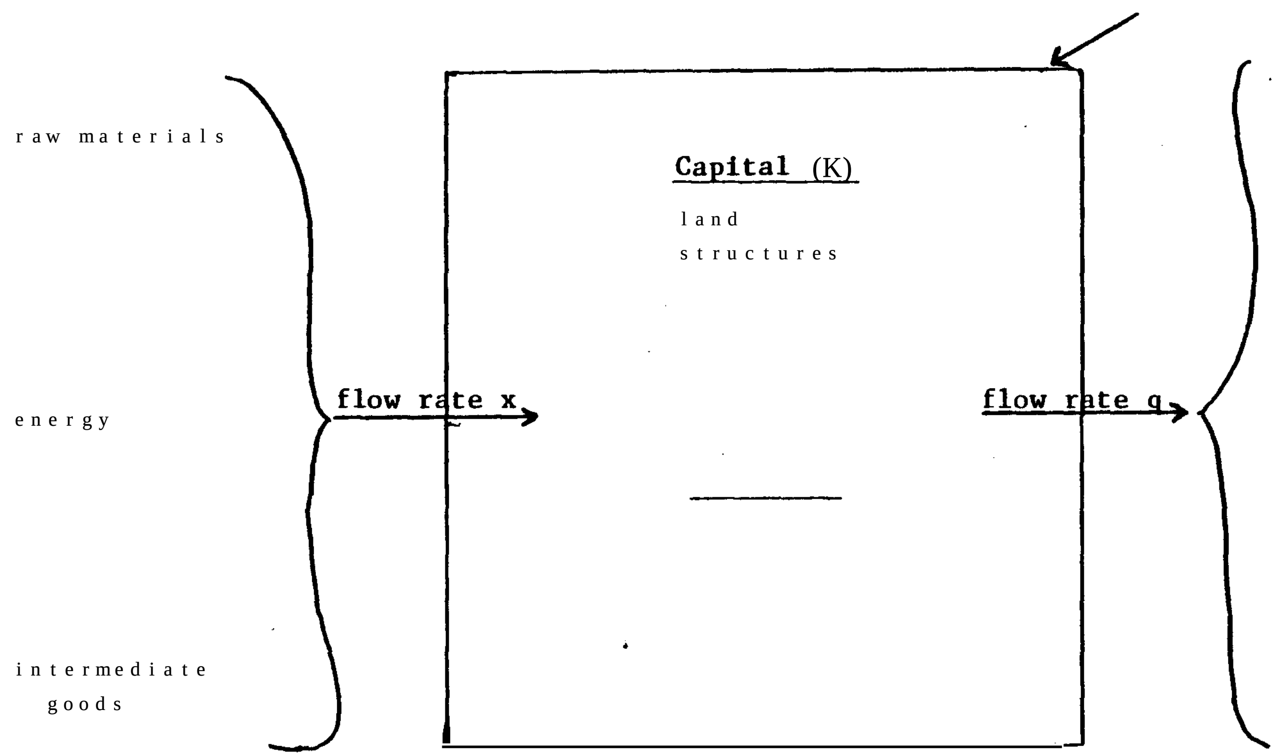
PROCESS ANALYSIS, CAPITAL UTILIZATION, AND THE EXISTENCE OF DUAL COST AND PROW

be determined can be structured in any number of ways. Daily pro-
duction, however, is immediately appl[illegible]

The process analysis of a factory begins by surrounding the factory with an analytical boundary. This boundary is an imaginary barrier placed around the factory at the discretion of the investigator in order to separate the process to be observed from the rest of the world. The investigator may also choose the placement of this boundary as it best suits the goals of the analysis. Let us choose our analytical boundary such that all raw materials, energy, and previously processed (i.e., intermediate) inputs mst mve across this boundary directly into the factory process, and all outputs cross the boundary immediately upon exit from the process. This has been done schematically in Figure 1.

We can observe that at any moment there is some flow of inputs across the boundary and some associated flow of output and "waste products" across the b undary in the opposite direction. We also observe some factors of production present inside the boundary whenever the factory is in operation. 1hese factors are what we commonly think of as labor and capital, plus the goods in process present at each work station along the assembly line. Thus the flow of output produced is related to pln


raw materials
energy
intermediate goods
flow rate x
Capital (K)
land
structures
flow rate q

process at time T. Notice that q can b a vector of output rates, excluding the residual waste products, for the case of joint saleable outputs; that K is a census of machines and structures of each types present at T; that H is a census of workers of each type present at T; and x is a vector of input flow rates at time T.

