Contracting, Exclusivity and the Formation of Supply Networks with Downstream Competition

Paolo Ramezzana

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Abstract

This paper studies the endogenous formation of supply networks in bilateral oligopoly by analyzing a model of contracting with transfers in which each firm optimizes its entire set of contracts across multiple bilateral negotiations. Because of downstream competition, industry profits are not necessarily maximized when all supply links are active and the supply networks that constitute coalition-proof Nash equilibria of the contracting game may differ from those that maximize industry profits. I first demonstrate that, in the absence of public commitment, all marginal input prices in any self-enforcing supply network are equal to the marginal cost of production. I then explore how a number of factors – such as supplier and retailer differentiation and the availability of exclusive contracts – affect the structure of equilibrium supply networks, profits and welfare. I also explore how the analysis changes if firms cannot use transfers or long-term contracts at the network-formation stage and must instead engage in ex-post bilateral bargaining with the associated hold-up problems.

Keywords: Bilateral oligopoly, contracting with transfers, exclusive contracts, supply network formation.

JEL Classification: D43, D85, L13, L14.

U.S. Federal Trade Commission, 600 Pennsylvania Avenue N.W., Washington, DC 20580; pramezzana@ftc.com. I would like to thank Jonathan Baker, Bhaskar Dutta, Dan O'Brien, David Schmidt, Marius Schwartz, Michael Vita and Alison Weingarden for helpful comments on this or previous versions. Any opinions expressed in this paper are mine alone and do not represent those of the Federal Trade Commission or its Commissioners.

1 Introduction

In many industries a number of differentiated but competing upstream firms ("suppliers") distribute their products through a number of differentiated but competing downstream firms ("retailers"). In some of these industries most retailers distribute the products of most suppliers, i.e., most potential supply links are active. In other industries, instead, each retailer distributes the products of a different supplier and some important potential supply links remain inactive. Examples of the latter include the exclusive distribution of many sports events and films in the pay TV industry – see, e.g., OECD (2013) and Weeds (2016) – and, until a few years ago, of the iPhone and other models of smartphones in the wireless telecommunication industry – see, e.g., Sinkinson (2014). Moreover, in some industries the structure of supply networks appears to be changing over time. For example, the wireless telecommunication industry has recently moved away from smartphone exclusivity, whereas the healthcare industry appears to be moving in the opposite direction, with an increasing number of health insurance companies offering networks and the healthcare market (e.g., Gowrisankaran, Nevo and Town, 2015). This literature relies on a "Nash-in-Nash" approach, in which each pair of firms engages in Nash bargaining, taking as given the agreements reached by all other pairs (see Collard-Wexler, Gowrisankaran and Lee (2017) for a thorough discussion of, and theoretical foundations for, this approach). Although this approach can accommodate downstream competition, it relies on a number of fairly strong assumptions about contracting. In particular, it assumes that, when negotiating their contracts, firms take all other contracts (including other contracts to which they themselves are a party) as given, thus not making use of all the information at their disposal, and that there are gains from trade associated with every potential supply link. These assumptions make it possible to derive, and take to the data, precise and tractable implications regarding the division of surplus over given supply networks. They are, however, less well-suited to studying how firms may affect downstream competition by implementing networks in which some supply links may remain inactive.[1] For example, this approach does not account adequately for the fact that a pair of firms may not find it profitable to rescind a supply link if all other links remain active, but may find it profitable to do so if other supply links are also rescinded. Another limitation of this literature is that it typically constrains payments from retailers to suppliers to be either lump-sum (e.g., Gowrisankaran et al., 2015; Collard-Wexler et al., 2017) or linear without any lump-sum component (e.g., Horn and Wolinsky, 1988; Crawford and Yurukoglu, 2012).

In this paper I advance this literature by combining insights from the literature on network formation with transfers (e.g., Bloch and Jackson, 2007; Jackson, 2008; Bloch and Dutta, 2011) with insights from the literature on vertical contracting with a single supplier (e.g., O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004) or a single retailer (e.g., O'Brien and Shaffer, 1997; Bernheim and Whinston, 1998). Relative to the "Nash-in-Nash" literature discussed above, this approach provides less precise predictions regarding the division of surplus between upstream and downstream firms, but can shed some light on other important aspects, such as the structure of vertical contracts, the importance of multilateral deviations, the endogenous emergence of narrow supply networks, and the effects of the latter on the intensity of downstream competition.

Overview – The starting point of the analysis is an exploration of how the structure of sup-

[1] Liebman (2016) and Ho and Lee (2017) study situations in which health insurance companies commit ex-ante to exclude one or more health care providers from their networks to increase their bargaining leverage. However, they either do not allow for downstream competition (Ho and Lee, 2017) or do not clearly discuss the implications of exclusivity for such competition (Liebman, 2016). Other important differences, discussed in further detail in Section 6.2, are that my framework allows for both upstream and downstream exclusivity and does not allow for ex-ante commitment to exclude.

ply networks affects industry profits.[2] Networks in which a relatively large number of potential distribution channels remain inactive, because some retailers are excluded or because active retailers distribute different products, have two opposite effects on industry profits. On the one hand, when suppliers and retailers are differentiated, the absence of some product-retailer combinations reduces the demand or willingness to pay expressed by some consumers, thus lowering total industry revenues. On the other hand, it softens downstream competition by reducing the number or increasing the effective differentiation of active retailers. The relative importance of these two effects determines which type of network maximizes industry profits. For example, when retailers are close substitutes, softening downstream competition is more important the product from the supplier, ii) exclusivity clauses (if any), and iii) an upfront transfer to be paid by one party to the other at the time the contract is signed. A supplier and a retailer enter a contract only if their proposals regarding all three of these elements are consistent. Once all contracting is concluded, retailers with at least one supply contract compete in prices or quantities in the downstream market.

There are three main differences between my model and that of Bloch and Jackson (2007). First, in Bloch and Jackson's model, and in much of the literature on network formation, the only relevant choice for a pair of players is whether to form or sever a link. In the vertical contracting setting of this paper, instead, each supplier-retailer pair must also specify a wholesale price (see i) above), which affects the price or quantity chosen by the retailer and, through this channel, the payoffs of the network formation game. I show that, in any equilibrium with secret contracts all wholesale prices are equal to marginal cost. Second, the application of the model to a bilateral oligopoly setting implies some inescapable restrictions on payoffs, arising mainly from supplier and retailer substitutability, that make it impossible to rely on some of the assumptions (e.g., nonnegative externalities and link-separability of payoffs) used by Bloch and Jackson to derive some of their results. Finally, as discussed below, in order to deal with the pervasive horizontal externalities in my model, I use Bernheim, Peleg and Whinston's (1987) coalition-proof Nash equilibrium (CPNE) as a solution concept. CPNE allows for multilateral deviations and is a stronger equilibrium concept than the pairwise Nash equilibrium used by Bloch and Jackson (2007), in which firms can only add one new link at a time, or the contract equilibrium proposed by Crémer and Riordan (1987). I solve the model described above by first deriving some general results and then applying them to a bilateral duopoly with linear demand. This allows me to and make it possible to support equilibria with narrow networks when these maximize industry profits, although pure-strategy equilibria do not always exist. When exclusive contracts affect the equilibrium structure of supply networks, they always reduce variety and the intensity of downstream competition, resulting in lower consumer and overall welfare. Moreover, the availability of exclusive contracts, by affecting the disagreement payoffs of suppliers and retailers differently, redistributes profits from retailers to suppliers, even when such contracts are not adopted in equilibrium.

Finally, how do constraints on the firms' ability to use upfront transfers or long-term contracts and to create instantaneously new supply links at the network formation stage affect equilibrium outcomes? In the baseline model discussed above there are no constraints on such ability. As a result, the division of the profits generated by new supply links takes place at the same time that the network is formed and is not affected by hold-up problems. Instead, if firms must form a complete supply network, for example by making specific investments, *before* starting to negotiate supply contracts and agreeing on any transfers, the division of profits resulting from ex-post bargaining is affected by hold up, as in Lee and Fong (2013) and Rey and Vergé (2016) (discussed

I assume that firms cannot publicly commit to their contract proposals and show that all CPNE wholesale prices are equal to marginal cost. Similar results have already been established by the existing literature in different settings (e.g., a single supplier or a single retailer) and/or for different equilibrium concepts (e.g., PBE or contract equilibrium). Here I extend those findings to the CPNE of a bilateral oligopoly. I also show that, by requiring multilateral deviations to be self-enforcing, CPNE avoids the issues identified by Rey and Vergé (2004) for the existence of perfect Bayesian equilibrium wholesale prices with Bertrand downstream competition.

Other related literature – This paper is also related to Rey and Vergé (2016) and Lee and Fong (2013). As in this paper, Rey and Vergé (2016) allow firms to engage in multiple bilateral contract negotiations. However, contrary to this paper, they do not allow for the use of transfers or exclusive contracts at the stage in which supply relations are initially formed, limiting their attention to the case in which the surplus from any relation can only be divided ex-post under conditions of hold up. Lee and Fong (2013) present a model of dynamic network formation in which, at any point in time, firms can only bargain over the existing network and, like in Rey and Vergé (2016), cannot create new supply links. However, they allow firms to create new supply links in subsequent periods. When the cost from doing so is low and the time between periods is short, Lee and Fong's (2013) environment approaches the environment without hold up that I study in this paper. Notwithstanding this similarity, their model and mine are quite different. Lee and Fong (2013) emphasize intrinsically dynamic aspects, such as the response of networks to shocks in the presence of adjustment costs, and simplify other aspects by, e.g., assuming that firms can only use lump-sum transfers and that exclusive contracts are not available. Instead, I adopt a static model of simultaneous contracting and emphasize the role played by the structure of vertical contracts and the degree of supplier and retailer differentiation. By relying on coalition-proof Nash equilibrium, I also propose a more systematic refinement of the set of equilibria than Lee and Fong (2013). Finally, I provide an in-depth analysis of the role played by exclusive contracts, which are not addressed in Rey and Vergé (2016) and Lee and Fong (2013), and are more naturally studied in an environment like mine, in which firms can use upfront payments to purchase exclusivity.

