PRICING BEHAVIOR OF MULTIPRODUCT RETAILERS

Daniel Hosken

Federal Trade Commission

and David Reiffen[1]

Commodity Futures Trading Commission

Revised: June, 2007

ABSTRACT

This paper develops a model of competition among multiproduct retailers that is consistent with observed pricing regularities, e.g., virtually all products have large mass points in their price distributions and most deviations fall below the mass point. The basis of the model is that, because consumers prefer to buy a bundle of goods from the same retailer, a given discount on any one good in the bundle will have a similar effect on consumers' likelihood of visiting that retailer. This implies that discounts on goods sold by a single retailer are substitute instruments for retailers, and factors that influence one good's price will affect the pricing of other goods. Hence, if intertemporal price changes are a means of price discriminating (as suggested in the literature), the impact of these changes will be reflected in the prices of many goods, including those for which discrimination is not feasible.

keywords: Retailing, Pricing Dynamics, Sales

JEL codes: L81, L4, L11, D21

We would like to thank Cindy Alexander, Keith Anderson, Mark Armstrong,David Balan, Schmidt, John Simpson, and Charles Foster Thomas for their u and Sara Harkavy for providing us with excellent research assis paper are those of the authors and should not be viewed as thos

[1] Corresponding author: David Reiffen, Commodity Futures Trading Commission, 1155 21st St., NW, Washington, DC 20581, phone: 202-418-5602, e-mail:dreiffen@cftc.gov.

I. Introduction

In their capacity as consumers, many economists have no doubt wondered about the motivation behind the complex pricing strategies employed by supermarkets. Perhaps the most perplexing aspect of retail behavior is that the majority of supermarkets choose to offer a relatively small set of items (among the more than 35,000 items they typically carry) at a low “sale” price each week, and change that set virtually every week. Despite the high administrative costs of

---

[2] This concept of a sale contrasts with other kinds of systematic price reductions that have been documented. One such pattern is that prices for goods with a “fashion” element often systematically decline over a fashion season (see, e.g., Pashigian (1988), Pashigian and Bowen (1991), Warner and Barsky (1995)), as retailers learn which styles are popular with consumers. We view this type of sale as a fundamentally different phenomenon than that examined here.

they find that the only symmetric equilibria feature mixed strategies in prices. Unlike Varian, they show that for certain levels of inventory there is a mass-point at the upper support of the pricing distribution.

The second type of model views sales as a means of price discrimination, see, e.g., Conlisk et al. (1984), Sobel (1984), and Pesendorfer (2002). The basic intuition of this modeling approach is that consumers differ in their reservation values and in their willingness to wait (which is analytically similar to differences in inventory costs). Low-value consumers are more willing to wait for price reductions because the cost of waiting is higher for the high-value consumers. Hence, only low-value consumers wait for the periodic price reductions. As a result, periodic price reductions allow a retailer to charge a low price to all low-value customers, while most high-value customers purchase at a higher price.[3] Similar to Hong, et al., this model predicts that prices will normally be at a high level with periodic discounts.

Both literatures provide useful insights into the forces generating retail sales. Recent empirical work, however, suggests that these models fail to explain important aspects of retail sale behavior. Specifically, five regularities about supermarket pricing drawn from the recent empirical literature are particularly relevant in modeling retail pricing dynamics. First, there is a large mode in the pricing distribution for all types of goods, in particular, both goods that can easily be stored, e.g., cola, and those that are highly perishable, e.g., bananas (Aguirregabiria (1999) and Hosken and Reiffen (2004a)). That is, most products have "regular" price. Second, most deviations from a product's modal price are price reductions (Dutta et al. (2002) and Hosken and Reiffen (2004b)). Third, most price reductions are temporary (Pesendorfer (2002), Hosken and Reiffen (2004a)). Fourth, short-lived reductions in retail prices often represent a decrease in retail margins rather than wholesale prices or manufacturing costs; that is, sales are often the result of retailer rather than manufacturer behavior (MacDonald (2000), Levy et al. (2001), Dutta et al. (2002), and Chevalier et al. (2003)). Fifth, some consumers respond to sales by purchasing more than they will consume

---

[3] Lal and Matutes (1989) offer a similar explanation for why competing multiproduct retailers using different (static) pricing strategies for their array of goods. Because each retailer has a low price on a different good, retailers sell some items at high prices to high transportation-cost/high reservation-value consumers, while low transportation-cost consumers buy at more than one store each period in order to get low prices on all goods.

in the current period; that is, a subset of consumers respond to low prices by purchasing for household inventory (Pesendorfer (2002) and Hendel and Nevo (forthcoming)).