If T is the period over which the factory operates, then total production is given b

$$(2) \qquad Q = \int_0^T f(K(T), H(T), x(T))dT.$$

If the capital and labor present over the period and the flow rates of inputs are constant, and the productivities of the factors are not affected by the length of the period, we can simplify (2) to

$$(3) \qquad Q = T \cdot f(K, H, x).$$

This says that the total output of the factory is determined by the length of time the factory operates T, the capital and labor on hand over the period, and the flow of other inputs over the period. In addition (3) indicates that in order to produce the quantity Q of product the firm mst choose the capital and labor to hire, the flow of inputs to purchase, and the length of the period to operate, T. Therefore the process analysis of factory production provides us with a production mdel that requires the

supplies of services. Regular maintenance mst be undertaken to insure each machine's operating efficiency, of course, but this does not increase the fund represented by a machine. At some point the machine will become very expensive to maintain, at which time it will be scrapped.

There are two major characteristics of a fund factor of production that we can identify:

1. The fund factors are not quantitatively altered by the production process.
2. A period of time is required to exhaust the services represented by a fund factor.

It is obvious that the capital and labor elements of (3) represent fund factors in the factory process. Moreover, it is the F of fund service that appears in (3) and is denoted by K and H. This can be illustrated with a simple example. Suppose a factory employs 50 workers for a period of 10 hours. The quantity of labor services used over the period is given by (50 men)•(10 hours) = 500 manhours. The rate of service use is then the quantity of labor services used divided by the period of time over which the services are rendered, or (500 manhours)/(10 hours) = 50 men. The rate of fund service is just the number of each fund factor present at any time the factory is in operation.

Now reconsider the right hand side of (3). Since K and H are the census figures for machines and workers present while the factory operates, they are the number of each type of fund factor

therefore

addition, the vector

the flow-fund approach. While his treatment of the waste outputs and the relation of the inputs and outputs to plant capacity was uniquely insightful, it undermined the intuitive appeal and the empirical utility of his production model.

Fortunately it is possible to construct a model in the spirit of Georgescu-Roegen that can be represented mre simply. Equation (3) and a dose of Alfred Marshall's "sensitiveness of touch" are the min ingredients. The primary difficulty is in understanding the way in which substitution occurs between the flow and fund factors and its relation to the conservation principle. Examine equation (2) once again

$$q = f(K, H, x).$$

it

III. Duality In the Flow-

We now define the producer's E function C as the solution to the problem of minimizing the cost of producing at least output level u, given the input prices p, or

(4) $C(u, p) = \min_y \{p'y : F(y)) u\}$.

Assumption 1 below is sufficient to imply the existence of solutions to the cost minimization problem as stated in the lemma that follows.

Assumption 1: F is continuous from above; i.e., for every $u \in$ range F, $L(u) = \{y : y) 0_N, F(y)) u\}$ is a closed set.

Lemma 1: If F satisfies Assumption 1 with $p >> 0_N$, then for every $u \in$ range F, $\min_y\{p'y : F(y)) u\}$ exists.

Furthermore the following seven properties can be derived for the cost function C requiring only Assumption 1 on F.

Properties

C4: For every $u \in$ range F, C(u, p) is a <u>concave</u> function of p.

C5: For $u \in$ range F, C(u, p) is <u>continuous</u> in p, $p \gg 0_N$.

C6: C(u, p) is <u>non-decreasing</u> in u for fixed p; i.e., if $p \gg 0_N$, $u_0, u_1 \in$ range F and $u_0 \leq u_1$, then $C(u_0, p) \leq C(u_1, p)$.

C7: For every $p \gg 0_N$, C(u, p) is continuous from below in u; i.e. if $p^* \gg 0_N$, $u^* \in$ range F, $u_n \in$ range F for n = 1, 2, ..., $u_1 \leq u_2 \leq \cdots$ and $\lim u_n = u^*$, then $\lim_n C(u_n, p) = C(u^*, p^*)$.

input demand equations directly from the functional form of C. It is not necessary to find F nor to carry through the entire constrained [illegible] r-rat

(7)

