxqguhg shufhqw ri wkh sur w iurp khu surgxfw ehfdxvh wkh rwkhu pdqdjhu pd| qrw kdyh wkh ixqgv wr pdnh vxfk d wudqvihu/ dqg ehfdxvh lw jlyhv khu wrr juhdw dq lqfhqwlyh wr vderwdjh surgxfwlrq1 Zkhqhyhu L qg wkdw wkh rswlpdo α

Wkh rswlpdo α

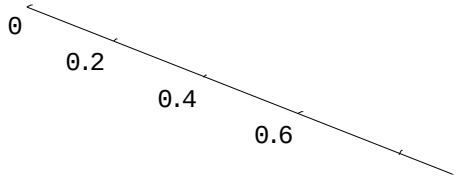
lw lv qrw qhfhvvdul| wkh fdvh/ dv wkh vlpxodwlrqv ehorz ghprqvwudwh/ wkdw juhdwhu vsllryhuv dozd|v lqgxfh odujhu α +pruh lqwud0 up frpshwlwlrq,1 Ehiruh surylglqj vrph judskv wkdw ghprqvwudwh erwk wkh udqjh ri srvvleoh pdjqlwxghv iru α dqg krz wkh rswlpdo α ydulhv zlwk ydulrxv sdudphwhuv/ L zloo vxppdul}h wkh nh| qglqjv iurp wkh vlpxodwlrqv1

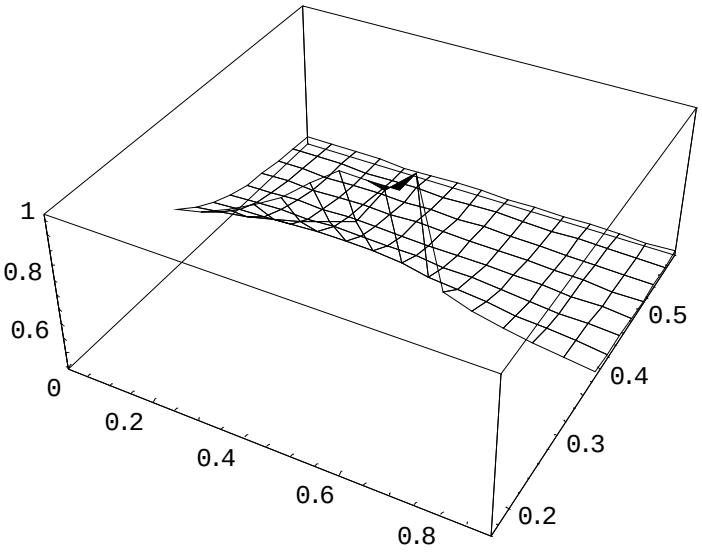
Simulation Results–Optimal α: *(A)* . ▲ α : *(1)* ( ) ▲ γ ▲ ▲ ▲ *; (2)* ▲ $_2$*; (3)* ▲ . *(B)* ▲ ▲ α ▲ $_2 > .9$. *( )* $_2 @ 4/9$, ▲ α ▲ $> :$▲ ▲ ▲ ▲ ▲ .;< $\leq 4, \gamma \leq .<8$. *( )*, ▲▲ ▲ ▲ α ▲ ▲ ▲ γ, , ▲ $_2$.