Comparing these recent empirical findings to the theoretical literature yields some inconsistencies between the theory and the evidence. Varian's model predicts an absence of mass points in the price distribution. The evidence, however, suggests that the price distributions of *all* types of products are characterized by relatively large point masses at the mode of the distribution, with short-lived discounts below this everyday price. The price discrimination models and Hong et al. generate pricing distributions consistent with these features of the data, and are consistent with recent evidence suggesting that consumers store the goods they buy at low prices for consumption in later periods. However, the price discrimination models only seem applicable for products that

[4] From a modeling standpoint, a good that is physically perishable, but for which consumers can "time" their consumption (fresh lobster, theater tickets) would be economically similar to a storable good. However, as Aguirregabiria (1999) and Hosken and Reiffen (2004a) show, mass points also appear in the pricing distributions of perishable goods that are typically purchased and consumed each period, e.g., milk, bread, bananas.

periodically, even though price discrimination through intertemporal price changes is only feasible for the storable. The model's pricing prediction are consistent with recent empirical findings: storable and perishable prices have high everyday (modal) prices with periodic discounts. In addition, we show that in equilibrium, price movements will be different for perishable and storable goods; storable pricing will feature long periods of stable prices, followed by significant but short-lived price reductions, whereas perishable prices will move more frequently, but by smaller amounts.

While our model and Varian's have different predictions about the shape of retail price distributions, the underlying intuition in both models is that retailers have sales in order to attract those consumers who choose between retailers on the basis of price. Both models find that, in equilibrium, retailers offer surplus to consumers every period, with the specific level of surplus drawn from an atomless, continuous distribution. Because Varian's retailers sell only one good, its price (and, equivalently, the surplus consumers obtain from each retailer) is drawn from an atomless distribution. By generalizing Varian's model to allow retailers to sell multiple products, we show that while surplus continues to be drawn from an atomless distribution, each product will have a mass point in its price distribution. The reason is that each product is an instrument for offering surplus, and it will generally be profitable to only use one instrument at any point in time. Hence, if the profitability of using one good as an instrument changes over time (e.g., as in the intertemporal price discrimination models), that can lead to changes in the relative profitability of using each good's price as an instrument for offering surplus. Thus, by incorporating the multiproduct nature of retailers' offerings into the model, we explain a richer set of observed retailer behavior.

II. A Model of Sales and Multiproduct Retailers

In this section we develop a model of competition among $N > 1$ multiproduct retailers. Each retailer sells the same two products to a unit mass of consumers. These products have different storage characteristics. The first good is *storable*, which means that consumers can purchase the good for current and later consumption. The second good is *perishable* and must be consumed during the period in which it is purchased.[5] We incorporate the multiproduct nature of the retailers by assuming that consumers can visit no more than one store each period. This implies that if consumers purchase both goods in a period they must purchase both from the same retailer.[6]

Our assumptions about consumer behavior are fairly standard.[7] We assume that all consumers consume at most one unit of each good in each period. Each consumer has measure zero and views prices as exogenous to his or her purchas

[5] The distinction between storable and perishable goods can be thought of in terms of storage costs; perishable goods are those with high storage costs. From this perspective, the dichotomous distinction in the text simplifies the analytics, while maintaining the economic substance of differing storage costs. From a practical standpoint, goods will in actually vary from highly perishable (very costly to store), like raspberries or bread to moderately perishable, like hot dogs or yogurt, to highly storable, like paper towels or canned fruit.

[6] This is a stronger form of the assumption in Lal and Matutes that every consumer prefers to make all of his or her purchases from the same retailer, reflecting a transaction cost of visiting each retailer.

[7] We discuss the empirical validity of several of our assumptions in Section III.

for perishable goods than for storables. Specifically, we assume that all consumers have a common reservation value of for the perishable (which is identical to the assumption in Varian). To reduce notational complexity, we interpret s_L, s_H and as the differences between consumers' reservation values and the constant marginal cost of selling the good, so that we normalize retailers' costs to of the storable as possible; that is, they purchase enough units of the storable such that their inventory holdings are M. Finally we solve for the equilibrium price distributions of the storable and perishable good. In contrast to previous models of sales, our model predicts (consistent with recent empirical evidence) that both perishable and storable goods will have mass points in their pricing distributions.

A. Consumers' Purchasing Behavior

The assumptions made above about consumers' reservation values, search costs and storage costs imply that store-loyals and shoppers react differently to a given set of prices. Let $P^j_{P,t}$ be retailer j's price for the perishable at time t, and $P^j_{S,t}$ be retailer j's the price for the storable at time t. Recalling that the reservation price for each good is independent of the price of the other good, a store-loyal customer will receive surplus of $\max\{(\quad P^j_{P,t}), 0\}$ from buying the perishable at retailer j, and $\max\{(s_H \quad P^j_{S,t}), 0\}$ from buying the storable at retailer j. It follows that a store-loyal will visit her preferred retailer and purchase one unit of the perishable if $P^j_{P,t}$, and one unit of the storable good if $P^j_{S,t} \quad s_H$ and one unit of each if both inequalities hold.

The assumption that consumers can visit no more than one store per period implies that if a shopper purchases both goods in any period, then she buys both from the same retailer. That is, in each period each shopper must determine which retailer to visit and how much to purchase of each good. A shopper's welfare-maximizing choice of retailer is the one that offers the greatest consumer surplus, summed across the two goods. For this reason, the prices of both goods may be relevant to a shopper's purchasing decision, even though his or her demand for each good is independent of the other good's price. Further, because shoppers can inventory the storable good, the choice of retailer that maximizes a shopper's welfare depends on the inventory the shopper had entering period t and future storable prices.

