Switching Costs and Equilibrium Prices

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Abstract. In a competitive environment, switching costs have two e ects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. I provide conditions under which switching costs decrease or increase equilibrium prices. Taken together, the suggest that, if markets are very com-

1. Introduction

Consumers frequently must pay a cost in order to switch from their current supplier to a di erent supplier (Klemperer, 1995; Farrell and Klemperer, 2007). These costs suggest some interesting questions: are markets more or less competitive in the presence of switching costs? Are prices higher or lower under switching costs? How do seller pro ts and consumer surplus vary as switching costs increase?

Empirical evidence regarding these questions is ambiguous: although most studies suggest that switching costs lead to higher prices, there is also evidence to the contrary. This ambiguity is mirrored by the theoretical literature: some papers provide su cient conditions such that switching costs make markets less competitive, with others predict the opposite e ect.

costs and conclude that switching costs should only raise concerns in concentrated markets. Four other recent papers on dynamics with switching costs are Arie and Grieco (2012); Biglaiser, Cremer and Dobos (2010); Somaini and Einav (2012); and Pearcy (2011). Arie or seller j are the same. This greatly simpli es the analysis, for even forward looking buyers need not compute any value function. (Sections 6 and 7 deal with asymmetric competition, and I will then need to explicitly compute the buyer's value function.) In each period the buyer chooses the insider seller if and only if

$$z_1 p_1 + u_1 z_0 p_0 + u_0 s$$
 (1)

where u_i is the buyer's expected discounted utility from being locked in to seller i and is the buyer's discount factor.³ By symmetry, the value, starting next period, of being locked in to the rm that the buyer is currently locked in is the same as the value of being locked in to the other rm (that is, assuming that the switching cost has already been paid). For this reason, $u_1 = u_0.^4$

De ne

z
$$z_1 z_0$$
 (2)
x $p_1 p_0 s$ (3)

Then (1) may be re-written as z = x. In words, x is the critical level of the buyer's relative preference z such that the buyer chooses the insider. De ne by q_1 and q_0 the probability that the buyer chooses the insider or the outsider, respectively. If z is distributed according to F(z), then we have

$$\begin{array}{rcl} \mathbf{q}_1 &=& 1 & \mathbf{F}\left(\mathbf{x}\right) \\ \mathbf{q}_0 &=& \mathbf{F}\left(\mathbf{x}\right) \end{array}$$

I make the following assumptions regarding the c.d.f. F and the corresponding density f :

Assumption 1. (i) F(z) is continuously di erentiable; (ii) f(z) = f(z); (iii) f(z) > 0; 8z; (iv) f(z) is unimodal; (v) F(z)=f(z) is strictly increasing.

Many distribution functions, including the Normal and the t, satisfy Assumption 1. In many of the results that follow, I will use repeatedly the following lemma, which characterizes several properties of F that follow from Assumption 1:

Lemma 1. Under Assumption 1, the following are strictly increasing in z:

$$\frac{F(z)^2}{f(z)}; \qquad \frac{F(z)}{f(z)}; \qquad \frac{2F(z)}{f(z)}$$

Moreover, the following is increasing in z i z > 0 (and constant in z at z = 0):

$$\frac{1 + F(z)^{2} + F(z)^{2}}{f(z)}$$

 ^{3.} I will denote the sellers' discount factor by . In many applications, it may make sense to assume
 a. However I distinguish between the buyer and the seller discount factor throughout. Among other reasons, this has the advantage of better highlighting the role of forward looking by buyers vs sellers.

^{4.} In Sections 6 and 7, where I explicitly consider an asymmetric duopoly, the equality $u_1 = u_0$ no longer holds and I will need to explicitly compute the buyer value functions.

Proof: See Appendix. ■

In the next sections, I o er sets of conditions under which switching costs lead to an increase or a decrease in prices (and average price). In the next section, I show that average price decreases (resp. increases) in switching costs if the initial value of the switching cost

Figure 1

anti-competitive e ect.