Organization of the paper – Section 2 introduces a formal model with upfront transfers. Section 3 presents the solution method and some general results. Sections 4 through 6 study supply networks with and without exclusive contracts in a bilateral duopoly model with linear demand. Section 7 explores the effects of ex-post bargaining and hold up. Section 8 concludes. All proofs are in Appendix A. Supplemental material is contained in Appendix B (available online).[4]

[4]This online appendix will be available shortly at `https://sites.google.com/site/paoloramezzana/`.

2 Model and equilibrium concept

This section first introduces a model of contracting and competition in bilateral oligopoly and then discusses why Bernheim, Peleg and Whinston's (1987) coalition-proof Nash equilibrium (CPNE) is an appropriate solution concept for this model.

2.1 Model

There are S 2 suppliers, each producing a different product at constant marginal cost c. The products are imperfect substitutes and are distributed to consumers by R 2 differentiated and competing retailers at no additional costs besides their payments to suppliers (introduced further below). With a slight abuse of notation, S and R

$\ell = \langle \ell_{sr} \rangle_{s \in S, r \in R}$ of all supply links, or lack thereof, gives rise to a supply network $g = g(\ell)$.

When bilateral contracting is concluded, retailers observe the resulting supply network g, but not the wholesale prices in the contracts signed by other retailers, and compete in the downstream market in stage 2. Since the general principles of the analysis that follows apply equally well to Cournot or Bertrand downstream competition, I allow for either mode of competition. I assume that for any supply network g and profile of wholesale prices $w \in \mathbb{R}^{S \times R}$, downstream competition results in a unique equilibrium profile of retail prices $p(g,w)$ and quantities $q(g,w)$. Given a profile of contracts $x = \langle t, w, q \rangle$, and the resulting supply network g, the payoffs of supplier s and retailer r are therefore, respectively,

$$\pi_s(g,t,w) = \sum_{r \in R} \ell_{sr}\left[t_{sr} + (w_{sr} - c)\, q_{sr}(g,w)\right], \tag{1}$$

$$\pi_r(g,t,w) = \sum_{s \in S} \ell_{sr}\left[(p_{sr} - w_{sr})\, q_{sr}(g,w) - t_{sr}\right]. \tag{2}$$

Throughout the paper it will also be helpful to keep track of the total vertical profits (gross of any payment to suppliers) generated by retailer r, which are given by

$$\Pi_r(g,w) = \sum_{s \in S} \ell_{sr}(g)\left[p_{sr}(g,w) - c\right] q_{sr}(g,w). \tag{3}$$

For future reference it is important to note that, because of downstream competition, total industry profits, $\sum_{r \in R} \Pi_r(g,w)$, are not necessarily maximized when all suppliers trade with all retailers. Networks in which some supply links are not active, such as downstream monopoly or pairwise exclusivity, sacrifice some variety but reduce the intensity of downstream competition between retailers and may thus yield higher overall industry profits.

In order to derive some of the results in subsequent sections, it is also helpful to ensure that secret changes in

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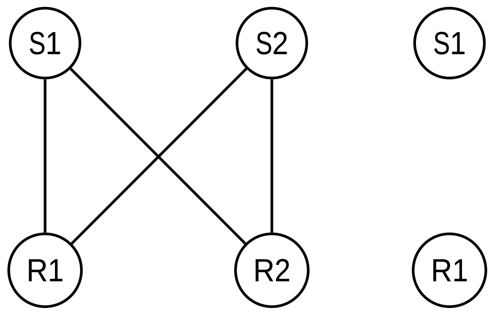
changes in quantities occur and thus $dq_{sj}/dw_{sr} = 0$ for all $j \neq r$. The only “bite” of Assumption 1 in this case is therefore to ensure that $dq_{sr}/dw_{sr} < 0$ and thus $dq_s/dw_{sr} < 0$. When competition is Bertrand, the changes in p_r induced by changes in w_{sr} cause instead the quantities sold by other retailers to change, even if the retail prices charged by those other retailers do not change, so that $dq_{sj}/dw_{sr} > 0$ for all $j \neq r$. In this case Assumption 1 ensures that the direct effect, $dq_{sr}/dw_{sr} < 0$, dominates the indirect effects, $\sum_{j \neq r} dq_{sj}/dw_{sr} > 0$, of a change in w_{sr} on the demand for product s and thus that the overall market demand for product



(a) Double common agency

find it profitable to sign up $S1$ to exclusivity if he can also sign up $S2$ to exclusivity, thus implementing a downstream monopoly as in Figure 1(d), but may not find it profitable to do so if he must continue in its nonexclusive contract with $S2$ (as mandated by contract equilibrium), thus but rather whether there exist deviations to contracts that implement the same supply network with different wholesale prices.

Proposition 1 (Equilibrium wholesale prices) *When firms cannot publicly commit to their contract proposals, for any supply network g there always exists a unique coalition-proof Nash equilibrium profile of wholesale prices $w(g)$, with $w(g) = c$, regardless of the mode of downstream competition.*

Proposition 1 extends results obtained in different settings and for different equilibrium concepts by the vertical contracting literature (briefly discussed below) to the CPNE of the bilateral oligopoly model studied in this paper. Its logic can be understood in two steps. First, since any jointly profitable bilateral deviation by s and r to a different w_{sr} is self-enforcing, a CPNE must be immune to any such deviation and is therefore also a contract equilibrium. By extending O'Brien and Shaffer's (1992) analysis of contract equilibria with a single supplier to a setting with multiple suppliers, one can therefore prove that in any CPNE it must be $w_{sr} = c$ for all s and r. If this were not the case, any supplier-retailer pair for which $w_{sr} > c$ could profitably engage in a bilateral deviation to $w_{sr} = c$, which would allow this pair to appropriate some of the retail margins of other retailers. Second, the above implies that any multilateral deviation that changed the wholesale prices of two or more retailers away from $w = c$ at the same time would never be self enforcing, since it would always be blocked by a further self-enforcing bilateral deviation to marginal cost pricing, regardless of the mode of downstream competition.

This last aspect is what distinguishes Proposition 1 from existing literature, especially for the case of Bertrand downstream competition. As shown by Rey and Vergé (2004), when retailers compete à la Bertrand, are sufficiently close substitutes and hold passive beliefs, multilateral deviations in which suppliers raise the wholesale prices of two or more retailers above marginal cost at the same time may become profitable. If one adopts an equilibrium concept such as perfect Bayesian equilibrium, which allows for such multilateral deviations without requiring them to be self enforcing, this implies that pure-strategy equilibria may fail to exist, because there exist profitable deviations both when all wholesale prices are equal to marginal cost and when some of them are not. The vertical contracting literature typically addresses these equilibrium existence issues by limiting attention to contract equilibrium concepts that allow only for bilateral deviations (see, e.g., O'Brien and Shaffer (1992) and McAfee and Schwartz (1994) for models without upstream competition and Rey and Vergé (2016) for a bilateral oligopoly model with upstream competition) or to Cournot competition (see, e.g., Nocke and Rey (2016) for a bilateral oligopoly model with upstream competition).[8] The CPNE solution concept adopted in this paper

[8]The existence issue does not arise with Cournot downstream competition because in that case multilateral deviations do not present any advantage relative to bilateral deviations.

does not face this stark choice, as it overcomes the existence problems associated with Bertrand competition while allowing for multilateral deviations.

3.2 Equilibrium supply networks

Having shown that, when firms cannot commit to their wholesale price proposals, equilibrium wholesale prices are equal to marginal cost in all possible networks, the next step is to determine which of these networks can be supported as equilibria by some profile of transfers. As will become clear below, when a network g can in fact be supported as an equilibrium there typically exists a (possibly broad) range of transfers t^g for which this is the case. The model is, therefore, better suited to shedding light on what types of supply networks arise as equilibria under different conditions, rather than to predicting exactly how profits will be split between firms in those equilibria. As the division of profits resulting from t^g is irrelevant for consumer and overall welfare, because it does not directly affect product or retailer variety or the intensity of downstream competition, I do not view this as a significant shortcoming of the model.[9] Moreover, as shown in Section 6.2, certain changes in the environment, such as the availability of exclusive contracts, cause the range of suppliers or retailers' equilibrium profits to shift entirely to the right or the left of their initial range. When this is the case, the approach taken in this model is sufficient to determine unambiguously the distributional effects of those changes.

In order to predict the exact level of t^g one would have to study a different model in which suppliers and retailers engage in some form of coalitional bargaining at the contracting stage.[10] However, most existing models of coalitional bargaining – see, e.g., Shapley (1953) for a cooperative model and Chatterjee et al. (1993) for a noncooperative model – assume that the grand coalition always forms and that there are no externalities between coalitions; or, when they allow for externalities between coalitions (e.g., Ray and Vohra, 1999), they rely to a large extent on symmetry among all players to obtain tractable results.[11] These assumptions do not describe well the environment studied in this paper, in which externalities resulting from downstream competition and exclusive contracts, as well as asymmetries between suppliers and retailers,

[9] It should, however, be noted that the division of profits among firms can have welfare effects if it affects the investment incentives of firms, an aspect that is not modeled in this paper.

[10] Such a model would be different, and have different implications, from the model studied in Section 7, in which firms first form a supply network in the absence of upfront transfers and then engage in ex-post bilateral bargaining over the existing supply network. Whereas hold-up plays no role in the coalitional bargaining game with upfront transfers described in the text above, it plays a crucial role in the model of ex-post bilateral bargaining of Section 7.

[11] For a discussion of these issues see Bloch and Dutta (2011) and Maskin (2003). In particular, Maskin argues that the assumption that the grand coalition always forms is one of the main shortcomings of noncooperative game theory.

play a crucial role. In light of this, developing a full-fledged coalitional bargaining game that is well-suited to this complex environment, only for the purpose of obtaining precise predictions regarding transfers, would be well beyond the scope of this paper and is not attempted here.[12]

Having clarified the scope of the analysis, I can proceed with a characterization of equilibrium supply networks. I first introduce formal definitions of self-enforcing agreements and of CPNE supply networks in the specific model with transfers of this paper, and then derive some helpful results that allow me to make these definitions operational.