Specifically, let $_{j,t}$ be the consumer surplus consumer k (who is a shopper) gets from

[8] Formally, $_{j,t}$ is the difference between the surplus associated with consumer k's having the opportunity to buy at retailer j's prices in period t, and the surplus from not making any purchases in period t, for any given set of expected future prices. See Appendix B for details.

[9] Note that, in contrast to M, which is the storage capacity of shoppers, m is an

purchase M + 1 - I_{t-1} units of the storable, because if she buys fewer units, she will stock out of the storable before the next sale. Under those conditions, the closed-form expression for $_{j,t}$ is

$$\max\{0, \sum_{\tau=I_{t-1}}^{M} [\delta^{\tau} s_L - P^{j}_{S,t}]\} + \max\{0, \beta - P^{j}_{P,t}\} \qquad \text{for all } I_{t-1} < M.$$

A particularly tractable case is where all shoppers believes that a sale on the storable can only occur if $I_{t-1} = 0$ for virtually all shoppers (that is, all shoppers except perhaps for a set of shoppers with measure zero). We let $\mathbf{I_{t-1}}$ (in bold) be the vector of all consumers' inventories entering period t, so that the condition can be written as $\mathbf{I_{t-1}} = 0$. In that case, if shopper k has $I_{t-1} = 0$ and visits retailer j who has set

[10] The assumptions in conditions (1) - (4) lead to an equilibrium in which shoppers have a simple purchasing rule. This allows us to obtain closed-form solutions for prices. However, the key condition for most of our results is that an equilibrium exists in which aggregate purchases by shoppers are increasing in the amount of time since the most recent sale on the storable. That condition is shown to characterize consumer behavior in a model with exogenous price shocks, but in which $\mathbf{I_{t-1}} = 0$ is not a necessary condition for a storable sale by Hendel and Nevo.

selling products to store-loyals versus charging low prices and potentially selling to shoppers as well. Profits from loyals are maximized at $P^{j}_{S,t}$ s_H and $P^{j}_{P,t}$ (and, consequently, retailers will never charge more than s_H and). As described above, a shopper's choice of retailer depends on the

[11] By assumption, all shoppers have the same inventory entering period 1, implying that $_{j,1}$ is the same across shoppers. In general, if shoppers had different inventory levels in period t, then the $_{j,t}$ associated with any set of prices may differ across shoppers.

symmetric equilibrium features all retailers playing a mixed strategy with respect to . The distribution of , G($|\mathbf{I}_{t-1}$)

A. Has no mass point.

B. Has a lower support of zero

Proof

Thus, G( ) constitutes an equilibrium. In every period, expected retailer profits are equal to (+ s_H)/N, independent of shoppers' inventory holdings of the storable good. The logic is that retailers are essentially homogeneous Bertrand competitors in selling to shoppers, and hence, in equilibrium, do not earn profits from selling to shoppers. Although it would be profitable, in expectation, to price discriminate by occasionally lowering the price of the storable *if* other retailers kept their storable price at s_H, competition between retailers to attract shoppers when inventories are low results in a dissipation of the gains to a firm from price discriminating (as in Sobel).

Propositions 1 and 2 relate to the equilibrium property of the symmetric distribution of .
We now turn to the relationship between and prices.

Since all retailers have at least one product on sale every period, we next consider the profitability of alternative types of sales. There are three kinds of sales; a sale on the perishable only, a sale on the storable only, and a sale on both goods. Retailer j's profits from having a sale on the perishable only (i.e., $P^j_{P,t}$, $P^j_{S,t}$ s_H) are the profits from the store-loyals, plus the expected profits from the shoppers, or:

$$\frac{\gamma(s_H + P^j_{P,t})}{N} \quad (1\text{-}\gamma)P^j_{P,t} * Pr(\Psi_{j,t})$$

where Pr($_{j,t}$) is the probability that retailer j is offering more surplus than all of the other N–1 firms at time t.

The other two possibilities are to have a sale on the storable only, or to have a sale on both goods. In either case, the firm's profits will depend on the number of units of the storable shoppers buy at the sale price, which in turn depends on shoppers' storable good inventory holdings. In Section II.A, we noted that for certain parameter values, shoppers will rationally believe that if $\min_j \{P^j_{S,t})$ $s_L \delta^{m-1+I_{t-1}}$, then there will not be a sale for the next M + 1 - I_{t-1} periods. As such, consumers will purchase M + 1 - I_{t-1} units whenever $\min_j \{P^j_{S,t}\}$ $\delta^M s_L$. Conditions (1)-(4), stated below, provide sufficient conditions for these beliefs to hold in equilibrium.

