

$$\begin{aligned} & \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{v}^T \mathbf{v} \right) + \mathbf{v}^T \nabla \cdot \mathbf{u} + \frac{1}{2} \mathbf{u}^T \mathbf{u} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{w}^T \mathbf{w} \right) + \mathbf{w}^T \nabla \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \mathbf{v} \right] \end{aligned}$$

$$\begin{aligned} & \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{d}^T \mathbf{d} \right) + \mathbf{d}^T \nabla \cdot \mathbf{w} + \frac{1}{2} \mathbf{w}^T \mathbf{d} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{w}^T \mathbf{d} \right) + \mathbf{w}^T \nabla \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \mathbf{d} \right] \end{aligned}$$

$$\begin{aligned} & \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{v}^T \mathbf{v} \right) + \mathbf{v}^T \nabla \cdot \mathbf{u} + \frac{1}{2} \mathbf{u}^T \mathbf{v} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{w}^T \mathbf{v} \right) + \mathbf{w}^T \nabla \cdot \mathbf{d} + \frac{1}{2} \mathbf{d}^T \mathbf{v} \right] \end{aligned}$$

$$\begin{aligned} & \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{d}^T \mathbf{v} \right) + \mathbf{d}^T \nabla \cdot \mathbf{w} + \frac{1}{2} \mathbf{w}^T \mathbf{v} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{w}^T \mathbf{d} \right) + \mathbf{w}^T \nabla \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \mathbf{d} \right] \end{aligned}$$

$$\begin{aligned} & \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{d}^T \mathbf{d} \right) + \mathbf{d}^T \nabla \cdot \mathbf{w} + \frac{1}{2} \mathbf{w}^T \mathbf{d} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{w}^T \mathbf{w} \right) + \mathbf{w}^T \nabla \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \mathbf{w} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{v}^T \mathbf{v} \right) + \mathbf{v}^T \nabla \cdot \mathbf{u} + \frac{1}{2} \mathbf{u}^T \mathbf{v} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{d}^T \mathbf{v} \right) + \mathbf{d}^T \nabla \cdot \mathbf{w} + \frac{1}{2} \mathbf{w}^T \mathbf{v} \right] \\ & + \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{w}^T \mathbf{d} \right) + \mathbf{w}^T \nabla \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \mathbf{d} \right] \end{aligned}$$

$$\begin{array}{c} KE \qquad \qquad RE \quad E \qquad \qquad \gamma_1 \gamma_2 \\ \hline \left[\frac{\partial}{\partial t} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = \mathbf{d}_1 \right] \qquad \left[\frac{\partial}{\partial t} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = \mathbf{d}_2 \right] \qquad \left[\frac{\partial}{\partial t} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = \mathbf{d}_3 \right] \\ \left[\frac{\partial}{\partial t} \mathbf{w}_1 + (\mathbf{v} \cdot \nabla) \mathbf{w}_1 - \nu \Delta \mathbf{w}_1 + \nabla q_1 = \mathbf{d}_4 \right] \qquad \left[\frac{\partial}{\partial t} \mathbf{w}_2 + (\mathbf{v} \cdot \nabla) \mathbf{w}_2 - \nu \Delta \mathbf{w}_2 + \nabla q_2 = \mathbf{d}_5 \right] \qquad \left[\frac{\partial}{\partial t} \mathbf{w}_3 + (\mathbf{v} \cdot \nabla) \mathbf{w}_3 - \nu \Delta \mathbf{w}_3 + \nabla q_3 = \mathbf{d}_6 \right] \end{array}$$

,

*F**F R E**E**REF R*

KE RE E

RE E

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E *R* *E*

F R E

FRE

E REF R

V

W. H. D. W. H. D.

<i>F</i>	<i>F R E</i>	<i>E</i>	<i>REF R</i>
<i>d</i>	<i>d</i>	<i>w</i>	<i>w</i>
<i>v</i>	<i>v</i>	<i>y</i>	<i>y</i>

KE RE E

RE E

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$$E \qquad \qquad R$$

	<i>F</i>	<i>F R E</i>	<i>E</i>	<i>REF R</i>
	<i>R</i>	<i>R</i>	<i>E</i>	<i>w</i> <i>w</i> <i>d</i>