Denote by $Z_{g \to h}$ the set of all coalitions that, starting from a network g, can implement a network $h \neq g$ without requiring the consent of firms outside the coalition (i.e., without needing firms outside the coalition to modify their strategies). One can then define a self-enforcing nonbinding agreement as follows.

Definition 1 (Self-enforcing agreement) *A nonbinding agreement among the members of coalition $Z \subseteq S \cup R$ to implement a supply network g with transfers t^g is self-enforcing if there does not exist any other self-enforcing agreement among the members of any subcoalition $Z' \subseteq Z$, $Z' \in Z_{g \to h}$, that implements a supply network $h \neq g$ with transfers t^h such that, for all $i \in Z'$,*

$$\pi_i(h, t^h) > \pi_i(g, t^g) \tag{4}$$

A few aspects of Definition 1 are worth noting. First, the definition is recursive: an agreement is self-enforcing if and only if it cannot be improved upon by another self-enforcing agreement. The problem remains, however, well defined because deviations from agreements are restricted to subcoalitions Z' of the coalition Z that reaches the original agreement. This limits the number of successive deviations that one needs to consider and a solution can be reached in a finite number of steps. In the bilateral duopoly applications discussed in Sections 5 through 7 this number of steps is generally small and the problem remains tractable.

Second, the ability of firms to implement a new supply network h starting from a supply network g, and the size and composition of the coalitions that can do so, captured by $Z_{g \to h}$, depend on a number of factors, such as the extent to which firms are allowed to communicate with one another and whether exclusive contracts are allowed or not. For example, it is generally

[12] One can, however, conjecture that an efficient coalitional bargaining game in which coalitions always reach agreements that are mutually profitable relative to alternative *self-enforcing* outcomes would yield the same equilibrium networks as the simultaneous contracting game that I study in this paper. Specifically, this should be the case for bargaining games in which firms i) can continue to make proposals to each other until all possibilities have been explored and ii) cannot credibly commit ex-ante to deal with a limited number of

more difficult for firms to implement a new supply network when nonbinding communication is partially restricted (e.g., when communication between firms on the same side of the market is prohibited). As for exclusive contracts, their adoption makes it mode difficult to deviate from a given network g

Lemma 1 is used below to construct an algorithm to solve for the CPNE of the model. Before doing so, however, it may be helpful to discuss a few intuitive aspects that play a role in that solution. The left-hand side of (7) represents the incremental gross vertical pro ts generated by the retailers that participate in a deviation from network g to network h. These incremental pro ts can be positive or negative. The right-hand side represents, instead, the net loss of transfers from that can implement this deviation).

The second type of deviations of interest is one in which the members of Z rescind their links with some firms that do not participate in the deviation, i.e., in which $\ell_{sr}^{g} = 1$ and $\ell_{sr}^{h} = 0$ for some $s \in Z$ and $r \notin Z$ or some $s \notin Z$ and $r \in Z$. This type of deviations is mutually profitable for the members of Z if, given an initial profile of transfers t^{g}, the firms dropped from network g

g is a CPNE. If the answer is no, the original deviation to h is self-enforcing and g is not a CPNE.

The solution algorithm introduced above is well defined for any arbitrary numbers of suppliers, S, and retailers, R. In particular, since successive deviations are limited to subcoalitions, it always converges to an end point. However, using it to solve a model with more than a few firms on each side of the market would be unwieldy because the numbers of possible supply networks and coalitions grow exponentially with S and R. Related to this, large values of S and R would also give rise to an unmanageably large number of deviations from deviations that would need to be checked, i.e. of iterations of Step 2 above. For this reason, in the rest or the paper I restrict attention to a more tractable bilateral duopoly model with two symmetrically differentiated suppliers and two symmetrically differentiated retailers that satisfies the assumptions laid out in Section 2.

4 A bilateral duopoly model

The possible types of supply networks that can arise in bilateral duopoly are listed below, together with the maximum vertical profits P_r^g that each retailer r can generate in supply network g when all retailers obtain products at wholesale prices $w(g) = c$ and compete in the downstream market.

Double common agency (denoted by $g = dca$ and illustrated in Figure 1(a)). Both retailers deal with both suppliers. Given the symmetry of the model, each retailer generates the same vertical profits $\mathrm{P}_r^{dca} = \mathrm{P}^{dca}$. *DMixd*

Upstream monopoly ($g = um$, Figure 1(e)). Both retailers only deal with the same supplier, excluding the other supplier. Each retailer generates the same vertical profits $\Pi_r^{um} = \Pi^{um}$.

Bilateral monopoly ($g = bm$, Figure 1(f)): A retailer and a supplier only deal with each other, while the other retailer and supplier are excluded. The active retailer generates vertical profits Π^{bm}, whereas the excluded retailer generates zero vertical profits.

Although one could in principle use a fairly general demand system to rank unambiguously the vertical profits generated by retailers under some of the supply networks listed above, this is not the case for all of these networks.[15] Moreover, even if one could obtain an ordinal ranking of vertical profits for all supply networks, this would be insufficient because the analysis in subsequent sections involves linear functions of these vertical profits and requires these profits to be cardinally comparable. In the interest of concreteness, therefore, I use the following (inverse) linear demand system, which allows me to parametrize the degrees of supplier and retailer differentiation

$$p_{sr} = v - (q_{sr} + a q_{s'r}) - b\left(q_{sr'} + a q_{s'r'}\right), \tag{8}$$

where $v > c$ and $a, b \in [0, 1]$.[16] Lower values of a and b indicate, respectively, higher supplier and retailer differentiation, with $a = 0$ and $b = 0$ corresponding to the extreme case in which, respectively, suppliers or retailers are completely independent in demand and $a = 1$ and $b = 1$ to the extreme case in which they are perfect substitutes. Note that supplier differentiation and retailer differentiation are "cumulative", in that the coefficient ab on $q_{s'r'}$ in (8))t 2.4-227(.Note)-281(that)-282(s

This framework is used in Figure 2 to characterize the supply networks that maximize total industry profits, conditional on retailers competing in the downstream market. Comparisons between these networks and the networks that arise in equilibrium will prove particularly helpful in the sections that follow, as they will provide insights into the nature and magnitude of the externalities that prevent equilibrium networks from maximizing industry profits. Figure 2 divides

(a) Bertrand competition (b) Cournot competition

Figure 2: Supply networks that maximize industry profits.

the unit square representing all possible combinations of supplier substitutability, a, and retailer substitutability, b, into three regions and shows that industry profits are maximized by a different supply network in each of these regions. Specifically, denoting with $\overline{b}_m(a)$ the value of b for which $2P^{pe} = 2P^{dca}$ (i.e., for which industry profits are the same under pairwise exclusivity and double common agency) and with $\overline{\overline{b}}_m(a)$ the value of b for which $P^{dm} = 2P^{pe}$ (i.e., for which industry profits are the same under downstream monopoly and pairwise exclusivity), the figure shows that industry profits are maximized by double common agency for b $\overline{b}_m(a)$, pairwise exclusivity for $\overline{b}_m(a)$ b $\overline{\overline{b}}_m(a)$, and downstream monopoly for b $\overline{\overline{b}}_m(a)$. Industry profits under other supply networks, such as a mixed network or upstream monopoly, are always dominated by industry profits under one or more of the three networks shown in the figure.

The intuition for Figure 2 can be explained by noting that the elimination of supply links has two opposite effects on industry profits. On the one hand, it reduces the intensity of downstream competition. Specifically, a move from double common agency to pairwise exclusivity increases the degree of effective retailer differentiation, since the two retailers go from both carrying the same products to each carrying a different product, whereas a move from pairwise exclusivity to downstream monopoly completely eliminates any residual downstream competition. On the other hand, when both suppliers and retailers are differentiated, eliminating supply links causes loss of variety. As b increases, more intense downstream competition causes greater dissipation of industry profits and thus the former effect (softening of downstream competition) becomes progressively more important than the latter effect (loss of variety). Therefore, as b increases, industry profits are maximized by networks with less downstream competition and less variety.

5 Equilibria without exclusive contracts

Having characterized the supply networks that maximize industry profits, one can study the extent to which bilateral contracting between suppliers and retailers can implement these networks under different assumptions about upfront transfers and exclusive contracts. In this section, I start by considering the case in which firms can use upfront transfers but exclusive contracts are banned or not enforceable.

Proposition 2 (No exclusive contracts) *Consider a bilateral duopoly model where firms cannot publicly commit to contract proposals and face the symmetric linear demand system in (8). When firms can use transfers but not exclusive contracts there exists a unique coalition-proof Nash equilibrium supply network $g_{t,ne}$ with*

1. $g_{t,ne} = pe$ *and transfers* t_{ne}^{pe} 2 $(0, P^{pe} \quad P^{um})$ *if and only if*

$$P^{pe} \quad P^{mix2}. \tag{9}$$

2. $g_{t,ne} = dca$ *and* t_{ne}^{dca} 2 $\quad 0, P^{dca} \quad P^{mix1}$ *otherwise.*

With differentiated Bertrand competition (9) holds if and only if b $\bar{}$

(a) Bertrand competition (b) Cournot competition

Figure 3: Equilibria with transfers and no exclusive contracts.

result, in equilibrium firms tend to form "too many" supply links from the point of view of industry profit maximization.