In general, when she has an inventory of I_{t-1}, shopper k's maximum willingness to pay for the mth unit is $P^j_{S,t}$ s_L $^{m\ 1\ I_{t\ 1}}$ since shoppers will not be consuming the m + I_{t-1} unit until period

$t + m - 1 + I_{t-1.}$ This implies that, conditional on an initial inventory of I_{t-1}, the revenue a firm can obtain from a shopper $\quad {}^{j}_{S,t}$ is less than or equal to We assume that

[12] In the context of supermarket purchases this relationship is plausible; "periods" should be thought of as weeks, so that would be close to 1, and the maximal number of weeks of storage (M) would be a relatively small (<10) number.

(> 1) units, and an increase of in $P^j_{P,t}$ accompanied by a decrease in $P^j_{P,t}$ of /(M + 1 - I_{t-1}) increases retailer j's profits from loyals without lowering , or his profit from shoppers, conditional on offering the highest . This implies that whenever $P^j_{S,t} \leq$ $^M s_L$ the retailer will set $P^j_{P,t}$ = . Hence, having only one good on sale dominates having both on sale (i.e., $P^j_{P,t}$ < and $P^j_{S,t} \leq s_L$).⊠

The intuition behind Proposition 3 is that the cost of offering any given level of consumer surplus ($_{j,t}$) to shoppers is the foregone profits that c

[13] The implication that no more than one product will be on sale at any point in time derives in part from the assumption that shoppers necessarily visit no more than one retailer in each period. As we discuss in Section III, in a model in which shoppers can (at some cost) visit multiple retailers, equilibrium might consist of multiple goods being on sale.

maximized at $P^{j}_{S,t} \quad s_L \quad {}^{M}$. In combination, these conditions imply that a sale on the storable can only be profitable if:

$$\frac{(\ {}^{M}s_L \quad)}{} \quad (1 \quad)((\quad L \quad {}_{t\ 1} \quad {}_{L}\ {}^{M} \qquad \frac{(s_H \quad)}{}$$

retailers to choose $P^j_{S,t}$ s_H is that $\mathbf{I_{t-1}}$ = 0. Perishable prices will be the same as in the Varian model when $\mathbf{I_{t-1}} > 0$. In this case the surplus that retailer j offers consumers is $\Psi_{j,t}$ β-$P^j_{P,t}$.

When $\mathbf{I_{t-1}} = 0$ retailer j could place the perishable or the storable on sale. If retailer j places the perishable on sale, the surplus he offers is $\Psi_{j,t}$ β-$P^j_{P,t}$. In general, it is not possible to derive a closed form for $\Psi_{j,t}$ if retailer j places the storable is on sale. We can, however, derive a closed form for the surplus retailer j offers consumers in a special case. Assume that shopper k enters period t with her inventory $I_{t-1} = 0$, observes $\min_j (P^j_{S,t}) \leq$ $^M s_L$ and also believes that $\min_j (P^j_{S,t})$ will be equal to s_H for periods t+1 through M+t. In this case, it will be rational for her to buy M+1 units of the storable in period t if she buys from retailer j. If she were to buy m < M+1 units, she would expect to receive zero surplus on the storable in each period t+ , where (m, M), rather than s_L - $P^j_{S,t}$ in each. Hence, when her $I_{t-1} = 0$, buying M+1 units if she visits retailer j is individually rational for shopper k when j storable price is below $^M s_L$. This means that when the storable is on sale, we have a closed-form expression for $\Psi_{j,t}$:

$$\Psi_{j,t} = {}_{\tau=0}^{M} \delta^t s_L - (M+1)P^j_{S,t}$$

In the next subsection we will used these expressions for consumer surplus in deriving retailer's equilibrium pricing when $\mathbf{I_{t-1}}$=0. In particular, we show that when $\mathbf{I_{t-1}}$=0 retailers may place either the perishable or storable product on sale depending on how much surplus they choose to offer consumers.

C. Equilibrium Pricing

The previous subsection showed that equilibrium when $\mathbf{I_{t-1}} > 0$ is characterized by sales on the perishable only. Hence, if shoppers purchase more than one unit of the storable when it is on sale, then a sale on the storable is never followed by another sale on the storable.

Our next result establishes that either good may be on sale when $\mathbf{I_{t-1}} = 0$. From Proposition 1 we know that firms play a mixed strategy with respect to the amount of surplus ($_{j,t}$) offered. The choice of whether to place the storable or perishable on sale to generate a given when $\mathbf{I_{t-1}} = 0$ depends on the level of $\Psi_{j,t}$ the retailer chooses to offer shoppers. Lemma 1 defines a break-even consumer surplus, denoted $\overline{\Psi}$, such that the most profitable way to offer small (i.e., $< \overline{\ }$) is to put the perishable on sale, and to put the storable on sale when offering large . The intuition for this result is that offering any surplus to shoppers requires reducing profits from loyals. Retailers choose prices in such a way as to minimize the reduction in profits from loyals for any given For small amounts of , setting4Ā’¥Pp@@

[14] We assume this inequality holds in what follows. If this inequality is not satisfied, then only the perishable will be on sale. Since we want to explain the observed pattern of sale behavior, we assume conditions hold that make a sale on the storable profitable.