In sum, Proposition 2 suggests that the e ect of switching costs on prices is largely an empirical question. Dube, Hitsch and Rossi (2006) claim that, for various products, the

Figure 2

Switching cost and equilibrium average price as a function of the sellers' discount factor p



(As I mentioned earlier, this is just the <code>\elasticity rule"</code> with the added element that sellers <code>\subsidize"</code> their cost by V.) We also saw that the seller value functions are given by

$$v_1 = 1 F(p)$$

Figure 3 In regionA

5. Pro ts and welfare

So far I have been dealing with the the impact of switching costs on average price, one of the central questions in the academic and public policy debate. What can be said about seller pro ts and consumer welfare? Tentatively, Propositions 2 and 3 suggest that under some conditions buyers are better o , and sellers worse o , with switching costs than without. However, paying a lower price is only half of the story for a buyer. To the extent that there is private information about preferences, switching actually occurs along the equilibrium path. We must therefore subtract the costs from switching when considering expected buyers surplus. As to the sellers, the fact that average price declines does not imply that sellers are uniformly worse o with switching costs. In fact, as shown earlier, the insider may be able to increase its price as a result of a higher switching cost. In sum, it is not obvious whether switching costs bene t buyers and sellers. My next result provides some answers to this question.

Proposition 4. If s is small, then there exist ${}^{0}(s)$ and ${}^{0}(s)$, where $0 < {}^{0}(s) < {}^{0}(s) < 1$, such that an increase in switching cost s leads to

- 1. An increase in the insider's value if and only if $< ^{0}(s)$.
- 2. A decrease in the outsider's value for all .
- 3. A decrease in industry value (that is, the joint value of insider and outsider) for all
- 4. An increase in consumer surplus if and only if > 0(s).
- 5. A decrease in welfare for all .

Proof: See Appendix. ■

Note that one of the implications of Proposition 4 is that, if ${}^{0}(s) < {}^{\circ}(s)$, then all agents (insider, outsider, buyer) are worse o with switching costs than without.⁶ More generally, Proposition 4 raises an interesting question: if switching costs are frequently created by sellers; and if largely sellers lose as a result of switching costs; then why do sellers create switching costs? Part of the answer to the question is given by point 1 in Proposition 4: if is su ciently small, then an increase in switching costs increases the value of the incumbent seller. In Section 7 I take this issue of step further by considering the possibility that one of the sellers unilaterally increases the cost of switching away from its product.

6. Customer recognition

In the preceding sections I have assumed that, other than the switching cost, sellers believe that the consumer is on average indi erent between the two sellers. In real-world customer markets, however, sellers typically know something about buyer preferences. I now explicitly consider the possibility of *customer recognition*, that is, the possibility that sellers have

^{6.} Biglaiser, Cemer and Dobos (2010) also show that an increase in the switching costs of all consumers can lead to a decrease in the prots of the incumbent. However, this takes place in a di erent context and for di erent reasons. See also Section 8.

some information about buyer preferences.⁷ I model this by assuming that the buyer has a preference for seller i that is given by z_i , distributed according to cdf $_i(z_i)$. In the previous sections, I assumed that $z_i = z_j$ is distributed according to a cdf F(z) which is symmetric about zero. Now I assume that $z_A = z_B = d + z$, where d is a constant common knowledge to sellers and z is distributed according to a cdf F(z) which is symmetric about zero.

We now have buyer preferences that are serially correlated. This changes the problem substantially. In particular, a buyer is no longer indi erent between being attached to one seller or the other, for a buyer anticipates that, on average, one of the sellers will provide higher utility. Speci cally, a buyer currently locked in to seller

high enough, then the harvesting e ect dominates and average price increases as the result of an increase in \mathbf{s} .

Proposition 5 is in line with the nding in Fabra and Garcia (2012) that \switching costs should only raise concerns in concentrated markets." In fact, a market with high **d** is a \concentrated" market, if we focus on the particular customer with high **d**. But Proposition 5 is about the value of **d**, not the distribution of market shares. It is conceivable that overall market shares are 50{50 and still the e ect of switching costs is very anticompetitive.

