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AN APPLICATION OF THE LE CBATELIER PRINCIPLE

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Uncertainty and the Value of Information: An Application of the Le Chatelier Principle

by

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It is well accepted that accurate information has a non-negative value. 1
It is more difficult to determine how that value changes when the uncertainty faced by a decision maker changes. Our intuitive response is that the relation should be positive: an increase in uncertainty increases the expected value of information. This hypothesis is of interest because it may have certain implications for the information-gathering activities of firms and consumers.

For example, suppose a seller's offering is of uniform quality but con-sumers are uncertain of what this level of quality is. The decision of how much to buy from such a seller must then be made subject to the uncertainty about quality. Suppose, however, that perfect information about the quality can be made available to consumers at some cost. For example, consumers may be able to buy a device that can measure with perfect accuracy the quality of the good before purchase; alternatively, the seller may (at some cost to consumers) be able to "guarantee" claims about the quality of the seller's current offering. If the cost of consumer measurement or of the seller guarantee is independent of the potential variance in quality, the hypothesis above suggests that information produced in either way is more likely to appear the greater is the potential variance of quality.

Unfortunately, the hypothesis that uncertainty and the value of information are positively related is not a straightforward implication of maximization behavior under uncertainty. Instead, such a positive relation can be esta-blished only by imposing

The local nature of the Le Chatelier principle necessarily limits this convexity to a

is itself a random variable with a distribution derived from f(A I). The ex ante value of information, or the expected value of information before it is received, is therefore just the expected ex post value of information.

Formally, consider the following maJ!:imization problem where the Xi are chosen after Al is revealed:

(1) Maximize Z = h(XI, .. ,Xn,Al> .. ,Am).
Xl'·· 'Xn

Assuming a unique solution exists for each A = (AI' .. ,Am)' the optimal Xi
are derived from the following system of first-order equations:

where hi = dhjdxj

• Let (hjj) be the hessian matrix of cross partials of h
with respect to the Xi. The sufficient second order conditions are that all principal minors I(hi)1 of order k have sign (_I)k at the maximum point.
Let the choice functIOns and the indirect payoff function be X~(AI' .. ,Am) o f

Because the X~ are not functions of AI' they are constant in equation (5) with respect to AI'

The ex post value of information on AI' VeAl' .. ,Am)' is the the

just-binding point, AB.12 By applying Silberberg's Generalized Envelope Theorem13 to the functions Z* and zr evaluated at AB, we obtain:

(lOa) Z* = zr

(lOb) Z*

Al = zr

Al

(IOc) *

and so the maximization problem represented by equation (3) is equivalent to that of equation (1) for Al = AI' Linearity of h in AI' however, clearly is not a necessary condition for the application of the Le Chatelier principle to hold.

Hess (1982) states a theorem that uses a set of sufficient conditions (apparently) less restrictive than Gould's to prove that the relation between uncertainty and the expected value of information is positive. This set is, essentially, that V(A I) is everywhere convex in AI' On the one hand, his sufficient conditions are less restrictive than the ones used here because they they

When I first formulated the Le Chatelier theorem twenty years ago, I had hopes of proving the non-local property. But for many years a proof eluded my most determined efforts. Finally it dawned on me that the theorem was not true in the large ....
LQJur paradox in the large could be illustrated by the following
type of example: Near the critical point where labor and land go
from being substitutes to being complements, as measured by the
sign of off -diagonal elements in the profit Hessian matrix [A ij],
we could find a clever counterexample in which the long-run arc
elasticity of demand for labor was more inelastic than the short-run.I5

I believe his remarks can be applied to the problem here in the following way.

Consider the case where only one choice variable, X, is constrained by uncertainty about a single parameter, A. If uncertainty about A is a binding constraint on the choice of X and if there exists a just binding point, then dX* jdA f= 0

and a p.d.f. f(a) for which a mean-preserving spread decreases the expected value of information. An example is presented in Table I:

Probability
of �S�t�a�t�~�
1 2 3

.40 .20 .40
.45 .10 .45

Table 1
A Counterexample

ER* ERE

$101.66 $90.15
69.30 62.32

Expected
Value of
Inf orma tion 17

$11.51
6.98

d = c = b = I a1 =.1 a2 = I as = 1.9 p = 100

To rule out the existence of a second just-binding point, it is sufficient to restrict X· to be a strictly monotonic function of a; however, this apparently is not sufficient to rule out the occurence of any concave portions of V(a). Hence, it may be that, for global convexity, we must appeal to the broad sufficient conditions of Hess (1982).

There is another set of circumstances under which the Le Chatelier prin-ciple may not hold in the large. Gould (1974) presents a counterexample in which the expected value of information and uncertainty are negatively re-lated. The example consists of a firm maximizing profits by choosing the level of a single input, X, subject to

result that as uncertainty goes to zero around the mean, the expected value of information approaches a strictly positive value. This is because the discon-tinuity creates a "point" of uncertainty. In fact, the value approached is equal to the average ex post value of information at the discontinuity, calculated by taking the limit from above and below.20

The second important effect is to create an interval where the ex post value of information is concave. As A approaches the discontinuity from below, it goes through a just binding point, where the unconstrained choice of X, X*, which is positively related to A, is equal to the choice of X under uncertain-
ty, X E. Through this point, the ex post value of information is convex. The discontinuity, however, changes the first order condition of the firm's profit maximization problem and decreases X* (discontinuously, of course). As A continues to increase, it goes through a second just binding point where X* = XE again.

Figure 2 illustrates this result. Strictly above or below the discon-tinuity, V(A) is convex. In at least part of the region between the two just binding points, Al and A2, however, the ex post value of information is a concave function of A. Hence, for small levels of

To ensure that the Le Chatelier principle holds in the large, what

Appendix

This appendix discusses the sufficient second order conditions associated with the maximization problem of equations (7) and (8).

Let (Hr r) be the matrix of the cross partials of the first r choice variables. These are the choice variables in the constraints of equation (8). Let (Hn_r n-r) be the matrix of the cross partials of the last (n-r) choice variables and (Hr n_r)be the matrix of the cross partials between the two groups. Finally,

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Figure 1
Multiple Just-Binding Points
h(X,A)

Figure 2
A Discontinuity in