Then, if shoppers believe that $\min_j (P^j_{S,t})$ will be equal to s_H for periods t+1 through M+t,

a. $\bar{\ }$ > 0,

b. $_P($ $_{j,t}) >$ $_S($ $_{j,t})$ for all $_{j,t} < \bar{\ }$,

c. If $\sum_0^M$ s_L $(M$ $1)\underline{P}_S$ $\bar{\ }$ then to offer surplus $_{j,t}$, it will be more profitable to put the storable on sale than the perishable for $_{j,t}$ such that $\sum_0^M \delta^\tau s_L$ $(M+1)\underline{P}_S$ $\Psi_{j,t}$ $\bar{\Psi}$, where

$$\underline{P}_S = \frac{s_H\text{-N}(1- \ )}{+\ N\ (1- \ )\ (M+1)}$$

(i.e., the lowest price a retailer could profitable charge for the storable).

Proof: See Appendix A.

Lemma 1 indicates that in the symmetric equilibrium $\bar{\ }$ is always positive. Since the lower support of $G(\ )$ is zero, this means that when $\mathbf{I}_{t-1} = 0$, it will be profit maximizing for the retailer to discount the perishable to generate small levels of consumer surplus $\bar{\ }$

exceed those of storing tuna, and hence consumers' capacity to store tuna would be greater than for soda; that is $M_{tuna}>M_{soda}$. This implies that consumers will stock out of goods like soda more frequently than items like tuna. Thus, storable products with lower maximum inventory holdings (M) will have more frequent sales. Another, more subtle implication of Lemma 1 concerns the perishable price distribution. Since $\overline{\Psi}$ is decreasing in M, the maximum discount on the *perishable* product when $I_{t-1} = 0$ is decreasing in M. Consequently, Lemma 1 implies that the maximum possible discount on the perishable falls as the storage costs of the storable falls.

Another implication of Lemma 1.C is that whenever the storable is on sale, the surplus shoppers receive from buying the storable will be greater than the surplus they could get from

Given that all shoppers will buy M+1 units as long as $P^{j}_{S,t} \quad {}^{M}s_{L}$, retailers will find it profitable to offer $P^{j}_{S,t} \quad {}^{M}s_{L}$ when $\mathbf{I_{t-1}}$ =0. To see why, assume $\mathbf{I_{t-1}}$ =0, and to the contrary, that no retailer is offering $P^{j}_{S,t} \quad {}^{M}s_{L}$. Then retailer j's profit from setting $P^{j}_{S,t} \quad {}^{M}s_{L}$ is

$$\frac{(\quad {}^{M}s_{L} \quad)}{N} \quad (1 \quad)((M \quad 1)s_{L} \quad {}^{M} \quad)$$

which is greater than the profits from a sale on the perishable, by condition (3). Therefore, offering a sale would be profitable, contradicting the premise that having a sale on the storable yields lowers profits. It follows that in

a. then retailer j puts the storable on sale with probability $\quad =1\text{-}G(\bar{\ })$.

$$1 \quad \left(\frac{\frac{\ }{N}(s_H \quad P_S(\bar{\ }))}{(1 \quad)[(M \quad 1)P_S(\bar{\ }) \quad]} \right)^{\frac{1}{N \quad 1}}$$

for P_P (/(+N(1-)), - ¯).

Proof: See Appendix A.

Proposition 6 shows that when $\mathbf{I_{t-1}}$ = 0, each retailer randomizes over which good to put

[15] That is, the profit that can be earned from each shopper when P_S $^M S_L$ includes both profit from the storable (as in the single-product case) and profit from the perishable ( ). Hence, more intense competition on the storable arises in the multiproduct case.

storable good prices will be at non-modal levels for shorter periods of time than at modal levels.[16]

Finally, the model has several additional implications for price distributions. For example, it implies that a storable is more likely to have the same price in consecutive periods than a perishable, and conditional on a price reduction occurring, the average change will be larger for the storable. To see this last point, note that the maximal possible discount off the regular perishable price will be - /(+N(1-)) = N (1-)/(+N(1-)), while the minimum possible discount on the storable is s_H - $^{M}s_L$ which is greater than N (1-)/ (by the condition that > M) and this in turn is greater than N (1-)/(+N(1-)).

Another impliction concerns the relationship between M and the size and frequency of discounts. As noted above, higher M goods will have less frequent sales. In addition, the lower bound on the distribution of storable price ($\underline{P}_S$) is decreasing in M, so that larger discounts will be observed on low storage-cost products. Casual empiricism is consistent with the former prediction. Bulky products that are consumed frequently, such as soft drinks, go on sale frequently. Products for which it is feasible to store a sufficient quantity to cover demand for a long period of time, such as laundry detergent, go on sale less frequently. Hendel and Nevo's finding that soft-drinks are discounted much more frequently than laundry detergent supports this premise.[17]

III. Discussion

---

[16] Generalizing the model to allow reservation values or costs to vary over time, the logic of the model suggests that prices below the mode will be more common than prices above it.