Zkhq wkh vsllryhu h hfwv iurp surgxfw lpsurylqj h ruw duh yhu| vpdoo/ lqfuhdvlqj wkh vsllryhu h hfw zloo lqfuhdvh wkh srvlwlyh h{whuqdolw| dvvrfldwhg zlwk surgxfw lpsurylqj h ruw1 Wkxv/ wkh lpsruwdqfh ri uhgxflqj wkh h ruw glvwruwlrq ehfrphv pruh lpsruwdqw1 Dv wkh vsllryhu ehqh w iurp h ruw ehfrphv forvhu wr wkh vl}h ri wkh gluhfw ehqh w/ krzhyhu/ wkh glvwruwlrq dvvrfldwhg zlwk odujh ghjuhhv ri sur w0vkdulqj ehfrphv vpdoohu1 Lq wkh h{wuhph fdvh/ zkhuh h ruw e| rqh pdqdjhu lpsuryhv wkh txdolw| ri erwk khu surgxfw dqg wkh rwkhu surgxfw e| wkh vdph dprxqw/ wkh h ruw glvwruwlrq lv xqd hfwhg e| wkh ghjuhh ri sur w vkdulqj1 Vr zkhq γ lv yhu| odujh/ plqlpl}lqj sulfh frpshwlwlrq ehfrphv uhodwlyho| pruh lpsruwdqw wkdq plqlpl}lqj wkh h ruw glvwruwlrq1 Wklv h hfw fdq eh vhhq lq wkh irorzlqj jxuhv1

Iljxuh 4 ghslfwv wkh rswlpdo α +wkh khljkw ri wkh vxuidfh, iru uhodwlyho| odujh ydoxhv ri 1 Ehfdxvh L gr qrw dorz $\alpha > 4$/ wkhuh duh sodwhdxv lq wkh vxuidfh dw rqh +zkhuh $\alpha @ 4$ surylghv pruh xwlolw| wkdq dq| vpdoohu α,1 Iru vpdoohu ydoxhv ri / zkhq $_2 @ 4/9$/ wkh rswlpdo α lv rqh ryhu vxfk d odujh udqjh ri γ wkdw lw lv kdughu wr vhh wkh uhodwlrqvkls ehwzhhq wkh ghjuhh ri vsllryhuv dqg wkh rswlpdo ghjuhh ri lqwud0 up frpshwlwlrq1 Wkh qh{w jxuhv vkrz wkh vdph uhodwlrqvkls iru d odujhu ydoxh ri $_2$ dqg vpdoohu ydoxhv ri 1 Wkh uhodwlrqvkls orrnv txlwh vlplodu1

Rqh srlqw wkdw fdq eh vhhq d olwwoh pruh fohdu| lq Iljxuh 5 wkdq lq Iljxuh 4 lv wkdw rqfh wkh frpelqdwlrq ri $_2$


0
0.2
0.4
0.6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0
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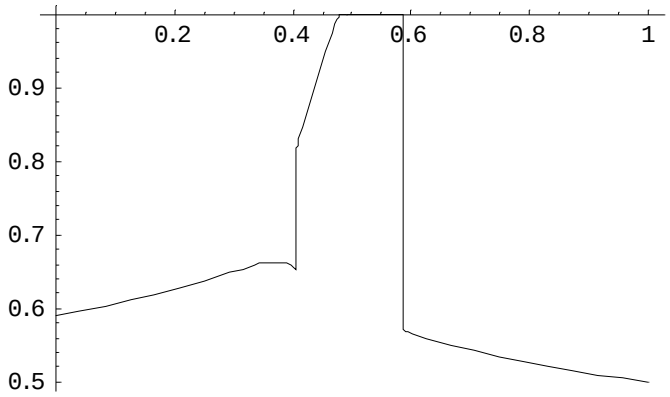
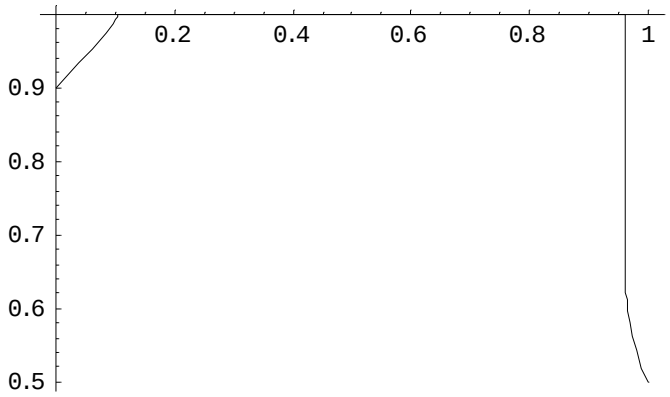
0.4
0.6
0.8
0.2
0.3
0.4
0.5