	<i>F</i>	<i>F</i>	<i>RE</i>	<i>E</i>	<i>REF</i>	<i>R</i>
1	[d	[[[[
2	d	[v	d	v	v
3	[v	d	[v	v
4	v	[v	v	[v
5	v	[v	v	v	[
6	v	[v	v	v	v
7	v	[v	v	v	v
8	v	[v	v	v	v
9	v	[v	v	v	v
10	v	[v	v	v	v
11	v	[v	v	v	v
12	v	[v	v	v	v
13	v	[v	v	v	v
14	v	[v	v	v	v
15	v	[v	v	v	v
16	v	[v	v	v	v
17	v	[v	v	v	v
18	v	[v	v	v	v
19	v	[v	v	v	v
20	v	[v	v	v	v
21	v	[v	v	v	v
22	v	[v	v	v	v
23	v	[v	v	v	v
24	v	[v	v	v	v
25	v	[v	v	v	v
26	v	[v	v	v	v
27	v	[v	v	v	v
28	v	[v	v	v	v
29	v	[v	v	v	v
30	v	[v	v	v	v
31	v	[v	v	v	v
32	v	[v	v	v	v
33	v	[v	v	v	v
34	v	[v	v	v	v
35	v	[v	v	v	v
36	v	[v	v	v	v
37	v	[v	v	v	v
38	v	[v	v	v	v
39	v	[v	v	v	v
40	v	[v	v	v	v
41	v	[v	v	v	v
42	v	[v	v	v	v
43	v	[v	v	v	v
44	v	[v	v	v	v
45	v	[v	v	v	v
46	v	[v	v	v	v
47	v	[v	v	v	v
48	v	[v	v	v	v
49	v	[v	v	v	v
50	v	[v	v	v	v
51	v	[v	v	v	v
52	v	[v	v	v	v
53	v	[v	v	v	v
54	v	[v	v	v	v
55	v	[v	v	v	v
56	v	[v	v	v	v
57	v	[v	v	v	v
58	v	[v	v	v	v
59	v	[v	v	v	v
60	v	[v	v	v	v
61	v	[v	v	v	v
62	v	[v	v	v	v
63	v	[v	v	v	v
64	v	[v	v	v	v
65	v	[v	v	v	v
66	v	[v	v	v	v
67	v	[v	v	v	v
68	v	[v	v	v	v
69	v	[v	v	v	v
70	v	[v	v	v	v
71	v	[v	v	v	v
72	v	[v	v	v	v
73	v	[v	v	v	v
74	v	[v	v	v	v
75	v	[v	v	v	v
76	v	[v	v	v	v
77	v	[v	v	v	v
78	v	[v	v	v	v
79	v	[v	v	v	v
80	v	[v	v	v	v
81	v	[v	v	v	v
82	v	[v	v	v	v
83	v	[v	v	v	v
84	v	[v	v	v	v
85	v	[v	v	v	v
86	v	[v	v	v	v
87	v	[v	v	v	v
88	v	[v	v	v	v
89	v	[v	v	v	v
90	v	[v	v	v	v
91	v	[v	v	v	v
92	v	[v	v	v	v
93	v	[v	v	v	v
94	v	[v	v	v	v
95	v	[v	v	v	v
96	v	[v	v	v	v
97	v	[v	v	v	v
98	v	[v	v	v	v
99	v	[v	v	v	v
100	v	[v	v	v	v

KE *RE* *E* *T* *T* *T* *T*

KE RE E

RE E

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E R

w d [] d d w [] w d [] v d v k d v f d v w d [] w d / w

the present system of the *Principia* is not to be regarded as a final one.

KE RE E

RE E

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$$\in \mathbb{R}$$

E

$$\frac{d}{v} \left(\frac{v}{w} \right) = \frac{dv}{v^2} - \frac{dw}{w^2} \quad \text{and} \quad \frac{d}{w} \left(\frac{v}{w} \right) = \frac{dv}{vw} - \frac{dw}{w^2}$$

$$KE = RE - E \quad \text{and} \quad \gamma_1 + \gamma_2 = 1$$

$$\frac{1}{\mu} \frac{d}{dt} \left(\frac{1}{2} \rho \mathbf{v}_T^T \mathbf{v}_T \right) = - \nabla \cdot (\mathbf{v}_T \mathbf{E}) + \frac{1}{\rho} \nabla p + \frac{1}{\rho} \mathbf{v}_T \cdot \nabla p + \frac{1}{\rho} \mathbf{v}_T^T \mathbf{v}_T$$