[17] Hendel and Nevo find that soft drinks are discounted at least 5% from its regular price about twice as frequently as for laundry detergent.

Like the previous literature developed to explain retail sales, our model relies on

Specifically, the typical supermarket sells over 35,000 items, and consumers cannot possibly know all of the prices that will be relevant to their decision-making without visiting each retailer, or having retailers list all their prices in a public forum. As a practical matter, it is costly for consumers to visit retailers and for retailers to advertise their prices, so that consumers will be less than fully informed about prices. Nevertheless, as Lal and Matutes (L&M, 1994) show, even without knowledge of every price, consumers can be well informed about the surplus they will receive at each retailer. L&M show that when retailers advertise a subset of prices, consumers can draw correct inferences about the remaining prices, and therefore calculate the surplus they will receive that each retailer. Similarly, in our model, consumer would correctly infer the price of any non-sale item.

The empirical counterpart to the assumption that retailers advertise a subset of their products is the advertising circular that most chain supermarkets in the U.S. provide to virtually all consumers in a metropolitan area.[18] The circular informs consumers about the prices of the several hundred products that will be sold at bep(theum)9(l)-6(e)-5ular 25

---

[18] The obvious exception to this pattern is WalMart, now the U.S.'s largest retailer. WalMart's strategy is to charge low everyday prices and avoid sales. WalMart arguably has very different pricing incentives than other food retailers because so much of its product selection contains consumer durables, e.g., tires, clothing, hardware, and consumer electronics.

[19] Hosken and Reiffen (2001) also consider the effect of allowing consumers to shop at more than one retail outlet in a period.

could "cream skim"; visiting multiple retailers and buying only low-priced goods. To avoid this, in the L&M equilibrium, retailers choose to spread the aggregate discount across enough goods to mitigate the cream skimming potential.

While the L&M model explains the number of items listed in the circular, it is static and consequently does not explain why the composition of items in a circular changes from week to week. Our model can form a basis for understanding this practice. When a retailer carries multiple storable goods with different inventory patterns (i.e., different M), each good will have its own sale frequency. Hence, the number and identity of storables on sale will change from week to week. Extending the logic of Proposition 3 to this environment, this suggests that in weeks in which the number of storables that are appropriate for putting on sale is low (i.e., consumer inventories are high), the number of perishables on sale will increase.

We also assume that all shoppers have identical inventory holdings, and that the time since the last sale on a storable product is a sufficient statistic for the level of that inventory. Neither of these assumptions is literally true. Even among individuals who do inventory storables, there will be heterogeneity in inventory behavior due to differential storage costs. Such heterogeneity will result in more complex pricing variation than is modeled here. However, retailers likely have reasonably accurate information about average inventory. Because retailers communicate sale prices through weekly circulars, it is not costly for a retailer to monitor rivals' recent sale behavior. This information, along with their own recent pricing history and information on average consumer consumption behavior, can allow retailers to develop reasonable expectations about average consumer inventory holdings.

IV. Conclusion

With the increasing availability of high-quality data on retail prices and quantities, economists (as well as marketing professionals and others) have enthusiastically begun to estimate economic magnitudes, such as demand elasticities. It is well understood that identifying these magnitudes requires variation in some independent variable, such as price. What is perhaps less well appreciated is the relevance of the source of this variation. Empirical evidence suggests that sales account for 25-50% of the annual price variation for popular categories of grocery products. Because these temporary reductions are such an important source of price variation, understanding why these changes occur is critical to interpreting econometric estimates which use this data.

Our model implies that the multiproduct aspect of a supermarket's offerings influences how its prices change over time. Consumers who are price-sensitive shoppers likely examine weekly supermarket circulars and choose the retailer offering the best (utility maximizing) set of prices for that consumer. This implies that prices of other goods sold by a retailer will influence

References

Aguirregabira, Victor (1999) "The Dynamic Markups and Inventories in Retailing Firms," *Review of Economic Studies*; 66, pp.275-308.

Chevalier, Judith, Anil Kashyap, and Peter Rossi (2003) "Why Don't Prices Rise During Periods of Peak Demand? Evidence from Scanner Data," *American Economic Review*; 93, pp.15-37

Conlisk, John, Eitan Gerstner, and Joel Sobel (1984) "Cyclic Pricing by a Durable Goods Monopolist," *Quarterly Journal of Economics;* 99, pp. 489-505

DeGraba, Patrick (2006) "The Loss Leader is a Turkey: Targeted Discounts from Multi-product Competitors," *International Journal of Industrial Organization*; 24, pp.613-628

Dutta, Shantanu, Mark Bergen, and Daniel Levy (2002) "Price Flexibility in channels of Distribution: Evidence from Scanner Data," *Journal of Economic Dynamics & Control*; 26, pp. 1845-00.

Hendal, Igal, and Aviv Nevo "Sales and Consumer Inventory," *Rand Journal of Economic*s, forthcoming.