Specifically, as illustrated in Figure 3, downstream monopoly can never be supported as an equilibrium, even though it maximizes industry profits for $b > \bar{\bar{b}}_m(a)$, because, without exclusive contracts, a supplier can accept an offer from the excluded retailer, r', without having to forego the profits he earns from the other retailer, r, and can thus always be induced to do so by r'. Analogously, pairwise exclusivity cannot be supported as an equilibrium in large parts (with Bertrand competition) or the totality (with Cournot competition) of the region with $b \in \left[\bar{b}_m(a), \bar{\bar{b}}_m(a)\right]$ in which it maximizes industry profits. The relevant deviation from a candidate equilibrium with pairwise exclusivity is one in which a supplier, say s, opens a new link with a second retailer, say r', thus giving rise to a mixed network in which r', who now carries two products, generates vertical profits Π^{mix2}. This deviations is mutually profitable and self enforcing whenever $\Pi^{mix2} > \Pi^{pe}$, which is the case for $b < \bar{b}_{t,ne}(a)$ with Bertrand competition and always the case with Cournot competition. The difference between Bertrand and Cournot competition is explained by the fact that, for any given degree of intrinsic retailer substitutability b, the former is generally more competitive than the latter. The softening of downstream com-

The results in Proposition 2 have been obtained under the assumption that firms can engage in *any* type of nonbinding pre-play communication, including nonbinding (but possibly self enforcing) reciprocal agreements between firms on the same side of the market not to deal with their competitor's suppliers or retailers. Since these agreements effectively amount to market allocation agreements, which are illegal in most jurisdictions even when nonbinding, one may wonder how the results would change if one prohibited deviations relying on such agreements. The following remark, which maintains the same assumptions as Proposition 2 except for the types of nonbinding agreements that are allowed, addresses this question.

Remark 1 (No exclusive contracts, no market allocation agreements) *When firms cannot discuss (nonbinding) market allocation agreements, in addition to the equilibria in Proposition 2 there also exist equilibria with $g_{t,ne} = dca$ when condition (9) holds. This gives rise to multiple equilibria for Bertrand competition and b $\bar{b}_{t,ne}(a)$ in the shaded area in Figure 3(a).*

As one would expect, limiting the type of nonbinding communication that can take place expands the set of equilibria by reducing the ability of firms to coordinate deviations. In the specific case of the bilateral duopoly model of this section, when firms can engage in any type of communication, as was the case in Proposition 2, and condition (9) holds, the two retailers can reach a mutually profitable and self-enforcing nonbinding agreement to deviate from a candidate equilibrium with $g = dca$ to a network $g = pe$ in which they allocate the market by each refusing to sell a different product. This is, however, no longer possible under the restriction in Remark 1.

6 Equilibria with exclusive contracts

In this section I study the implications of allowing firms to use exclusive contracts. In principle one could adopt a general framework in which a contract between firm i and firm j can be made contingent on all other supply links in the market, i.e., on all the links that firm i has with other firms $h \neq j$, all the links that firm j has with other firms $k \neq i$, and all the links that other firms $h, k \neq i, j$ have with one another. Such contracts are, however, rarely enforceable for practical and legal reasons. Therefore, I focus on the case in which the only contingencies allowed in a contract between i and j are those requiring i and / or j to be exclusive to the other.[17]

[17] I also assume that the tariff in a nonexclusive contract between a firm i and a firm j cannot be made contingent on the volume traded by either firm with third parties $k \neq i, j$, as would instead be the case with market-share discounts or retail price parity agreements. These restraints would not make a difference in

be supported by different combinations of exclusive clauses (including, possibly, no exclusive clauses at all). Since a link between two firms can remain inactive also in the absence of contractual exclusive clauses, provided that at least one of the two firms refuses to trade with the other, exclusive clauses are not necessary to obtain any give supply network g. For example, the

6.1 Effects of exclusive contracts on equilibrium supply networks

Under the assumptions introduced above one can establish the following result.

Proposition 3 (Exclusive contracts) *In a bilateral duopoly model with the same assumptions as in Proposition 2, but in which firms can use exclusive contracts, there exist pure-strategy coalition-proof Nash equilibria with the following supply networks and transfers:*

1. $g_{t,e} = dm$ *and* $t_e^{dm} = \Pi^{dm}/2$ *if and only if*

$$\Pi^{dm} \geq 2\Pi^{pe}, \tag{10}$$

which is the case if and only if $b \geq \bar{\bar{b}}_m(a)$ *(see Figure 4).*

2. $g_{t,e} = pe$ *and* $t_e^{pe} \in \left[\Pi^{dm} - \Pi^{pe}, \min\{2(\Pi^{pe} - \Pi^{um}), \Pi^{pe}\}\right]$ *if and only if (10) fails and*

$$2(\Pi^{pe} - \Pi^{um}) \geq \Pi^{dm} - \Pi^{pe}, \tag{11}$$

which is the case if and only if $\bar{\bar{b}}_{t,e}(a) \leq b < \bar{\bar{b}}_m(a)$ *(see Figure 4).*

3. $g_{t,e} = dca$ *and* $t_e^{dca} \in \left[\left(\Pi^{dm} - \Pi^{dca}\right)/2, \Pi^{dca} - \Pi^{mix1}\right]$ *if and only if*

$$2\left(\Pi^{dca} - \Pi^{mix1}\right) \geq \Pi^{dm} - \Pi^{dca}, \tag{12}$$

which is the case if and only if $b \leq \bar{b}_{t,e}(a)$ *(see Figure 4).*

There exist no other pure-strategy equilibria.

For those combinations of the supplier and retailer substitutability parameters, a and b, for which a pure-strategy equilibrium supply network exists, such a network is unique and maximizes industry profits.[21] The latter can be seen by noting that the lines $\bar{b}_m(a)$ and $\bar{\bar{b}}_m(a)$ in Figure 4 are the same as those used in Figure 2 to illustrate the supply networks that maximize industry profits for different values of a and b. The intuition for this result is as follows.

Starting from a candidate equilibrium with some inactive supply links (e.g., $g_{t,e} = pe$), consider a deviation in which two firms i and j want to form a new link. In the absence of exclusive contracts, they can do so without having to ask for permission from, or forego their relationship

combination of exclusive clauses. However, deviations that use the most restrictive combination of exclusive clauses are the most likely to be self enforcing, and therefore play a prominent role in the proofs of the results that follow.

[21] Indeed, as can be seen from the proof of Proposition 3, the CPNE of the model with exclusive contracts correspond to its strong equilibria. In a model with transferable utility like the present one, when strong equilibria exist, they must always maximize the sum of the players' payoffs (i.e., of the firms' profits).

(a) Bertrand competition

(b) Cournot competition

Figure 4: Equilibria with exclusive contracts and upfront transfers.

with, any other firm $k \neq i, j$. This is indeed the reason that, as shown in Section 5, equilibria with downstream monopoly or pairwise exclusivity might not be supportable without exclusive contracts even when they maximize industry profits. Things are, however, quite different if in the candidate equilibrium one or both of i and j have committed to be exclusive to other firms. For example, assume that firm i is part of the candidate equilibrium network and has committed to be exclusive to firm $k \neq j$. If i wishes to deviate by forming a supply link with j it must now either i) obtain consent from k, possibly in exchange for compensation, or ii) forego its relationship with k, effectively swapping j for k. Starting from equilibria with $g = dm$ and $g = pe$, i) and ii) make forming a link with j unprofitable for i whenever it is unprofitable for the supply network as a whole. Specifically, starting from $g = dm$ a supplier may wish to form a link with the excluded retailer, and starting from $g = pe$ any supplier or retailer may wish to form a second supply link. However, exclusive contracts, through the effects in i) and ii) discussed above, make this unprofitable when $g = dm$ and $g = pe$ maximize industry profits.

Besides supporting equilibria with some sort of exclusivity when these maximize industry profits, exclusive contracts also eliminate (pure-strategy) equilibria with nonexclusive networks, such as $g = dca$, when these networks do not maximize industry profits, as is the case for $\bar{b}_m(a) \leq b < \bar{b}_{t,ne}(a)$ in Figure 3, where exclusive contracts were not available. The reason for this is that exclusive contracts tend to make deviations that rescind some links, such as deviations to $g = dm$ or to $g = pe$, self-enforcing.

The latter is also the reason that pure-strategy equilibria may fail to exist for high values of a and intermediate values of b.[22] For these parameter values, the availability of exclusive contracts makes both deviations in which firms exclude one of the retailers (such as a deviation to $g = dm$) and deviations in which they exclude one of the suppliers (such as a deviation to $g = mix$) become self enforcing. As preventing the first type of deviation requires large transfers t^g from retailers to suppliers (to ensure that retailers do not extract to much), whereas preventing the second type of deviations requires small transfers t^g (to ensure that suppliers do not extract too much), there may exist no t^g that can prevent all deviations.

This is, instead, not a problem for low values of a

(or some other assumption with a similar function) is necessary for a realistic analysis of the long-term structure of supply networks.

6.2 Effects of exclusive contracts on welfare and firms' profits

The results obtained above have the following welfare implications.

Proposition 4 (Welfare) *Whenever exclusive contracts are adopted and affect the equilibrium structure of supply networks they reduce consumer and overall welfare.*

The result in Proposition 4 is straightforward (a proof relying on the utility function underlying the inverse linear demand in (8) is provided in the online appendix enclosed with this submission). Specifically, with Bertrand downstream competition, exclusive contracts cause the equilibrium network to switch from double common agency to pairwise exclusivity for $\overline{\overline{b}}_{t,e}(a) \quad b < \overline{b}_{t,ne}(a)$ and from pairwise exclusivity to downstream monopoly for $b \quad \overline{\overline{b}}_{m}(a)$. With Cournot downstream competition, instead, they cause the equilibrium network to switch from double common agency to pairwise exclusivity for $\overline{b}_{m}(a) \quad b < \overline{\overline{b}}_{m}(a)$ and from double common agency to downstream monopoly for $b \quad \overline{\overline{b}}_{m}(a)$. In all these cases they soften downstream competition, thus leading to higher prices, and reduce the variety of supplier retailer combinations available in the market. Both effects unambiguously reduce consumer and overall welfare.[23]

Less straightforward are, instead, the effects of the availability of exclusive contracts on the equilibrium profits earned by individual suppliers and individual retailers. Although, as explained in Section 3.2, the simultaneous contracting model used in this paper does not yield exact predictions regarding the equilibrium level of transfers, it nevertheless provides ranges within which such transfers, and thus the equilibrium profits of individual suppliers and retailers, must lie. If a change in the environment, such as the availability of exclusive contracts, causes these ranges to shift entirely to the right or the left of their initial position, the model adopted in this paper is sufficient to determine the distributional effects of that change. This is the approach used in deriving the following result.