■ Closed-form solution. For most of the paper I have made minimal assumptions regarding the distribution of F(z). The disadvantage of that approach is that no analytical closed form solution is feasible. If we make additional assumptions, then a closed-form solution may be possible. Speci cally, suppose that z is uniformly distributed. Without further loss of generality, suppose that $F(z) = \frac{1}{2} + z$.⁸ Suppose also that z = 0. Then a closed-form solution can be derived for all values of d (sel0ues of



Figure 4 Switching cost and average price: linear case(z) = $\frac{1}{2}$ + z, = :5, = 0)

7. Endogenous asymmetric switching cost

Suppose that only seller A creates a switching cost. In other words, it costs s_A for a consumer to switch from seller A to seller B, but it costs zero for the consumer to switch from seller A. As in the previous section, I need to keep track of the seller's identity. Moreover, I need to explicitly compute the buyer's value functions. A buyer who is currently locked-in to seller A, chooses seller A again if and only if

 $z_A \hspace{0.1in} p_{1A} \hspace{0.1in} + \hspace{0.1in} u_{A} \hspace{0.1in} s_A \hspace{0.1in} + \hspace{0.1in} z_B \hspace{0.1in} p_{0B} \hspace{0.1in} + \hspace{0.1in} u_{B}$

If the buyer is locked-in to seller B, however, then he chooses seller B if and only if

 z_B z_A p_{1B} p_{0A} u_B + u_A

This implies that the critical values of the buyer's relative preference leading to a switch away from the insider seller are now given by

the insider seller sells with probability 50%. This is one of the important di erences of my

9. Conclusion

In a competitive environment, switching costs have two e ects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. In this paper, I derived conditions under which switching costs decrease or increase equilibrium prices. Overall, the paper's message is that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by \competitive" I mean a market that is close to a symmetric duopoly or one where the sellers' discount factor is very high.

Appendix

() f(Proof of Lemma 1: First notice that

$$\frac{F(z)^{2}}{f(z)} = F(z) \frac{F(z)}{f(z)}$$

Since F(z) is increasing and $\frac{F(z)}{f(z)}$ is strictly increasing (by Assumption 1), itnw269(strt)]TJ-270(t)1(heJ-270(p-31(y)-duct)))

su ciently low. Speci cally, at $\mathbf{s} = 0$ we have

$$\frac{@v_0}{@s} = \frac{1}{1} \quad \frac{@x}{@s}$$
$$\frac{@v_1}{@s} = 1 + \frac{1}{1} \quad \frac{@x}{@s} = \frac{1 \cdot 2}{1} \quad \frac{@x}{@s}$$
$$\frac{@(v_0 + v_1)}{@s} = \frac{2}{1} \quad \frac{@x}{@s}$$

Since @x = @s < 0, we conclude that, at s = 0, both v_0 and $v_0 + v_1$ are decreasing in s, whereas v_1 is increasing in s if and only if $< \frac{1}{2}$. Next consider consumer welfare. Recall that the distribution of z_i (i = A; B) is given by (z_i)

and de ne Ζ

$$\begin{array}{cccc} \mathsf{E}\left(z\right) & & z_{i} & d & (z_{i}) \, d & (z_{j}) \\ & & z_{i} & z_{j} & z \end{array}$$

In words, E(z) is the buyer's expected valuation given that he chooses a particular seller by using the threshold z of di erences in valuations (times the probability of choosing that particular seller). Per period expected consumer surplus is given by

$$u = E(x) + E(x) - 1 - F(x) - p_1 - F(x)(p_0 + s)$$
(21)

Notice that if E(x) is the expected value of z_i given that $z_i = z_i = x$, then E(-x) is the expected value of z_i given that $z_i = z_i$ \mathbf{x} , where the latter is the same condition as $\mathbf{z}_i = \mathbf{z}_j = \mathbf{x}$, that is, the negation of $z_i = z_i - x$. De ne

$$e(z) = \frac{d E(z)}{d z}$$

Di erentiating (21) with respect to s_i , we get

$$\frac{@u}{@s} = e(x) \quad e(-x) + f(x)(p_1 - p_0 - s) \quad \frac{@x}{@s} = 1 \quad F(x) \quad \frac{@p_1}{@s} = F(x) \quad \frac{@p_0}{@s} = F(x)$$