Hong, Pilky, R. Preston McAfee, and Ashish Nayyar (2002) "Equilibrium Price Dispersion with Consumer Inventories" *Journal of Economic Theory*; 105, pp. 503-517.

Hosken, Daniel and David Reiffen (2001) "Multiproduct Retailers and the Sale Phenomenon," *Agribusiness*; 17, pp.115-137

___ (2004a) "Patterns of Retail Price Variation," *RAND Journal of Economics*; 35, pp. 128-146.

___ (2004b) "How do Retailers Determine Sale Products: Evidence from Store-Level Data," *Journal of Consumer Policy*; 27, pp.141-177.

Klein, Benjamin and Keith Leffler (1981) "The Role of Market Forces in Assuring Contractual Performance," *Journal of Political Economy*; 89, pp. 615-41.

MacDonald, James (2000) "Demand, Information, and Competition: Why Do Food Prices Fall at Seasonal Demand Peaks?," *Journal of Industrial Economics*; 48, pp.27-45.

Lal, Rajiv and Carmen Matutes (1989) "Price Competition in Duopoly Markets," *RAND Journal of Economics*; 20, pp. 526-37.

___ (1994) "Retail Pricing and Advertising Strategies," *Journal of Business*; 67, pp. 345-70

Levy, Daniel, Mark Bergen, Shantanu Dutta, and Robert Venable (1997) “The Magnitude of Menu Costs: Direct Evidence from Large U.S. Supermarket Chains,” Q*uarterly Journal of Economics*; 112, pp. 791-825

Pashigian, B. Peter (1988) "Demand Uncertainty and Sales: A Study of Fashion and Markdown Pricing," *American Economic Review;* 78, pp. 936-53.

___ and Brian Bowen (1991) "Why are Products Sold on Sales?: Explanations of Pricing Regularities," *Quarterly Journal of Economics*; 106, pp.1014-1038.

Pesendorfer, Martin (2002) "Retail Sales: A Study of Pricing Behavior in Super Markets," *Journal of Business;* 75, pp.33-66

Sobel, Joel (1984) "The Timing of Sales," *Review of Economic Studies*; 51, pp. 353-68.

Varian, Hal R. (1980) "A Model of Sales," *American Economic Review*; 70, pp. 651-9.

Ward, Michael R. and Lee, Michael J. (2000) "Internet Shopping, Consumer Search and Product Branding,"

Appendix A

To prove Proposition 1, first note that when all shoppers begin a period with the same inventory, they all will receive the same from any given set of prices. Hence, is unambiguously defined in this case. We can therefore define **P( )** as the set of (P_{Pt}, P_{St}) that yield maximal profits among the set of prices that result in surplus . That is, while there are multiple pairs of P_{Pt} and P_{St} that yield any given level of surplus, the present value of expected profits could differ across these pairs. It follows that for any level of , firms will always choose to offer that by selecting prices in the set **P( )**. Proposition 1 derives the properties of the symmetric distribution of , assuming prices are drawn from the set **P( )**.

Proposition 1: The only symmetric equilibrium must feature all retailers playing a mixed proposed equilibrium. Suppose that firm i deviates from this proposed equilibrium by charging prices $\hat{P}_{St}$ and $\hat{P}_{Pt}$. Conditional on firm i having the highest , this change does not lower expected future profits since it yields the same I_t as the pair $\hat{P}_{St}$ and $\hat{P}_{Pt}$. To see the effect on i's profits, note that since the present value of expected profits from all pairs of prices in **P($\hat{}$)** are identical, the change in i's expected profits from this deviation is equal to the change relative to charging $\hat{P}_{St}$ and $\hat{P}_{Pt}$. We see that such a deviation results in retailer i offering surplus $\hat{}$ + with probability , and $\hat{}$ with probability 0. Compared to the pair ($\hat{P}_{St}$, $\hat{P}_{Pt}$), setting P_{St} = $\hat{P}_{St}$ and P_{Pt}= $\hat{P}_{Pt}$ does not reduce i's future profits, and therefore the change in the present value of firm i's profits from that deviation is equal to the change in period t profits,

$$\Pr(\ _j\ \hat{}\ \ , \mathit{for\ all}\ j,\ _j\ \hat{}\ \mathit{for\ any}\ j)\ \ (\hat{P}_{S,t}, \hat{P}_{P,t}\ \ )\quad \Pr(\ _j\ \hat{}, \mathit{for\ all}\ j)\ \ (\hat{P}_{S,t}, \hat{P}_{P,t})$$

$$\Pr(\ _j\ \hat{}\ \ , \mathit{for\ some}\ j)\frac{}{N}(\hat{P}_{S,t}\ \ \hat{P}_{P,t}\ \ )\quad \Pr(\ _j\ \hat{}, \mathit{for\ some}\ j)\frac{}{N}(\hat{P}_{S,t}\ \ \hat{P}_{P,t})$$

$$\sum_{k\ 1, k\ i}^{N} \Pr(\ _j\ \hat{}\ \ , \mathit{for\ all}\ j,\ _j\ \hat{}\ \mathit{for}\ k\ \mathit{firms})\ \ (\hat{P}_{S,t}, \hat{P}_{P,t}\ \ )$$

$$\sum_{k\ 1, k\ i}^{N} \Pr(\ _j\ \hat{}\ \mathit{for\ all}\ j,\ _j\ \hat{}\ \mathit{for}\ k\ \mathit{firms})[\frac{}{N}(\hat{P}_{P,t}\ \ \hat{P}_{S,t})\ \ \frac{1}{k\ \ 1}(\hat{P}_{P,t}\ \ m(\hat{P}_{S,t})\hat{P}_{S,t})]$$