Proposition 5 (Distribution of profits) *When exclusive contracts become available and are adopted in equilibrium, so that the resulting equilibrium supply network is $g_{t,e} = pe$ or $g_{t,e} = dm$, they make*

[23] As is well known, under certain conditions (e.g. when the seller and / or the buyer can make relationship-specific investments) exclusive contracts can enhance welfare by aligning investment incentives. In order to focus on the competitive effects of exclusive contracts, these potential efficiencies are not addressed in this paper.

suppliers strictly better off and retailers strictly worse off. When exclusive contracts become available but are not adopted in equilibrium, so that the equilibrium supply network remains $g_{t,e} = dca$, they make suppliers no worse off (and possibly strictly better off) and retailers no better off (and possibly strictly worse off).

The intuition for this results can be understood by noting that, besides possibly affecting the level of equilibrium industry profits, the availability of exclusive contracts may also affect the shares of any given amount of profits that suppliers and retailers are able to extract. Specifically, the upper bound on the profits that the firms on one side of the market can extract in a given equilibrium are determined by the extent to which the firms on the opposite side of the market can profitably drop them from their supply network *in deviations that are self enforcing.* In other words, the profits of the firms on one side of the market are determined by the credible disagreement payoffs available to the firms on the opposite side of the market. The availability of exclusive contracts makes a broader range of deviations self enforcing relative to an environment without exclusive contracts. For example, deviations to $g = um$, in which a supplier is excluded, or to $g = dm$, in which a retailer is excluded, can be self enforcing only if exclusive contracts are available. As shown in the proof of Proposition 3, whereas deviations to $g = um$, which limit the bargaining power of suppliers, are not sufficiently profitable to be self-enforcing (i.e., credible), deviations to $g = dm$, which limit the bargaining power of retailers, are generally sufficiently profitable to be self enforcing. Loosely speaking, this is the case because deviations to $g = um$ sacrifice product variety without softening downstream competition, whereas deviations to $g = dm$, though also costly in terms of variety, can increase profits by eliminating downstream competition. As a result, by making deviations to $g = dm$ self enforcing, the availability of exclusive contracts improves the credible disagreement payoffs of suppliers and shifts the balance of power in their favor.

Note that, when exclusive contracts are allowed, retailers have no way of preventing this outcome. Each individual retailer would have incentives to accept contracts that made her a downstream monopolist if she were offered such contracts, and (at least in this model) retailers cannot credibly commit not to accept such contracts. One would, however, expect that if retailers were given a say in public policy towards exclusive contracts before the contracting game is played, they would generally oppose the availability of such contracts.

The mechanism by which the availability of exclusive contracts can affect the distribution of profits in the present model is different from the mechanism discussed in O'Brien and Shaffer side of the market (typically downstream firms) can extract higher profits by (credibly) committing ex-ante to accept only a limited number of offers and honoring that commitment by actually excluding some firms in equilibrium. The mechanism in this paper does not rely on ex-ante com-some differences and additional contributions relative to Rey and Vergé (2016). Specifically, I assume that retailers observe the full set of active supply links (i.e., the prevailing supply network) before competing in the downstream market,[25] and also study the cases, not covered by Rey and Vergé, in which firms can adopt exclusive contracts and in which downstream competition is Cournot.

Given the gross vertical profits generated by downstream competition in stage 3, which remain the same as in Sections 4 through 6 (and are derived in the online appendix for the case of linear demand), one can solve for the contracts that result from bilateral negotiation in stage 2. As in previous sections, all wholesale prices are equal to marginal cost. The profits earned by a supplier s in supply network g correspond, therefore, to the sum of fixed fees, $\sum_{r\in R}\ell_{sr}F_{sr}^{g}$ charged by s in that network. For each network g in which $\ell_{sr}=1$ the fixed fee F_{sr}^{g} is determined according to the generalized Nash bargaining solution by solving the following equation for F_{sr}^{g}, with $gnsr$ denoting the supply network obtained from g when s and r do not form a supply link,

$$F_{sr}^{g}+\sum_{j\neq r}\ell_{sj}F_{sj}^{g}=\sum_{j\neq r}\ell_{sj}F_{sj}^{g}+\beta\left[\left(\sum_{j\neq r}\ell_{sj}F_{sj}^{g}+\Pi_{r}^{g}-\sum_{i\neq s}\ell_{ir}F_{ir}^{g}\right)-\left(\sum_{j\neq r}\ell_{sj}F_{sj}^{g}+\Pi_{r}^{gnsr}-\sum_{i\neq s}\ell_{ir}F_{ir}^{g}\right)\right],$$

which reduces to

$$F_{sr}^{g}=\beta\left(\Pi_{r}^{g}-\Pi_{r}^{gnsr}\right). \tag{13}$$

Using (13) one can calculate the following supplier (left column) and retailer (right column) profits for each possible supply network in the bilateral duopoly model introduced in Section 4.

$$\begin{aligned}
\pi_s^{dca} &= 2\beta\left(\Pi^{dca}-\Pi^{mix1}\right), & \pi_r^{dca} &= \Pi^{dca}-2\beta\left(\Pi^{dca}-\Pi^{mix1}\right),\\
\pi_s^{pe} &= \beta\Pi^{pe}, & \pi_r^{pe} &= (1-\beta)\,\Pi^{pe},\\
\pi_s^{dm} &= \beta\left(\Pi^{dm}-\Pi^{bm}\right), & \pi_r^{dm} &= \Pi^{dm}-2\beta\left(\Pi^{dm}-\Pi^{bm}\right),\\
\pi_s^{um} &= \beta\Pi^{um}, & \pi_r^{um} &= (1-\beta)\,\Pi^{um},\\
\pi_s^{bm} &= \beta\Pi^{bm}, & \pi_r^{bm} &= (1-\beta)\,\Pi^{bm},\\
\pi_s^{mix1} &= \beta\left(\Pi^{mix2}-\Pi^{um}\right), & \pi_r^{mix1} &= (1-\beta)\,\Pi^{mix1},\\
\pi_s^{mix2} &= \beta\left(\Pi^{mix1}+\Pi^{mix2}-\Pi^{pe}\right), & \pi_r^{mix2} &= \Pi^{mix2}-\beta\left(2\Pi^{mix2}-\Pi^{pe}-\Pi^{um}\right).
\end{aligned} \tag{14}$$

[25] Rey and Vergé (2016) assume that retailers do not observe the prevailing downstream market structure before setting retail prices and, following an unobserved deviation involving one of their rivals, behave as if the market structure was the one prescribed by the candidate equilibrium of the supply network formation game.

The profits in (14) constitute the payoffs of the link formation game played by firms in stage 1, which I turn to study next, first for the case in which commitment to exclusivity at the network formation stage is not possible and then the case in which it is.[26]

7.1 No ex-ante commitment to exclusivity

The main difference between the contracting game with upfront transfers studied in Section 5 and the game with ex-post bargaining studied in this subsection lies in the ability of firms to induce other firms to participate in deviations *in which new supply links are formed*. Specifically, when firms can use upfront transfers as in Section 5 and a coalition of firms would find it jointly profitable to enter into new supply contracts, there always exists a profile of upfront transfers that makes it individually profitable for each firm in the coalition to enter into those contracts. In the game with ex-post bargaining studied in this section, instead, the division of the joint profits resulting from the formation of a new supply link is determined by the firms' relative ex-post bargaining power and may not make each firm better off even if forming the new link is jointly profitable.

In the absence of exclusive contracts, upfront transfers do not play any role in facilitating deviations *in which firms rescind one or more supply links*. Without exclusive contracts, any agreement between a supplier s and a retailer r that, say, supplier s

formation stage. There always exists a unique coalition-proof Nash equilibrium supply network $g_{nt,ne}$ *with*

1. $g_{t,ne} = pe$ *if and only if*

$$2P^{pe} \quad P^{mix1} + P^{mix2}. \tag{15}$$

2. $g_{nt,ne} = dca$ *otherwise.*

Condition (15) holds for $b \quad \bar{b}_{nt,ne}(a)$, *where,* Ex-post bargaining does, instead, make a difference for the regions in which pairwise exclusivity and double common agency can be supported as equilibria. Specifically, with ex-post bargaining the region of parameters for which pairwise exclusivity arises in equilibrium expands, with the lower bound on b in Figure 5 shifting downward from $\bar{b}_{t,ne}(a)$ to $\bar{b}_{nt,ne}(a)$ (where $\bar{b}_{t,ne}(a) = 1$ for Cournot competition) and pairwise exclusivity replacing double common agency as the unique equilibrium outcome in the shaded regions.

The intuition for this is closely related to the discussion that precedes Proposition 6. Specifically, just as in the case with upfront transfers, the relevant self-enforcing deviation from a candidate equilibrium with pairwise exclusivity involves a supplier s starting to trade with a second retailer r^{0} (or vice versa), thus implementing a mixed network. Such a deviation yields incremental joint profits of $\mathrm{P}^{mix2} \quad \mathrm{P}^{pe}$ for s and r^{0}. With upfront transfers, s and r^{0} can find a way to profit individually from this deviation provided that $\mathrm{P}^{mix2} > \mathrm{P}^{pe}$ (see Proposition 2). With ex-post bargaining, instead, the formation of the new link between s and r^{0} also affects the network formation with externalities has also been stressed in a different context by Bloch and Jackson (2007).[28]

7.2 Ex-ante commitment to exclusivity

Assume now that, when firms announce which other firms they are willing to form a link with at the network formation stage, they can make that announcement contingent on the different type of exclusivity studied in Section 6. For example, a supplier s can announce that he is willing to form a link with a retailer r only if he obtains exclusivity from r

have shown that, when contracts are secret, all (coalition-proof) equilibria are characterized by marginal input prices equal to marginal cost. Moreover, when exclusive contracts are not available and retailers are sufficiently close substitutes, equilibrium supply networks tend to have more supply links and more intense downstream competition than the networks that maximize industry profits. Exclusive contracts make it easier to support equilibrium networks with fewer supply links, thus eliminating the divergence between equilibrium and industry-profit maximizing networks and harming welfare through lower variety and higher prices. Finally, if the division of profits must take place through ex-post bargaining, for example because firms must make specific investments before starting to negotiate, it is more difficult for firms to organize mutually profitable deviations to broader networks and is thus easier to support equilibria with fewer supply links relative to an environment with upfront transfers or long-term contracts.