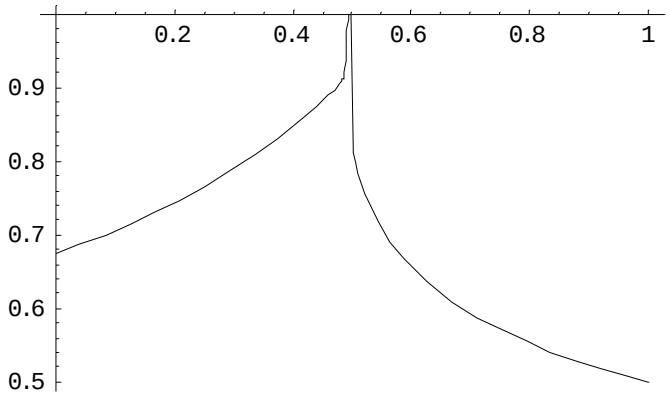
Ilj xuh 9=



Ilj xuh ;= @ 4, $_2$ @ 4/9› + , , d{hv @ +γ, α,

1<81

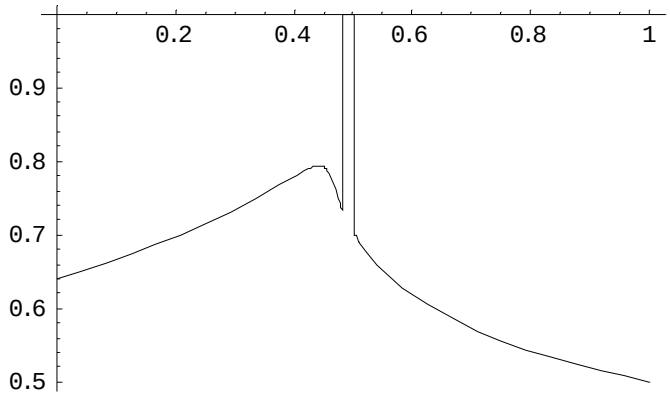
Wklv yd0xh ri rqh0vl{wk iru wkh furvv sulfh ghulydwlyh kdv qrw ehhq slfnhg dw udqgr p/ exw zdv fkrvhq edvhg rq vwdqgdug dqwlwuxvw pdunhw gh qlwlrq sudfwlfhv1 Dffruglqj wr wkh X1V1 Ghsduw phqw ri Mxvwlfh dqg wkh Ihghud0 Wudgh Frp plvvlrq*v Krul}rqwd0 Phujhu Jxlgh0lqhv +4<<:,/ wkh wzr fo rvhvw



Ilj xuh <= @ :.779, $_2$ @ 4/9> + , , d{hv @ +γ, α,

xqghuhvwlpdwhv wkh sur w jdlq iurp d sulfh lqfuhdvh1 Vr/ zkhq ghpdqg lv ghvfulehg e| +45,/ $^i \geq$,/ dqg $_2$ @ 4/9/ li lqgxflqj h ruw zhuh qrw dq lvvxh/ d phujhu ri wkhvh wzr surgxfwv zrxog d0prvw fhuwdlq0| eh ghhphg dqwl0frpshwlwlyh1 Zkhq pdqdjhuld0 prud0 kd}dug lv lpsruwdqw/ krzhyhu/ wkh vlpx0dwlrq uhvx0wv uhyhd0 wkdw wkhuh zl00 eh vxevwdqwld0 lqwud0 up frpshwlwlrq hyhq li wkhuh duh qr vsl00ryhuv1 Dqg/ li wkhuh duh hyhq vpd00 h ruw vsl00ryhuv/ vxfk d phujhu pljkw qrw uhgxfh wkh ghjuhh ri sulfh frpshwlwlrq ehwzhhq wkh wzr surgxfwv dw d00 surylghg wkh frvw ri h ruw lv qrw wrr kljk1 Wkxv/ wkh txhvwlrq ri zkhwkhu ru qrw sur w vkdulqj pxvw eh 0lplwhg wr prwlydwh pdqdjhuv fdq eh ri fulwlfd0 lpsruwdqfh lq dqd0|}lqj wkh frpshwlwlyh h hfwv ri vrph phujhuv1