Evaluating at $\mathbf{s} = 0$, we get

$$\frac{@u}{@s}_{s=0} = \frac{@p}{@s} = \frac{1}{2}$$

Di erentiating (8) we get $@x = @s = \frac{1}{3}$. Di erentiating (9), we get $\frac{@p}{@x} = 2$. It follows that

$$\frac{@u}{@s} = \frac{2}{3} \qquad \frac{1}{2}$$

It follows that consumer surplus increases if and only if > $\frac{3}{4}$. Finally, the result regarding total welfare is trivial: since the market is covered, all price e ects are simply a transfer between buyers and sellers. The net e ects on consumer welfare come from \transportation cost" (an e ect of second order at s = 0 and switching costs (a rst-order e ect). This implies that an increase in s has a rst-order negative e ect on total welfare. ■

Proof of Proposition 5: Seller i's value functions (i = A; B; j & i) are given by

The corresponding rst-order conditions are

Solving for $p_{1i}; p_{0i}$, we get

$$p_{1i} = \frac{1 - F(x_i)}{f(x_i)} \qquad V_i$$

$$p_{0i} = \frac{F(x_i)}{f(x_i)} \qquad V_i$$
(23)

where $V_i = v_{1i} = v_{0i}$. Substituting (23) for p_{ki} in (22), I get

$$v_{1i} = \frac{1 |F(x_i)|^2}{f(x_i)} + v_{0i}$$

$$v_{0i} = \frac{F(x_j)^2}{f(x_j)} + v_{0i}$$
(24)

and so

$$V_{i} = v_{1i} \quad v_{0i} = \frac{1 |F(x_{i})|^{2}}{f(x_{i})} - \frac{F(x_{j})^{2}}{f(x_{j})}$$
(25)

Substituting (25) for V_i in (23), we get

$$p_{1i} = \frac{1}{f(x_i)} \frac{F(x_i)}{f(x_i)} - \frac{1}{f(x_i)} \frac{F(x_i)}{f(x_i)} + \frac{F(x_j)^2}{f(x_j)}$$

$$p_{0i} = \frac{F(x_j)}{f(x_j)} - \frac{1}{f(x_i)} \frac{F(x_i)}{f(x_i)} + \frac{F(x_j)^2}{f(x_j)}$$
(26)

This parallels the derivation starting in (4), only that now value functions and prices are indexed by seller identity.

Next consider the buyer value functions, u_i , which I measure *before* the buyer learns his valuations z_A ; z_B . The values of u_i are recursively given by

$$u_{i} = E(x_{i}) + E(x_{i}) + 1 \quad F(x_{i}) \quad d_{i} \quad p_{1i} + u_{i} + F(x_{i}) \quad d_{j} \quad s \quad p_{0j} + u_{j}$$
(27)

i = A;B; j € i.

Recall that the preference thresholds are given by

Notice that, as $d\,!\,1$, $x_A\,!\,1$ and $x_B\,!\,+1$.^11 This implies that, in the limit as $d\,!\,1$, $F\,(x_A)\,!\,$ 0 and $F\,(x_B)\,!\,$ 1.

In what follows, I use the notation, for a generic variable x,

$$\underline{x}$$
 lim $\frac{dx}{ds}$

^{11.} To see why, suppose that x_A and x_B remain bounded while d ! 1 . Then from (26) and (27) all prices and value functions are bounded, which by (28) contradicts the hypothesis that x_i are bounded.