The model presented in this paper has been developed for the main purpose of studying the determinants and welfare effects of different supply networks, not of precisely predicting the division of profits between suppliers and retailers. As a result, it only characterizes lower and upper (wooundih62)-20774heferentt muTsoac]TJ 0 -34 -19.559 Td [(pe)-37040ufficiently37041ofects mi18(onment)-250(of)n281(takh-282(diry-ibuon)-26781f)-281(2r)18(ofits.)-3640Fin examplefoit281(2r)18(oictins

willg abll-319(6huntil28317ll)-308(6hse)sibibre3r6h02rms 6het,dib supply networks because its potential to soften downstream competition and increase industry profits is greater in certain networks than in others. Moreover, since it typically yields wholesale prices that are different from the marginal cost of production, it introduces externalities between suppliers, in addition to the externalities between retailers that are also present in the environ-

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APPENDIX A

Proofs of Lemmas and Propositions

Proof of Proposition 1 – I first show that, for any network g, any profile of self-enforcing wholesale prices must be equal to marginal cost, i.e., $w(g) = c$. I then argue that this implies that there does not exist any self-enforcing (bilateral or multilateral) deviation from $w(g) = c$ and therefore that there always exist a unique CPNE profile of wholesale prices with $w(g) = c$.

A CPNE profile of wholesale prices must be immune to jointly profitable bilateral deviations by any supplier-retailer pair sr. Such bilateral deviations are always self-enforcing when jointly profitable because, in a CPNE, further deviations can only be undertaken by s alone or r alone and cannot modify the wholesale prices in the contracts with other firms. Therefore, if in their initial bilateral deviation s and r agreed on a contract $\langle t_{sr}, w_{sr} \rangle$ that left both of them better off given the strategies of all other players, neither s nor r can improve on that contract by deviating unilaterally.

The above implies that a CPNE profile of wholesale prices is always a contract equilibrium (see O'Brien and Shaffer, 1992), i.e., it maximizes the joint profits of each supplier-retailer pair, taking as given the contracts entered into by other pairs. In any contract equilibrium, and thus in any CPNE, it must be $w_{sr}(g) = c$ for all $s \in S$ and $r \in R$ with $\ell_{sr} = 1$. This can be demonstrated as follows. Consider a deviation by a coalition that involves only s and r. Given the assumptions of the model and the equilibrium concept adopted, the only wholesale price that can be changed by this bilateral deviation is w_{sr}. This is the case because, given the wholesale price proposals $w^s_{r'}$ by any other retailer $r' \neq r$, s cannot unilaterally change the wholesale price $w_{sr'}$ in his contract with r'. Moreover, since r' is not part of the original deviating coalition $\{s, r\}$, and CPNE restricts further deviations to subcoalitions, s cannot orchestrate further deviations to persuade r' to change her proposal $w^s_{r'}$. The fact that in this deviation s cannot change wholesale prices other than w_{sr}, and the fact that this is common knowledge, makes it unnecessary to specify beliefs for r in this deviation. Finally, since the change in w_{sr} is unobserved to other retailers, it does not affect the quantities (with Cournot competition) or prices (with Bertrand competition) chosen by those retailers. A small change in w_{sr} has, therefore, the following effect on the joint profits $\pi_{sr} = \pi_s + \pi_r$ of s and r,

$$\frac{d\pi_{sr}}{dw_{sr}} = \sum_{i \in S_r} \frac{\partial \pi_r}{\partial x_{ir}} \frac{dx_{ir}}{dw_{sr}} + q_{sr} + \sum_{j \in R_s} \left(w_{sj} - c \right) \frac{dq_{jr}}{dw_{sr}} - q_{sr}, \tag{A-1}$$

where x_{ir} represents the choice variable of retailer r and is equal to q_{ir} for Cournot competition and p_{ir} for Bertrand competition, and S_r is the set of suppliers with which retailer r has a link (i.e., $s \in S_r$ if $\ell_{sr} = 1$) and R_s is the set of retailers with which supplier s has a link (i.e., $r \in R_s$ if $\ell_{sr} = 1$). In a contract equilibrium, and thus in a CPNE, the wholesale price w_{sr} must maximize the joint profits of any sr pair with $\ell_{sr} = 1$, otherwise there would exist a profitable and self-enforcing bilateral deviation for at least one of these pairs. It must therefore be $d p_{sr} / d w_{sr} = 0$ for all s and r. Moreover, profit maximization by retailer r implies that $\partial p_r / \partial x_{ir} = 0$ for all $i \in S_r$, therefore the first term in (A-1) is always equal to zero by the envelope theorem. This implies that the following $R_s \times R_s$ system of equations must hold in a CPNE for all $s \in S_r$

$$\frac{dp_{sr}}{dw_{sr}} = \sum_{j \in R_s} (w_{sj} - c) \frac{dq_{sj}}{dw_{sr}} = 0, \quad \text{for all } r \in R_s. \tag{A-2}$$

If downstream competition is Cournot, $dq_{sj} / dw_{sr} = 0$ for all $j \neq r$ and, for all $s \in S_r$, (A-2) reduces to

$$(w_{sr} - c) \frac{dq_{sr}}{dw_{sr}} = 0, \quad \text{for all } r \in R_s. \tag{A-3}$$

Since $dq_{sr} / dw_{sr} < 0$ this implies $w_{sr} = c$ for all $s \in S_r$ and $r \in R_s$.

If downstream competition is instead Bertrand, $dq_{sj} / dw_{sr} \geq 0$ for $j \neq r$. In particular, although $dp_{ij} / dw_{sr} = 0$ for $j \neq r$, one still has $dp_{ir} / dw_{sr} \neq 0$ for all $i \in S_r$, and changes in the prices p_{ir} affect all quantities, including q_{sj} for $j \neq r$. The relevant conditions remain therefore those given in (A-2). These conditions can be written in matrix form as $(\mathbf{w}_s - c)\,\mathbf{M} = \mathbf{0}$, where $\mathbf{w}_s$ is a $1 \times R_s$ vector, and $\mathbf{M}$ is an $R_s \times R_s$ matrix with the expression in (A-2) constituting the typical element for column j and row r. Given Assumption 1, $\mathbf{M}$ has a dominant diagonal, i.e., $|dq_{sr} / dw_{sr}| > \sum_{j \neq r} |dq_{sj} / dw_{sr}|$ for all $r \in R_s$. By the Levy-Desplaques theorem $\mathbf{M}$ is therefore invertible and the unique solution to (A-2) is $w_{sr} = c$ for all $r \in R_s$.

The results above can be used to establish that $w(g) = c$ is not only a necessary, but also a sufficient, condition for $w(g)$w deviation is mutually profitable implies that, for all $s \in Z$,

$$\sum_{r \in R} \ell_{sr}^{h} t_{sr}^{h} > \sum_{r \in R} \ell_{sr}^{g} t_{sr}^{g}, \tag{A-4}$$

and, for all $r \in Z$,

$$\Pi_r^h - \sum_{s \in S} \ell_{sr}^{h} t_{sr}^{h} > \Pi_r^g - \sum_{s \in S} \ell_{sr}^{g} t_{sr}^{g}. \tag{A-5}$$

Note that, for $k \in \{h, g\}$, one can write $\sum_{r \in R} \ell_{sr}^{k} t_{sr}^{k} = \sum_{r \in Z} \ell_{sr}^{k} t_{sr}^{k} + \sum_{r \notin Z} \ell_{sr}^{k} t_{sr}^{k}$ and $\sum_{s \in S} \ell_{sr}^{k} t_{sr}^{k} = \sum_{s \in Z} \ell_{sr}^{k} t_{sr}^{k} + \sum_{s \notin Z} \ell_{sr}^{k} t_{sr}^{k}$. Using these identities and adding (A-4) over $s \in Z$ and (A-5) over $r \in Z$, one obtains

$$\sum_{s \in Z} \left[\sum_{r \in Z} \ell_{sr}^{h} t_{sr}^{h} + \sum_{r \notin Z} \ell_{sr}^{h} t_{sr}^{h} \right] > \sum_{s \in Z} \left[\sum_{r \in Z} \ell_{sr}^{g} t_{sr}^{g} + \sum_{r \notin Z} \ell_{sr}^{g} t_{sr}^{g} \right], \tag{A-6}$$

$$\sum_{r \in Z} \Pi_r^h - \left[\sum_{r \in Z} \sum_{s \in Z} \ell_{sr}^{h} t_{sr}^{h} + \sum_{s \notin Z} \ell_{sr}^{h} t_{sr}^{h} \right] > \sum_{r \in Z} \Pi_r^g - \left[\sum_{r \in Z} \sum_{s \in Z} \ell_{sr}^{g} t_{sr}^{g} + \sum_{s \notin Z} \ell_{sr}^{g} t_{sr}^{g} \right]. \tag{A-7}$$

Adding up (A-6) and (A-7) and rearranging terms, one obtains

$$\sum_{r \in Z} \left[\Pi_r^h - \Pi_r^g \right] > \sum_{s \in Z} \sum_{r \notin Z} \left(\ell_{sr}^{g} t_{sr}^{g} - \ell_{sr}^{h} t_{sr}^{h} \right) + \sum_{r \in Z} \sum_{s \notin Z} \left(\ell_{sr}^{g} t_{sr}^{g} - \ell_{sr}^{h} t_{sr}^{h} \right) \tag{A-8}$$

For all links sr in765 Tℓ^hd8 Tf819 0o Tf 7.r

The terms in square brackets in the left-hand side of (A-10) are independent from one another, since each refers to a different supplier $s \in Z$, and can be chosen freely, since the restrictions imposed by (A-9) constrain ℓ^h_{sr} and t^h_{sr} for $s \notin Z$ but not for $s \in Z$. Therefore, for $\sum_{r \in Z} \epsilon_r \to 0^+$, (A-10) implies that one can always find ℓ^h_{sr} and t^h_{sr} for each $s \in Z$ such that (A-4) holds for all $s \in Z$. This proves that, if (7) holds, one can always find a network h with profile of transfers t^h such that all suppliers and retailers in some coalition $Z \in \mathcal{Z}_{g \to h}$ are better off.