Wkh frqglwlrq wkdw $^i \geq$,



Iljxuh 43= @ 4.78;58, $_2$ @ 4/6> + , , d{hv @ +γ, α,

Iljxuh < vkrzv wkdw zkhq γ lv qhdu rqh0kdoi d vpdoo vkliw lq wkh vl}h ri wkh h ruw vsloryhuv +iru wkhvh sdudphwhu ydoxhv, fdq fkdqjh wkh rswlpdo α e| dq| zkhuh iurp 14 wr 151 Ri frxuvh/ vlqfh wkh ydoxh ri wklv fulwlfdo γ zloo fkdqjh zlwk dqg $_2$/ d vpdoo fkdqjh lq hlwkhu ri wkhvh sdudphwhuv fdq kdyh gudpdwlf h hfwv rq wkh rswlpdo α dv zhoo1 Wklv h hfw lv hyhq pruh surqrxqfhg iru odujhu $_2$/ dv wkh qh{w jxuh ghprqvwudwhv1 Lw dovr jlyhv dq h{dpsoh ri zkhuh wkh rswlpdo α lv qrw dozd|v uhodwhg wr γ lq wkh pdqqhu ghvfulehg lq sduw +D, ri wkh vxp ppdu| ri wkh vlpxodwlrq uhvxowv1

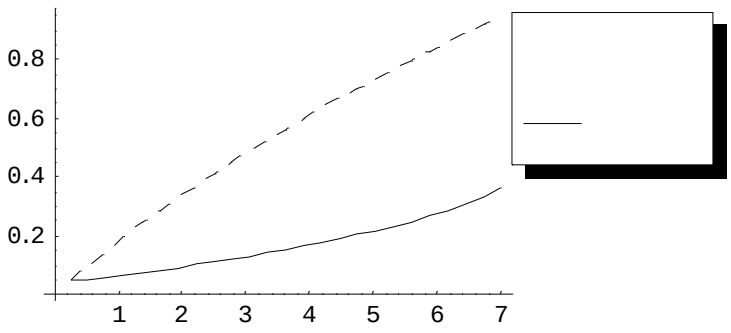
Zkdw lv jrlqj rq lq Iljxuh 43 lv wkdw iru γ ehwzhhq durxqg 178 dqg 18/ wkh xwlow| dw wkh lqwhulru orfdo pd{lpxp +zkhuh wkhuh lv ohvv sulfh frpshwlwlrq dqg ohvv h flhqw h ruw lqfhqwlyhv, lv yhu| forvh wr wkh xwlow| dw α @ 4 +zkhuh wkhuh lv pxfk pruh sulfh frpshwlwlrq exw pruh h flhqw h ruw lqfhqwlyhv,1 Iru α lq ehwzhhq wkh xwlow| lv pxfk ohvv1 Vr/ zkloh wkh xwlow| ri wkh pdqdjhuv lv qrw vhqvlwlyh wr vpdoo fkdqjhv lq sdudphwhuv lq wklv uhjlrq/ wkh zd| wkh| pd{lpl}h wkhlu xwlow| lv1 Ehfdxvh wkh zhoiduh lpsolfdwlrqv ri wkh rxwfrph lv dovr yhu| vhqvlwlyh wr wklv fkrlfh ri α/ lq wkhvh uhjlrqv jhwwlqj dffxudwh hvwlpdwhv ri wkh sdudphwhuv lv hvshfldoo| lpsruwdqw1 Ri frxuvh/ dv wkh sulru jxuhv +dqg pdq| rwkhu wkdw duh qrw vkrzq, ghprqvwudwh/ vlwxdwlrqv olnh wkhvh +hvshfldoo| wkh rqh lq Iljxuh 43, duh qrw frp prq1