Taking derivatives of (26) with respect to **s** and then limits as **d**!**1** | get

Taking derivatives of (27) with respect to **s** and then limits as **d**! **1** I get

where I note that $\lim_{x_i \ge 1} e(x_i) = \lim_{x_i \ge 1} e(x_i) = 0$. Taking derivatives of (28) with respect to s and then limits as d ≥ 1 I get

$$\underline{\mathbf{x}}_{A} \quad \underline{\mathbf{p}}_{1A} \quad \underline{\mathbf{p}}_{0B} \quad (\underline{\mathbf{u}}_{A} \quad \underline{\mathbf{u}}_{B}) \quad 1$$

$$\underline{\mathbf{x}}_{B} \quad \underline{\mathbf{p}}_{1B} \quad \underline{\mathbf{p}}_{0A} + (\underline{\mathbf{u}}_{A} \quad \underline{\mathbf{u}}_{B}) \quad 1$$

$$(31)$$

The system formed by (29), (30) and (31) includes 8 equations and 8 unknowns. Solving form p_{1A} , I get

$$p_{1A} = \frac{1}{3} (1 \quad 2) (1 +)$$

Since q_{1A} ! 1 and q_{0B} ! 1 as d ! 1, average price is determined by p_{1A} , whereas changes in average price are determined by p_{1A} . The result follows.

Proof of Proposition 6: The buyer's value functions, measured *before* the buyer learns his valuations z_A ; z_B , are recursively given by

In what follows, I use the notation, for a generic variable \mathbf{x} ,

$$\hat{\mathbf{x}} = \frac{\mathbf{d} \mathbf{x}}{\mathbf{d} \mathbf{s}_{\mathsf{A}}} \mathbf{s}_{\mathsf{A}} = 0$$

Note that, at $s_A = 0$, we have a symmetric outcome where $x_A = x_B = 0$, $u_A = u_B = u$, and $p_{1A} = p_{0B} = p_{0A} = p_{1B} = p$. Di erentiating the buyer value functions with respect to s_A at $s_A = 0$ and de ning $e(x) = \frac{dE(x)}{dx}$, I then get

$$\hat{u}_{A} = \mathbf{e}(0) \, \hat{\mathbf{x}}_{A} \quad \mathbf{e}(0) \, \hat{\mathbf{x}}_{A} + \frac{1}{2} \qquad \hat{\mathbf{p}}_{1A} + \hat{\mathbf{u}}_{A} \qquad \mathbf{f}(0) \, \hat{\mathbf{x}}_{A} \qquad \mathbf{p} + \mathbf{u} + \frac{1}{2} \qquad 1 \quad \hat{\mathbf{p}}_{0B} + \hat{\mathbf{u}}_{B} + \mathbf{f}(0) \, \hat{\mathbf{x}}_{A} \qquad \mathbf{p} + \mathbf{u} \\ = \frac{1}{2} \quad \hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B} \qquad \hat{\mathbf{p}}_{1A} \qquad \hat{\mathbf{p}}_{0B} \qquad 1 \\ \hat{\mathbf{u}}_{B} = \frac{1}{2} \quad \hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B} \qquad \hat{\mathbf{p}}_{1B} \qquad \hat{\mathbf{p}}_{0A}$$

$$(32)$$

This is intuitive: a buyer's expected valuation increases by the increase in future expected valuation, $p_1^1(\hat{u}_A) + \hat{u}_B$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}(\hat{p}_{1A} + \hat{p}_{0B})$ $p_1^1(\hat{u}_A) + \hat{u}_B$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}(\hat{p}_{1A} + \hat{p}_{0B})$ $p_1^1(\hat{u}_A) + \hat{u}_B$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}(\hat{p}_{1A} + \hat{p}_{0B})$ $p_1^1(\hat{u}_A) + \hat{u}_B$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}(\hat{p}_{1A} + \hat{p}_{0B})$ $p_1^1(\hat{u}_A) + \hat{u}_B$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}(\hat{p}_{1A} + \hat{p}_{0B})$ the buyer is attached to seller A, buyer welfare further decreases by an additional $\frac{1}{2}$ s, the probability that an immediate switch to seller B will take place.