Proof of Proposition 2 (No exclusive contracts, with transfers) – The proof proceeds by first characterizing the conditions under which there exist coalition-proof Nash equilibria ("equilibria", for short) with $g_{t,ne} = pe$ and $g_{t,ne} = dca$, and then proving that there never exist equilibria with $g_{t,ne} \in \{bm, dm, um, mix\}$.

Existence of equilibria with $g_{t,ne} = pe$

Consider a candidate equilibrium with $g = pe$ (see Figure 1(c)), in which no firm is willing to engage in "cross trade," i.e., in which s and r' refuse to trade with each other and the same applies to s' and r. Equilibria with $g = pe$ can also be supported by having only one of s and r' (or one of s' and r) refuse to trade with the other. However, such candidate equilibria would be easier to break than candidate equilibria supported by strategies in which both firms refuse to trade with the other. The restriction to the latter is, therefore, without loss of generality. Consider a symmetric profile of transfers $t_{sr} = t_{s'r'} = t^{pe}$ for the pairs of firms that agree to trade. This restriction is also without loss of generality because candidate equilibria with asymmetric transfers are easier to break than candidate equilibria with symmetric transfers, since, when transfers are asymmetric, the supplier and / or retailer with the lowest payoff would be more amenable to a deviation.

For there to be no profitable unilateral deviations in which a supplier or a retailer rescinds his refuse to of s

r, and mutually profitable, since $P^{dm} \quad P^{pe}$. For an equilibrium with $g = pe$ to exist it must therefore be $t^{pe} > 0$.

A deviation to $g = mix$, with r^0 agreeing to trade with both s and s^0, while s continues to trade with r. This deviation is self-enforcing (i.e., supportable as a Nash equilibrium of the two-player game between s and r^0) if and only if $t^{pe} \quad P^{mix2} \quad P^{um}$ and strictly Pareto dominates $g = pe$ if and only if $P^{mix2} > P^{pe}$. Therefore, there does not exist a mutually profitable and self-enforcing deviation to $g = mix$ if and only if at least one of the following holds

$$P^{pe} \quad P^{mix2} \tag{A-12}$$

$$t^{pe} > P^{mix2} \quad P^{um} \tag{A-13}$$

A deviation to $g = um$, with r^0 agreeing to start trading with s and stop trading with s^0, while s continues to trade with r. This deviation is self-enforcing if and only if $t^{pe} \quad P^{mix2} \quad P^{um}$ and Pareto dominates $g = pe$ if and only if

r. As explained above, the restriction to symmetric transfers is without loss of generality. For a supplier or retailer to have no incentives to rescind unilaterally one of their supply links, thus implementing a market structure with $g = mix$ as in Figure 1(b), it must be

$$0 \quad t^{dca} \quad P^{dca} \quad P^{mix1}. \tag{A-15}$$

Moreover, when (A-15) holds, in the candidate equilibrium a retailer earns $P^{dca} \quad 2t^{dca}$ $2P^{mix1} \quad P^{dca} > 0$ (where the last inequality follows from the products being substitutes) and has thus no incentive to withdraw from the market completely by rescinding both of its links (a supplier has obviously no incentives to do so for $t^{dca} \quad 0$.)

Consider a deviation to $g = pe$ by the grand coalition $f s, s^{0}, r, r^{0} g$ in which all firms refuse to engage in cross trade (e.g., s and r^{0} refuse to deal with each other and s^{0} and r refuse to deal with each other). Two aspects of this deviation are worth mentioning. First, as discussed in relation to Remark 1 in the main text, this deviation could be viewed as a (nonbinding) market allocation agreement. The consequences of prohibiting such agreements (and thus the deviation considered here) are discussed in Remark 1, which is proved further below in this appendix. For the time being, however, I allow for this type of nonbinding agreements. Second, a deviation to $g = pe$ can also be implemented by smaller coalitions (e.g., $f r, r^{0} g$ agreeing not to "cross trade" with suppliers) but such deviations would be less likely to be self enforcing than a deviation by $f s, s^{0}, r, r^{0} g$, as in such deviations not all firms would withdraw their cross offers (e.g., in a deviation by $f r, r^{0} g$ only the retailers would withdraw their cross offers, whereas the suppliers would continue to offer contracts to all retailers). Such deviations by smaller coalitions would thus not add anything to the analysis presented here. Having clarified these aspects, denote by $\tilde{t}^{pe}$ the transfers in this deviation. The deviation can be mutually profitable if there exists a $\tilde{t}^{pe}$ such that

$$2t^{dca} < \tilde{t}^{pe} < P^{pe} \quad P^{dca} + 2t^{dca} \tag{A-16}$$

The deviation is never self enforcing for $P^{mix2} > P^{pe}$ because, as shown above, when this condition holds a network with $g = pe$ is never self-enforcing even with unrestricted transfers t^{pe}. Therefore, it is a fortiori never self-enforcing with transfers $\tilde{t}^{pe}$ that are restricted as in (A-16). The deviation is, instead, always self enforcing for $P^{pe} \quad P^{mix2}$, as there always exists a $\tilde{t}^{pe} > 0$ that satisfies both (A-14) and (A-16). The latter requires $P^{pe} > P^{dca}$ (which is always the case when $P^{pe} \quad P^{mix2}$, as $P^{mix2} > P^{dca}$) and $2t^{dca} < P^{pe} \quad P^{um}$. For there to be no $\tilde{t}^{pe} > 0$ that satisfies (A-14) and (A-16) one would therefore need $2t^{dca} \quad P^{pe} \quad P^{um}$. However, with linear demand and when $P^{pe} \quad P^{mix2}$, there never exists a t^{dca} satisfying both this condition and condition (A-15) for no unilateral deviations.

Consider next a deviation to $g = dm$ by a coalition fs, s^{0}, rg. For $t^{dca} > 0$, this deviation is not self-enforcing in the absence of exclusive contracts, since either supplier would always have incentives to re-form his link with r^{0}. For $t^{dca} = 0$, this deviation is instead self enforcing, since no supplier would have incentives to reform his link with r^{0}, and mutually profitable, since P^{dm} P^{dca}. For an equilibrium with $g = dca$ to exist it must therefore be $t^{dca} > 0$.

Finally, consider a deviation to $g = um$ by a coalition $fs, r, r^{0}g$. This deviation is self enforcing if and only if $t^{dca} > P^{mix2}$ P^{um} and always mutually profitable whenever it is self enforcing. A necessary condition for there to exist an equilibrium with $g = dca$ is therefore t^{dca} P^{mix2} P^{um}. Since with linear demand P^{mix2} P^{dca} and $P^{um} = P^{mix1}$, this condition is never binding when (A-15) holds.

Taken together, all of the above implies that there exist equilibria with $g = dca$ and t^{dca} *2* 0, P^{dca} P^{mix1} if and only if $P^{mix2} > P^{pe}$.

not rely on market allocation agreements, prohibiting such agreements has no effects.

Regarding the existence of equilibria with $g = dca$, prohibiting market allocation agreements rules out the deviation to $g = pe$ by the grand coalition in the proof of Proposition 2. The only feasible deviation to $g = pe$ remains one by a supplier s and a retailer r

ing (A-17), this implies that there does not exist a self-enforcing deviation to $g = pe$ by $\{s, r\}$ if and only if

$$\Pi^{dca} \quad \Pi^{pe} \tag{A-19}$$

Deviations to $g = pe$ can also be implemented by broader coalitions, such as the grand coalition $\{s, s', r, r'\}$ in the proof of Proposition 2, but these deviations do not add anything to the conditions derived from the deviation by $\{s, r\}$ studied above.

Consider now a deviation to $g = um$ in which r and r' commit to be exclusive to s. This deviation is mutually profitable if and only if the joint profits that $\{s, r, r'\}$ can earn in the deviation, $2\Pi^{um}$, are greater than the joint profits that $\{s, r, r'\}$ earns in the candidate equilibrium, $2\Pi^{dca} \quad 2t^{dca}$, which is the case if and only if $t^{dca} > \Pi^{dca} \quad \Pi^{um}$. This condition never holds, and thus a deviation to $g = um$ is never profitable, when condition (A-15) for no unilateral deviations holds, since, with linear demand, $\Pi^{um} = \Pi^{mix1}$.

Finally, consider a deviation to $g = dm$ in which s and s' commit to be exclusive to r. This deviation is mutually profitable if and only if there exists a $\tilde{t}^{dm}$ such that

$$4t^{dca} < 2\tilde{t}^{dm} < \Pi^{dm} \quad \Pi^{dca} + 2t^{dca}. \tag{A-20}$$

exist no mutually profitable and self-enforcing deviation to $g = dm$, (A-20) must fail for any $\tilde{t}^{dm}$, which is the case if and only if

$$2t^{dca} \quad P^{dm} \quad P^{dca} \tag{A-21}$$

The analysis above implies that there exist equilibria with $g = dca$ if and only if (A-19) holds and there exists a t^{dca} such that (A-15) and (A-21) also hold. The latter is the case if and only if

$$2 \quad P^{dca} \quad P^{mix1} \qquad P^{dm} \quad P^{dca}. \tag{A-22}$$

Intuitively, for there to exist an equilibrium with $g = dca$ there must exist intermediate transfers t^{dca} that ensure that i) either supplier does not extract too much, otherwise one or both retailers would drop him and ii) either retailer does not extract too much, otherwise the other retailer could convince both suppliers to drop her. As can be seen in Figure 4, this is possible only if suppliers and retailers are sufficiently differentiated.