3.2 P_ic c

Wklv vhfwlrq orrnv dw wkh h hfw rq txdolw| dgmxvwhg sulfhv ri d srwhqwldo phujhu ehwzhhq wzr surgxfwv lq wkh olqhdu ghpdqg prgho1 Txdolw| dgmxvwhg sulfh lv gh qhg dv sulfh plqxv wkh lqfuhdvh lq surgxfw txdolw| wkdw rffxuv dv d uhvxow ri pdqdjhuldo h ruw1 Wkdw lv/ wkh txdolw| dgmxvwhg sulfh iru surgxfw

Iv= ^{i} @ ^{i} — ^{i} — γ^{-i} z khuh

wklv phujhu wr eh d phujhu wr prqrsro|/ hyhq txlwh vpdoo vsloryhuv zloo uhvxow lq d ghfuhdvh lq txdolw| dgmxvwhg sulfhv surylghg wkh frvw ri h ruw lv qrw wrr juhdw uhodwlyh wr wkh pdjqlwxgh ri wkh rzq sulfh ghulydwlyh1 Zkhq lqwudo up frpshwlwlrq pd| eh qhfhvvdu| wr prwlydwh pdqdjhuldo h ruw/ vwdqgdug phujhu dqdo|vlv pd| eh idu wrr vwulfw1 Ri frxuvh/ zkhwkhu ru qrw wklv lv wkh fdvh +iru dq| jlyhq ghjuhh ri surgxfw vxevwlwxwdelolw|, zloo ghshqg xsrq hvwlpdwhv ri wkh ghjuhh ri h ruw vsloryhuv dqg wkh frvw ri txdolw| lpsurylqj h ruw1 Lq rughu wr dffxudwho| dvvhvv wkh frpshwlwlyh lpsdfw ri d phujhu +zkhuh pdqdjhuv pxvw eh jlyhq sur w edvhg lqfhqwlyhv wr lqgxfh surgxfw vshfl f/ txdolw| hqkdqflqj h ruw,/ rqh pxvw hvwlpdwh qrw rqo| wkh ghpdqg sdudphwhuv dqg wkh v|qhujlhv iurp wkh frpelqdwlrq exw dovr wkh lqqrydwlrq frvw sdudphwhuv1 Ri frxuvh/ li hvwlpdwlqj wkh ghpdqg sdudphwhuv vkrzv wkdw wkh surgxfwv duh yhu| forvh vxevwlwxwhv/ wkhq +dv Wdeoh 4 lqglfdwhv, lw lv yhu| xqolnho| wkdw wkh phujhu zloo uhvxow lq d txdolw| dgmxvwhg sulfh ghfuhdvh qr pdwwhu zkdw wkh ydoxh ri wkh rwkhu sdudphwhuv1


0.8
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7

phqw rq frvwv dqg uhyhqxhv vhsdudwho|1 Iruflqj frqwudfwv/ zkhuh rqh glylvlrq jhwv doo wkh sur wv xqohvv frvwv duh dw ru ehorz wkh uvw ehvw ohyho/ pd| eh lqihdvleoh/ krzhyhu/ gxh wr wkh srvvlelolw| ri vderwdjh1 Dv d uhvxow/ wkh rqo| ihdvleoh frqwudfwv pd| eh frqwudfwv zkhuh hdfk glylvlrq jhwv d jlyhq vkduh ri lwv uhyhqxh dqg sd|v d +srvvleoh gl huhqw, vkduh ri lwv frvwv1 E| jlylqj wkh pdqdjhu d odujh iudfwlrq ri wkh ehqh wv iurp frvw uhgxfwlrq exw d vpdoohu iudfwlrq ri khu rzq uhyhqxh/ wkh up fdq ghfrxsoh frvw uhgxfwlrq lqfhqwlyhv iurp sulfh frpshwlwlrq lqfhqwlyhv1 Li/ krzhyhu/ d pdqdjhu fdq uhgxfh frvwv qrw rqo| e| h{huwlqj h ruw exw dovr e| uhgxflqj txdolw|/ wkhuh zloo eh dq h flhqf| orvv dvvrfldwhg zlwk wklv ghfrxsolqj1 Wkxv/ prudo kd}dug zloo vwloo lqgxfh ohvv uhyhqxh vkdulqj wkdq zrxog eh rwkhuzlvh rswlpdo/ wkrxjk suredo| wr d ohvvhu h{whqw wkdq zkhq wkh xqrevhuydeoh h ruw d hfwv txdolw| udwkhu wkdq frvwv1