As approaches zero, the differences on the first two lines approach zero, while the difference on the last two lines becomes unambiguously positive. That is, there is a finite probability that k other firms offer surplus $\hat{}$, and when that occurs, firm i's profits in period t are higher by

$$\frac{1}{\ \ 1} \qquad 1 \quad —$$

value of expected future profits of   (s_H +   )/(1 -   )N. It is also true that every strategy within G(|$\mathbf{I}_{t-1}$) must yield the same present value of expected future profits. In particular, the present value of expected future profits at time t must be   (s_H +   )/(1 -   )N, and the present value of expected future profits at time t+1 must also be   (s_H +   )/(1 -   )N for all   in the support of G(|$\mathbf{I}_t$

π

—

$$G(\Psi) = (\frac{\Psi\gamma}{N(1-\gamma)(\beta -}$$

Appendix B - Shoppers' Surplus Function

A shopper who enters period t with inventory, I_{t-1} seeks to maximize her utility, which is a function of her current and future consumption of the two goods. Shopper k's goal in time t is to pick the retailer (j) and make purchases of the perishable and storable to maximize the present purchases (j).[20]

We define the function as the difference between the H_{jt} associated

[20] Hereafter, $m_{P,t}$ and $m_{S,t}$ refer to a consumer's purchases at retailer j at prices $P^{j}_{S,t}$ and $P^{j}_{P,t}$.

$P^j_{S,t}$ and $P^j_{P,t}$, we can write $_{jt}$ as [21]

$$\text{(B.2)} \quad _{jt} \quad [\ q_{P,t} \quad s_L q_{S,t} \quad P^j_{P,t} q_{P,t} \quad P^j_{S,t}(I_t \quad I_{t\ 1} \quad q_{S,t}) \quad E(V(\mathbf{I}_t, \mathbf{P}_{t\ 1} \,|\, \mathbf{I}_{t\ 1}, \mathbf{P}_t))]$$
$$\text{-}s_L q_{S,t} \quad E(V(\mathbf{I}'_t, \mathbf{P}_{t\ 1} \,|\, \mathbf{I}_{t\ 1}, \mathbf{P}_t)$$
$$=[\ q_{P,t} \quad s_L \quad P^j_{P,t} q_{P,t} \quad P^j_{S,t}(I_t \quad I_{t\ 1} \quad 1) \quad E(V(\mathbf{I}_t, \mathbf{P}_{t\ 1} \,|\, \mathbf{I}_{t\ 1}, \mathbf{P}_t))]$$
$$\text{-}s_L \quad E(V(\mathbf{I}'_t, \mathbf{P}_{t\ 1} \,|\, \mathbf{I}_{t\ 1}, \mathbf{P}_t))$$

Equation (B.2) implies that if retailer j sets $P^j_{S,t} \quad s_L$ and $P^j_{P,t}$ then $_{jt} \quad P^j_{P,t}$, since $\mathbf{I}_{t-1} = \mathbf{I'}_{t-1}$. The relationship between $_{jt}$ and $P^j_{S,t}$ when $P^J_{S,t} \quad s_L$ is complicated by the intertemporal nature of the maximization in equation (B.1). In particular, in the general case there is no closed-form characterization of . However, we can obtain a closed-form expression for $_{jt}$ when consumer k has inventory I_{t-1}, observes $P^j_{S,t} \quad ^{M\ 1\ I_{t\ 1}} s_L$, and knows that storable prices will be s_L or higher for the next M - I_{t-1} periods . Under these conditions, and assuming $P^j_{P,t}$,[22]

---

[21] The expression is similar when I_{t-1}= 0. There is one potential change in the first term (the utility from shopping at retailer j) because $q_{S,t}$ may be zero when I_{t-1}=0 (i.e., the consumer may not consume a unit of the storable at time t). There are two changes in the second term (the counterfactual): the consumer does not consume the storeable in period t (i.e., s_L does not appear in the second term), and $I_t = I_{t-1}$, rather than I_{t-1} -1.

[22] Since retailers make zero sales of the perishable if $P^j_{P,t}$ this condition is always satisfied in equilibrium.

$m_{S,t}$ $I_{t\ 1}$ 1 M

j j i i

L t P,t S,t S,t L i P, t i P, t

jt 1 $m_{S,t}$ $I_{t\ 1}$

M 1

M t M t 1 t 1 t

$I_{t\ 1}$ 1

i