$$\begin{array}{cccc} KE & RE & E & \gamma_1 \gamma_2 \\ \hline \hline \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.2 & 1.2 & 1.2 & 1.2 \\ 1.4 & 1.4 & 1.4 & 1.4 \\ 1.6 & 1.6 & 1.6 & 1.6 \\ 1.8 & 1.8 & 1.8 & 1.8 \\ 2.0 & 2.0 & 2.0 & 2.0 \\ 2.2 & 2.2 & 2.2 & 2.2 \\ 2.4 & 2.4 & 2.4 & 2.4 \\ 2.6 & 2.6 & 2.6 & 2.6 \\ 2.8 & 2.8 & 2.8 & 2.8 \\ 3.0 & 3.0 & 3.0 & 3.0 \\ 3.2 & 3.2 & 3.2 & 3.2 \\ 3.4 & 3.4 & 3.4 & 3.4 \\ 3.6 & 3.6 & 3.6 & 3.6 \\ 3.8 & 3.8 & 3.8 & 3.8 \\ 4.0 & 4.0 & 4.0 & 4.0 \\ 4.2 & 4.2 & 4.2 & 4.2 \\ 4.4 & 4.4 & 4.4 & 4.4 \\ 4.6 & 4.6 & 4.6 & 4.6 \\ 4.8 & 4.8 & 4.8 & 4.8 \\ 5.0 & 5.0 & 5.0 & 5.0 \\ 5.2 & 5.2 & 5.2 & 5.2 \\ 5.4 & 5.4 & 5.4 & 5.4 \\ 5.6 & 5.6 & 5.6 & 5.6 \\ 5.8 & 5.8 & 5.8 & 5.8 \\ 6.0 & 6.0 & 6.0 & 6.0 \\ 6.2 & 6.2 & 6.2 & 6.2 \\ 6.4 & 6.4 & 6.4 & 6.4 \\ 6.6 & 6.6 & 6.6 & 6.6 \\ 6.8 & 6.8 & 6.8 & 6.8 \\ 7.0 & 7.0 & 7.0 & 7.0 \\ 7.2 & 7.2 & 7.2 & 7.2 \\ 7.4 & 7.4 & 7.4 & 7.4 \\ 7.6 & 7.6 & 7.6 & 7.6 \\ 7.8 & 7.8 & 7.8 & 7.8 \\ 8.0 & 8.0 & 8.0 & 8.0 \\ 8.2 & 8.2 & 8.2 & 8.2 \\ 8.4 & 8.4 & 8.4 & 8.4 \\ 8.6 & 8.6 & 8.6 & 8.6 \\ 8.8 & 8.8 & 8.8 & 8.8 \\ 9.0 & 9.0 & 9.0 & 9.0 \\ 9.2 & 9.2 & 9.2 & 9.2 \\ 9.4 & 9.4 & 9.4 & 9.4 \\ 9.6 & 9.6 & 9.6 & 9.6 \\ 9.8 & 9.8 & 9.8 & 9.8 \\ 10.0 & 10.0 & 10.0 & 10.0 \end{array}$$

The figure is a 3D surface plot representing a function of three variables: KE, RE, and E. The vertical axis represents the function value, ranging from -1.0 to 1.0. The horizontal axes are KE (x-axis, 0 to 1), RE (y-axis, 0 to 1), and E (z-axis, 0 to 1). The surface exhibits a complex, multi-peaked structure. A major peak is located at approximately (KE=0.5, RE=0.5, E=0.5) with a value of about 0.8. There are other smaller peaks along the diagonal where the sum of KE, RE, and E is approximately 1. The surface also contains deep valleys, particularly at the corners where one or more parameters are zero.

$$\begin{array}{ccccccc}
 & F & & F & R & E & \\
 \hline
 & \left[\frac{\partial}{\partial t} \psi + \nabla \cdot \mathbf{v} \psi - \frac{1}{2} |\nabla \psi|^2 + \frac{1}{2} \nabla^2 \psi \right] & & \left[\frac{\partial}{\partial t} \mathbf{v} \cdot \mathbf{z} + \nabla \cdot \mathbf{v} \mathbf{z} - \frac{1}{2} \nabla^2 \mathbf{v} \cdot \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla^2 \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla \mathbf{z} + \frac{1}{2} \mathbf{z} \cdot \nabla \mathbf{v} \right] & & E & \text{REF } R \\
 & \mathbf{d}_t \psi + \nabla \cdot \mathbf{v} \psi - \frac{1}{2} \nabla^2 \psi + \frac{1}{2} \mathbf{v} \cdot \nabla^2 \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla \mathbf{z} + \frac{1}{2} \mathbf{z} \cdot \nabla \mathbf{v} & & \mathbf{d}_t \mathbf{v} \cdot \mathbf{z} + \nabla \cdot \mathbf{v} \mathbf{z} - \frac{1}{2} \nabla^2 \mathbf{v} \cdot \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla^2 \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla \mathbf{z} + \frac{1}{2} \mathbf{z} \cdot \nabla \mathbf{v} & & E & \text{REF } R \\
 & \psi \mathbf{d}_t \psi + \mathbf{v} \cdot \nabla \psi - \frac{1}{2} \nabla^2 \psi + \frac{1}{2} \mathbf{v} \cdot \nabla^2 \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla \mathbf{z} + \frac{1}{2} \mathbf{z} \cdot \nabla \mathbf{v} & & \mathbf{v} \mathbf{d}_t \mathbf{v} \cdot \mathbf{z} + \nabla \cdot \mathbf{v} \mathbf{z} - \frac{1}{2} \nabla^2 \mathbf{v} \cdot \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla^2 \mathbf{z} + \frac{1}{2} \mathbf{v} \cdot \nabla \mathbf{z} + \frac{1}{2} \mathbf{z} \cdot \nabla \mathbf{v} & & E & \text{REF } R
 \end{array}$$