Existence of equilibria with $g_{t,e} = pe$

Consider an equilibrium with $g = pe$ supported by mutual exclusivity between s and r and between s^0 and r^0. Although equilibria with $g = pe$ can also be supported by one-way commitments to exclusivity, equilibria supported by mutual exclusivity are more difficult to break and thus more likely to exist. As in the proof of Proposition 2, focus without loss of generality on symmetric transfers $t_{sr} = t_{s^0 r^0} = t^{pe}$ for the pairs of firms that agree to trade. Suppliers and retailers do not have incentives to deviate unilaterally by becoming inactive if and only if (A-11) holds.

Consider first a deviation to $g = um$ in which r and r^0 commit to be exclusive to s. This deviation is mutually profitable if and only if there exists a $\tilde{t}^{um}$ such that $t^{pe} < 2\tilde{t}^{um} < 2\,(P^{um} \quad P^{pe} + t^{pe})$ and is (trivially) self enforcing whenever it is profitable (in the candidate equilibrium, supplier s^0 does not have a proposal out to retailer r and cannot be involved in further negotiations because it does not belong to the original deviating coalition that includes r). Therefore, for there to exist no mutually profitable and self-enforcing deviation to $g = um$ it must be

$$t^{pe} \quad 2\,(P^{pe} \quad P^{um}) \tag{A-23}$$

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Consider next a deviation to $g = dm$ iiginalbeusivrbeusivtytg cannot be involved in further negotiations because it does not belong to the original deviating coalition that includes s

in which the excluded retailer, r', steals the entire business of the active retailer, r, by offering both suppliers transfers $\tilde{t}^{dm} \in (t^{dm}, \Pi^{dm}/2)$ in exchange for exclusivity and earns profits Π^{dm} $2\tilde{t}^{dm} > 0$. Such a deviation would be self enforcing because the only feasible further deviation, i.e., r' and one of the two suppliers, say s', committing to mutual exclusivity, is not profitable. This is the case because, by the "no stranding" assumption, such a deviation would cause the two excluded firms, s and r, to enter a contract and would thus implement $g = pe$, and Π^{pe} Π^{dm} $\tilde{t}^{dm}$ for $\tilde{t}^{dm}$ $\Pi^{dm}/2$ when Π^{dm} $2\Pi^{pe}$.

Finally, when (A-25) fails (or holds with equality), one also has that $\Pi^{dm} > 2\Pi^{dca}$ and $\Pi^{dm} > \Pi^{mix1} + \Pi^{mix2}$ so that deviations in which the grand coalition deviates to $g = dca$ or a coalition of $\{s', r, r'\}$ deviates to $g = mix$ (with r' carrying product s' and s carrying both products) are not mutually profitable.

There do not exist equilibria with $g_{t,e} \in \{bm, um, mix\}$

There cannot exist an equilibrium with $g = bm$ because the excluded supplier and retailer would always have self-enforcing incentives to form a link and earn positive profits.

Consider a candidate equilibrium with $g = um$ in which r and r' commit to be exclusive to s and assume, without loss of generality, symmetric transfers $t^{um}_{sr} = t^{um}_{sr'} = t^{um}$. Unless t^{um} Π^{um} Π^{pe} there always exists a self-enforcing deviation in which the excluded supplier, s', and one of the retailers, say r, enter a mutually exclusive contract that implements $g = pe$. Since Π^{um} $\Pi^{pe} < 0$ and suppliers have incentives to rescind their candidate equilibrium supply link if

if

with $g_{t,e} = pe$

2 P^{dca} P^{mix1} without exclusive contracts and a lower bound p

Consider then a deviation by the coalition $\{s, s^0\}$. This coalition can carry out deviations that implement $g = dm$, $g = mix$ or $g = pe$. A deviation that implements $g = dm$ is never self enforcing, since $p_s^{pe} > p_s^{dm}$ and thus, when supplier s^0 trades only with r, supplier s prefers trading only with r^0 to also trading only with r72 0 24 Tf32 8(3g])-208e)]TJ /F113 1126353(implem353s)]TJ /F113 11.1182 Tf 150.
withwith

enforcing because, when s is willing to deal with both suppliers, r' prefers exclusivity with s' to exclusivity with s, since $\Pi^{pe} > \Pi^{um}$ implies $p_{r'}^{pe} > p_{r'}^{um}$. Finally, a deviation to $g = mix$ is self enforcing if and only if r' responds to the willingness of s to deal with both retailers by being willing to deal with both suppliers, and vice versa, i.e., if and only if $p_s^{mix2} \geq \max\left\{p_s^{pe}, p_s^{dm}\right\}$ and $p_{r'}^{mix2} \geq \max\left\{p_{r'}^{pe}, p_{r'}^{um}\right\}$. The first conditions corresponds to $p_s^{mix2} \geq p_s^{pe}$, since $p_s^{pe} \geq p_s^{dm}$, and is satisfied if and only if (A-33) holds. The second condition corresponds to $p_{r'}^{mix2} \geq p_{r'}^{pe}$, since $p_{r'}^{pe} \geq p_{r'}^{um}$, and, rearranging terms, yields

$$\left(2 - \frac{1}{b}\right)\left(\Pi^{mix2} - \Pi^{pe}\right) \geq \Pi^{um} \qquad \text{(A-34)}$$

Condition (A-34) always holds when (A-33) does. To see this note that, when (A-33) holds, $\Pi^{mix2} - \Pi^{pe} > 0$, given that $\Pi^{mix1} < \Pi^{pe}$. Since $(2 - 1/b)$ is increasing in b, this implies

Existence of equilibria with $g_{nt,e} = dca$

First, note that all the deviations that were profitable and self-enforcing without exclusive contracts remain so with exclusive contracts. Therefore, in light of Proposition 6, an equilibrium with $g = dca$ can exist only for $b \geq \bar{b}_{nt,ne}(a)$. Exclusive contracts may also render additional deviations self-enforcing.

A deviation by a coalition $\{s, r\}$ in which these two firms commit to mutual exclusivity, thus implementing $g = pe$, is profitable for s if and only if

$$\Pi^{pe} > 2\left(\Pi^{dca} - \Pi^{mix1}\right) \tag{A-35}$$

and for r if and only if

$$(1 - b)\,\Pi^{pe} > \Pi^{dca} - 2b\left(\Pi^{dca} - \Pi^{mix1}\right) \tag{A-36}$$

This deviation is never profitable when $b \geq \bar{b}_{nt,ne}(a)$. To see this, rewrite (A-36) as

$$(1 - b)\left[\Pi^{pe} - 2\left(\Pi^{dca} - \Pi^{mix1}\right)\right] > \Pi^{dca} - 2\left(\Pi^{dca} - \Pi^{mix1}\right) \tag{A-37}$$

When (A-35) holds the term in square brackets in the left-hand side of (A-37) is positive. The condition is therefore most likely to hold for $b = 0$, where it becomes $\Pi^{pe} > \Pi^{dca}$, which is therefore a necessary condition for the deviation to be profitable for r. Since $\Pi^{pe} > \Pi^{dca}$ never holds for $b \geq \bar{b}_{nt,ne}(a)$, this deviation to mutual exclusivity by $\{s, r\}$ does not further restrict the region of parameters for the existence of an equilibrium with $g = dca$ relative to the case without exclusive contracts.

Finally, deviations to $g = dm$ and $g = um$ are not self enforcing since, as will be proven below, there always exist profitable and self-enforcing deviations from these networks also with exclusive contracts.

Existence of equilibria with $g_{nt,e} = pe$

As in Proposition 3, consider an equilibrium with $g = pe$ supported by mutual exclusivity between s and r and between s' and r'. Consider first a deviation to $g = dca$ by the grand coalition. This deviation is profitable for suppliers and retailers if and only if, respectively, (A-35) and (A-36) fail (or, more precisely, if the sign in those conditions is changed from > to <). With a reasoning analogous to that conducted above in relation to (A-37) one can demonstrate that the deviation is always profitable for retailers when it is profitable for suppliers, which implies that there exists a jointly profitable deviation to $g = dca$ if and only if (A-35) fails, which is the case if and only if $b \geq \bar{\bar{b}}_{nt,e}(a)$. Since $\bar{\bar{b}}_{nt,e}(a) < \bar{b}_{nt,e}(a)$, the deviation to $g = dca$ is self-enforcing whenever it is profitable. Therefore, the existence of an equilibrium with $g = pe$ requires (A-35) to hold, i.e., $b > \bar{\bar{b}}_{nt,e}(a)$.

Consider next a deviation to $g = mix$ by the grand coalition. This deviation is never profitable for the supplier that would be left dealing with only one retailer, since it can be verified that with linear demand $b \quad P^{mix2} \quad P^{um} > bP^{pe}$. Moreover, it can be verified that a deviation to $g = dm$ is never profitable for suppliers, because $P^{dm} \quad P^{bm} \quad P^{pe}$, and a deviation to $g = um$ is never profitable for retailers, because $P^{um} \quad P^{pe}$. Intuitively, although a deviation to $g = dm$ may increase total industry profits when $P^{dm} > 2P^{pe}$, it reduces the bargaining power of suppliers, The analysis above implies that there exists an equilibrium with $g = mix$ if and only if (A-33) holds.

There do not exist equilibria with $g_{nt,e} \in \{bm, dm, um\}$

This part of the proof is analogous to that of the proof of Proposition 6. Specifically, there does not exist equilibria with $g = dm$ because p