Appendix

Wkh dujxphqw lq wkh wh{w suryhv wkdw vr orqj dv $'+4/5, > 3$ wkh uhvxow krogv1 Wr vkrz wkdw $'+4/5, > 3/$ L qhhg wr vkrz wkdw wkh pdujlqdo ehqh w ri h ruw lv lqfuhdvlqj lq α dw α @ 4/

$$\frac{+\ -\ \prime,+4-\gamma,+5\sigma_{11}\sigma_{11}-\sigma_{12},\ .\ +\ -\ \prime,+\sigma_{11}\ .\ \sigma_{22},\sigma_{11}\ .\ \sigma_{22}-\sigma_{12},,}{-+5+\sigma_{11}-\sigma_{12},\ .\ +\ -\ \prime,+\sigma_{11}\ .\ \sigma_{22}-\sigma_{12},,} \qquad +55,$$

Wklv lv srvlwlyh zkhqhyhu $\sigma_{11}\ .\ \sigma_{22}-\sigma_{12}<31$ T1H1G1

ff Vxevwlwxwlqj lq iru sulfhv xvlqj +47,/ gl huhqwldwlqj + , zlwk uhvshfw wr i zkhq ghpdqg lv jlyhq e| +45, dqg wkhq vroylqj wkh uhvxowlqj uvw rughu frqglwlrqv iru i jlyhv wkh iroorzlqj=

$$i\ @\ \frac{\begin{array}{c}-\alpha+\ -\ {}_1+4-\ {}_2,\prime,\{-7\alpha^2+\gamma\ .\ \alpha+4-\gamma,,-\ {}_2+4-\alpha,+4\ .\ \gamma,\\ .\ 5\alpha\ {}_2\wedge 5\alpha+4-\alpha,+4-\gamma,\ .\ \gamma'\ .\ {}_2^2\wedge 4\ .\ 7\alpha^2\ .\ \gamma-\alpha+6\ .\ \gamma,'\}\end{array}}{\begin{array}{c}{}_1\wedge 5\ +\ {}_2-5\alpha,^2+\ {}_2\ .\ 5\alpha,\ .\ \alpha+4-\ {}_2,+4\ .\ \gamma,\\ \{-7\alpha^2+\gamma\ .\ \alpha+4-\gamma,,-\ {}_2+4-\alpha,+4\ .\ \gamma,\\ .\ 5\alpha\ {}_2\wedge 5\alpha+4-\alpha,+4-\gamma,\ .\ \gamma'\ .\ {}_2^2\wedge 4\ .\ 7\alpha^2\ .\ \gamma-\alpha+6\ .\ \gamma,'\}'\end{array}} \qquad +56,$$