Di erentiating (18) with respect to s_A at $\dot{s_A}$ = 0, I get

$$\hat{\mathbf{x}}_{A} = \hat{\mathbf{p}}_{1A} \quad \hat{\mathbf{p}}_{0B} \qquad (\hat{\mathbf{u}}_{A} \quad \hat{\mathbf{u}}_{B}) \quad 1$$

$$\hat{\mathbf{x}}_{B} = \hat{\mathbf{p}}_{1B} \quad \hat{\mathbf{p}}_{0A} \qquad (\hat{\mathbf{u}}_{B} \quad \hat{\mathbf{u}}_{A})$$

$$(33)$$

The derivation of value functions and rst-order conditions is identical to those in the proof of Proposition 5, leading to (26), with the di erence that the values of x_i are now di erent. Di erentiating with respect to s_A at $s_A = 0$, and noting that $f^0(0) = 0$, I get

$$\hat{p}_{1i} = (1) \hat{x}_i + \hat{x}_j$$

$$\hat{p}_{0i} = \hat{x}_i$$

Equations (36) and (37) imply the result.

Closed form solution in linear case (cf Section 6). Substituting $F(x) = \frac{1}{2} + x$ in (25), we get

$$\mathbf{v}_{1i}$$
 $\mathbf{v}_{0i} = \frac{1}{2} \mathbf{x}_{i}^{2} \frac{1}{2} + \mathbf{x}_{j}^{2} = (\mathbf{x}_{i} + \mathbf{x}_{j})(\mathbf{x}_{i} - \mathbf{x}_{j} - 1)$ (38)

Substituting (38) for v_{1i} $~v_{0i}$ and F (x) = $\frac{1}{2}$ + x in (23), we get

$$p_{1i} = \frac{1}{2} \quad x_i \qquad (x_i + x_j) (x_i \quad x_j \quad 1)$$

$$p_{0i} = \frac{1}{2} + x_j \qquad (x_i + x_j) (x_i \quad x_j \quad 1)$$

which implies

$$p_{1A} \quad p_{0B} = \frac{1}{2} \quad x_A \quad (x_A + x_B) (x_A \quad x_B \quad 1) \qquad \frac{1}{2} + x_A \quad (x_A + x_B) (x_B \quad x_A \quad 1) \\ = 2 x_A \quad 2 \quad (x_A + x_B) (x_A \quad x_B) \\ p_{1B} \quad p_{0A} = \frac{1}{2} \quad x_B \quad (x_A + x_B) (x_B \quad x_A \quad 1) \qquad \frac{1}{2} + x_A \quad (x_A + x_B) (x_A \quad x_B \quad 1) \\ = 2 x_B \quad 2 \quad (x_A + x_B) (x_B \quad x_A)$$

Plugging this back into (16), we get

From this we get

$$x_A \quad x_B = 2(x_A \quad x_B) \quad 4(x_A + x_B)(x_A \quad x_B) \quad 2d_A + x_B = 2(x_A + x_B) \quad 2s_A$$

Solving the system with respect to $x_A\,+\,x_B\,$ and $x_A\,-\,x_B$, we get

$$\begin{array}{rcl} \mathbf{x}_{A} \,+\, \mathbf{x}_{B} \,=& \displaystyle \frac{2}{3}\,\mathbf{s} \\ \mathbf{x}_{A} \,& \mathbf{x}_{B} \,=& \displaystyle 2 \, 3 \, \frac{8}{3}\,\,\mathbf{s} \, \frac{1}{d} \\ \mathbf{x}_{A} \,=& \displaystyle \frac{1}{3}\,\mathbf{s} \, 3 \, \frac{8}{3}\,\,\mathbf{s} \, \frac{1}{d} \\ \mathbf{x}_{B} \,=& \displaystyle \frac{1}{3}\,\mathbf{s} \,+\, 3 \, \frac{8}{3}\,\,\mathbf{s} \, \frac{1}{d} \end{array}$$

Substituting these equations for \boldsymbol{x}_i in the above price equations and simplifying we get the expressions in the text.

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