$$\begin{array}{c}
 \hline
 & KE & RE & E & \gamma_1 & \gamma_2 \\
 \hline
 \end{array}$$

$$\begin{array}{c}
 \frac{\partial}{\partial t} \int_{\Omega} \frac{1}{2} |\nabla v|^2 + \int_{\Omega} \frac{1}{2} |\nabla w|^2 - \int_{\Omega} \mu \nabla v \cdot \nabla w \\
 - \int_{\Omega} v \nabla d \cdot \nabla v - \int_{\Omega} w \nabla d \cdot \nabla w \\
 - \int_{\Omega} \mu \nabla v \cdot \nabla w - \int_{\Omega} \mu \nabla w \cdot \nabla v \\
 - \int_{\Omega} v \nabla d \cdot \nabla v - \int_{\Omega} w \nabla d \cdot \nabla w \\
 - \int_{\Omega} v \nabla d \cdot \nabla w - \int_{\Omega} w \nabla d \cdot \nabla v
 \end{array}$$

<i>F</i>	<i>F R E</i>	<i>E</i>	<i>REF R</i>
<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>
<i>w</i>	<i>w</i>	<i>w</i>	<i>w</i>
<i>v</i>	<i>v</i>	<i>v</i>	<i>v</i>
<i>y</i>	<i>y</i>	<i>y</i>	<i>y</i>

KE *RE* *E*

	<i>F</i>	<i>F R E</i>	<i>E</i>	<i>REF R</i>	<i>T</i>
$\int_{\Omega} \nabla v \cdot \nabla w \, dx - \int_{\Omega} \nabla w \cdot \nabla v \, dx$	d	d	d	d	v
$\int_{\Omega} \nabla v \cdot \nabla w \, dx - \int_{\Omega} \nabla w \cdot \nabla v \, dx$	d	d	d	d	v
$\int_{\Omega} \nabla v \cdot \nabla w \, dx - \int_{\Omega} \nabla w \cdot \nabla v \, dx$	d	d	d	d	v

γ	KE	RE	E	$\gamma_1 \gamma_2$
0.0	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.1	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.2	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.3	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.4	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.5	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.6	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.7	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.8	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
0.9	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
1.0	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000

$$\frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \left(\frac{\partial u}{\partial t} \right)^2 dx + \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \left(\nabla u \right)^2 dx = KE - RE - E.$$

$$\frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \left(\frac{\partial u}{\partial t} \right)^2 dx + \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \left(\nabla u \right)^2 dx = KE - RE - E.$$

F *F RE* *E* *REF R* *N*

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) = \frac{d}{dt} \left[-\mu_i \left(q_i \right) + \frac{1}{2} \dot{q}_i^T M_i(q_i) \dot{q}_i + \frac{1}{2} \dot{q}_i^T P_i(q_i) \dot{q}_i \right] = -\mu_i'(q_i) \dot{q}_i + \frac{1}{2} M_i(q_i) \ddot{q}_i + \frac{1}{2} P_i(q_i) \ddot{q}_i$$

$$\begin{aligned}
& \frac{\partial}{\partial t} \int_{\Omega} \left[\frac{1}{2} \rho \left(\frac{|\nabla u|^2 + |\nabla v|^2}{2} - T \gamma \mathbf{x} \cdot \nabla u \right) \right] \mathrm{d}x + \int_{\Omega} \left[\frac{1}{2} \rho \left(\frac{|\nabla u|^2 + |\nabla v|^2}{2} - T \gamma \mathbf{x} \cdot \nabla u \right) \right] \mathrm{d}x \\
& + \int_{\Omega} \left[\frac{1}{2} \rho \left(\frac{|\nabla u|^2 + |\nabla v|^2}{2} - T \gamma \mathbf{x} \cdot \nabla u \right) \right] \mathrm{d}x = \int_{\Omega} \left[\frac{1}{2} \rho \left(\frac{|\nabla u|^2 + |\nabla v|^2}{2} - T \gamma \mathbf{x} \cdot \nabla u \right) \right] \mathrm{d}x
\end{aligned}$$

$$\frac{\partial}{\partial t} \left[\frac{d}{dt} \nabla_{\mathbf{v}_i} \cdot \nabla_{\mathbf{w}_i} T^{\alpha} \right] = \frac{\partial}{\partial t} \left[\nabla_{\mathbf{v}_i} \cdot \nabla_{\mathbf{w}_i} T^{\alpha} \right]$$