Vxevwlwxwlqj lq iru sulfhv xvlqj +47, dqg h ruw xvlqj +56,/ gl huhqwldwlqj + , zlwk uhvshfw wr α zkhq ghpdqg lv jlyhq e| +45, jlyhv wkh iroorzlqj uvw rughu frqglwlrq iru α=

$$\frac{\begin{array}{c};\ {}^2+\ -\ {}_1+4-\ {}_2,\prime,^2+\ {}_2-5\alpha,^2\{\ \ {}_2^2+5\alpha-4,+\ {}_2-5\alpha,+\ {}_2\ .\ 5\alpha,\\ .+4\ .\ \ {}_2,\alpha\wedge 49\alpha^4+4-\gamma,+4-\alpha+4-\gamma,,\ .\ ;\ {}_2\alpha\ +4-\gamma,+4-5\alpha+4-\alpha,+4-\gamma,\\ -\ {}_2^5\alpha+4-\gamma^2,-\ {}_2^4+4-\gamma,+5+5\ .\ \gamma,\ .\ 7\alpha^2+7\ .\ \gamma,-\alpha+4:\ .\ :\gamma,,\\ .\ 7\ {}_2^2\alpha+5-\gamma-7\alpha\ \gamma+4-\gamma,-\gamma^2\ .\ 7\alpha^2+5-\gamma-\gamma^2,-\alpha+<-8\gamma-7\gamma^2,,\\ .\ 5\ {}_2+4\ .\ \gamma-5\gamma^2\ .\ \alpha^2+43\ .\ 49\gamma-59\gamma^2,\\ -7\alpha\ +4\ .\ 6\gamma-7\gamma^2,-\alpha+8\ .\ :\gamma-45\gamma^2,,'\}\end{array}}{\begin{array}{c}{}_1\wedge 5\ +\ {}_2-5\alpha,^2+\ {}_2\ .\ 5\alpha,\ .\ \alpha+4-\ {}_2,+4\ .\ \gamma,\\ \{-7\alpha^2+\gamma\ .\ \alpha+4-\gamma,,-\ {}_2+4-\alpha,+4\ .\ \gamma,\ .\ 5\alpha\ {}_2\wedge 5\alpha+4-\alpha,+4-\gamma,\ .\ \gamma'\\ .\ {}_2^2\wedge 4\ .\ 7\alpha^2\ .\ \gamma-\alpha+6\ .\ \gamma,'\}'\end{array}} \qquad +57,$$

$$i,N\quad H\quad @\quad \frac{+\quad -+4-\quad _2,\ _1\prime,}{+4-\quad _2,\ _1}$$

$$\frac{\{5\quad _2+\ \frac{2}{2}-7,\ .\ +4-\quad _2,\wedge 7\gamma+5\ .\ \gamma,\ .\ \ \frac{2}{2}+4-5\gamma-\gamma^2,`\}}{\wedge 5-5\ _2-\ \frac{2}{2}\ .\ \ _2-\ \ +5-\quad _2,^2+5\ .\ \ _2,`+7\ \ -+4-\quad _2,+4\ .\ \gamma,^2,} \qquad +58,$$

$$\frac{i,N\quad H}{\gamma}\ @\ \frac{-7\ +\ \ -+4-\quad _2,\ _1\prime,+4\ .\ \gamma,}{\ _1+7\ \ -+4-\quad _2,+4\ .\ \gamma,^2,^2} < 3 \qquad +59,$$

Hydoxdwlqj +57, dw α @ 4 dqg γ @ 3/ lw e5Tnp0 1 . 49629 . 49Tmh . 49v=Tc. 5541 6 . 36r59

dq| jlyhq $_2$ zl00 rffxu dw ~@ 3/ zkhuh wkh shufhqwdjh sulfh lqfuhdvh lv= $\frac{K_2}{2}$

^4' Mxdq Fdurv Edufhqd0Uxl} dqg Pduld Sd} Hvslqrvd1 Vkrxog pxowlsurgxfw upv surylgh glyl0 vlrqdo ru frusrudwh lqfhqwlyhvB , ▲ ▲ ▲ , ▲ ▲ ▲ / 4:=:84~:97/ Mxo| 4<<<1

^5' Oxlv P1E1 Fdeudo1 U)g frrshudwlrq dqg surgxfw pdunhw frpshwlwlrq1, ▲ ▲ ▲ , ▲ ▲ ▲ / 4;+:,=4366~437:/ Rfwrehu 53331

^6' Udprq Idxol0Roohu dqg Pdjgdohqd Jludw1 Frpshwlwlrq dqg frrshudwlrq zlwklq d pxowlglyl0 vlrqdo up1 ▲ , ▲ / 76=::~<</ Pdufk 4<<81

^7' Sdror Ixojklhul dqg Odxulh Vlprq Krgulfn1 V|qhujlhv dqg lqwhuqdo djhqf| frq lfwv= Wkh grxeoh0hgjhg vzrug ri phujhuv1 LQVHDG Zrunlqj Sdshu <:2;;2 ILQ/ Vhswhpehu 4<<:1

^8' Pduwlq Jd|qru1 Frpshwlwlrq zlwklq wkh up= Wkhru| soxv vrph hylghqfh iurp phglfdo jurxs sudfwlfh1 . A ▲ / 53+4,=8<~:9/ Pdufk 4<;<1

^9' Pduwlq Jd|qru1 Lqwhuqdo qrq0sulfh frpshwlwlrq dqg rswlpdo frpshqvdwlrq lq sduwqhuvklsv1 Mrkqv Krsnlqv Ghsduwphqw ri Srolwlfdo Hfrqrp| Zrunlqj Sdshu 594/ Qryhpehu 4<<31

^:' Ehqjw Kropvwurp1 Prudo kd}dug lq whdpv1 . B ▲ / 46=657~673/ 4<;51

^;' Ndl D1 Nrqudg Kxfn/ Vwh hq dqg Zlhodqg Pxoohu1 Sur wdeoh krul}rqwdo phujhuv zlwkrxw frvw dgydqwdjhv= Wkh uroh ri lqwhuqdo rujdql}dwlrq/ lqirupdwlrq/ dqg pdunhw vwuxfwxuh1 FHVlir Zrunlqj Sdshu Qr1 768/ Pdufk 53341

^<' Hlwdq Pxoohu Ndplhq/ Pruwrq L1 dqg Lvudho]dqj1 Uhvhdufk mrlqw yhqwxuhv dqg u)g fduwhov1 . A ▲ / ;5+8,=45<6~4639/ Ghfhpehu 4<<51

^43' X1V1 Ghsduwphqw ri Mxvwlfh dqg wkh Ihghudo Wudgh Frpplvvlrq1 Krul}rqwdo phujhu jxlgholqhv/ 4<<:1

^44' Mrkq Erlhv Suhfkho/ Kduodqg dqg Wlp Zrrgv1 Ghew/ phujhuv dqg dftxlvlwlrqv/ lqvwlwxwlrqdo duudqjhphqwv dqg fkdqjh wr wkh pxowlod|huhg vxevlgldu| irup1 ▲ ▲ / ;3+4,=448~468/ Pdufk 4<<<1

^45' Mrkq Ylfnhuv1 Gholjdwlrq dqg wkh wkhru| ri wkh up1 . ▲ / <8=46;~47:/ Vxssohphqw 4<;81

^46‘ Qlfkrodv V1 Yrqruwdv1 Lqwhu0 up frrshudwlrq zlwk lpshuihfwo| dssursulde oh uhvhdufk1, ◮ - ◮ ◮ , ◮ ◮ ◮ / 45+6,=746~768/ Vhswhpehu 4<<71

^47‘ Pdu|]h| dqg Wdpl Vzhqvrq1 Wkh wudqvirupdwlrq ri wkh grplqdqw frusrudwh irup iurp pxowlglylvlrqdu| wr pxowlvxevlgldu|= Wkh uroh ri wkh 4<;9 wd{ uhirup dfw1 ◮ ◮ / 73+5,=574~59:/ Vsulqj 4<<<1

^48‘ Vwh hq]lvv1 Vwudwhjlf u)g zlwk vsllooryhuv/ frooxvlrq dqg zhoiduh1 ◮ , ◮ / 75+7,=6:8~6<6/ Ghfhpehu 4<<71