Since a dual cost function mst exist for f in (6) with given input prices by Lemma 1, a cost function dual to (7) should also exist, but it will not b identical to the neoclassical cost function due to the institutional differences in payments to capital vs. the other inputs. We take account of the firm's ability to purchase labor services and mterial input flows, but only the capital fund in the definition of the flow-fund cost function

$$(8) \qquad C0(0, t, p) = \min_{K,H,x} \{p_k K + t p_h H + t p_x x : t f(K,H,x) \geq 0\}$$

or equivalently

$$(9) \qquad C(q, P) = \min_{K,H,x} \{p_k K + t p_h H + t p_x x : f(K,H,x) \geq q, \; tq = 0\}$$

where P is [illegible]

for any

where 0 =

But here is a surprise. For the labor and flow inputs we get the usual result

(13) $\partial H/\partial p_x = \partial x/\partial p_h$,

while for the capital input we find

(14) $\partial K/\partial p_h = t(\partial H/\partial p_k)$, $\partial K/\partial p_x = t(\partial x/\partial p_k)$;

i.e., the usual conditions are now weighted by the utilization rate t. Georgescu-Roegen (1972) derives an equivalent result using the Lagrangean method: that the marginal rate of substitution of capital for labor is equal to the price ratio weighted by the utilization rate. He interprets this to mean that the budget line in K and H space is not tangent to the relevant isoquant when $t < 1$. We know that this is not possible for a (K, H) combination that is a solution to the cost minimization problem. A closer look at (14) resolves this paradox.

Recall, first of all, that p_h is defined as a daily price for labor, $p_h = 24w_h$ when w_h is the hourly wage. This means changes in p_h arise from changes in w_h alone, or

(15) $dp_h = (24)\cdot dw_h$.

And using this with (14) yields

(16) $(\partial K/\partial w_h)(1/24) = t(\partial H/\partial p_k)$,

which we rearrange to find

(17) $\partial K/\partial w_h = 24t(\partial H/\partial p_k) = T(\partial H/\partial p_k) = \partial(TH)/\partial p_k$,

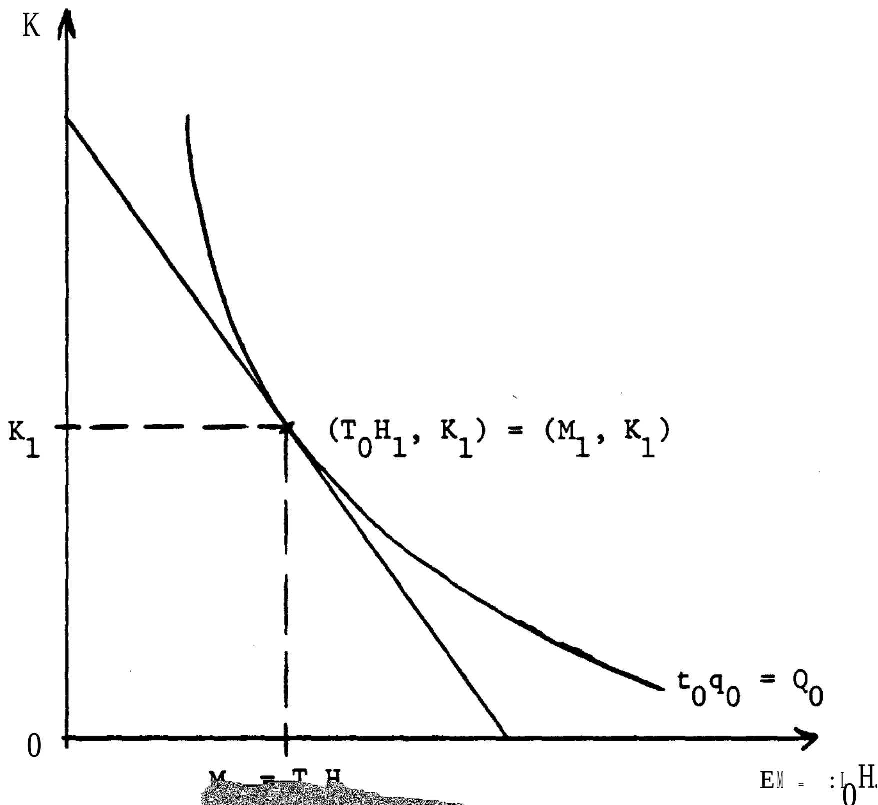
where T is the number of hours per day that the factory operates. Of course TH is nothing mre than the number of manhours, or the quantity of labor services, used by the firm in a working day. We can now write the demand for labor services as

$$(18) \quad M(T, q, P) = T \cdot H(q, D),$$

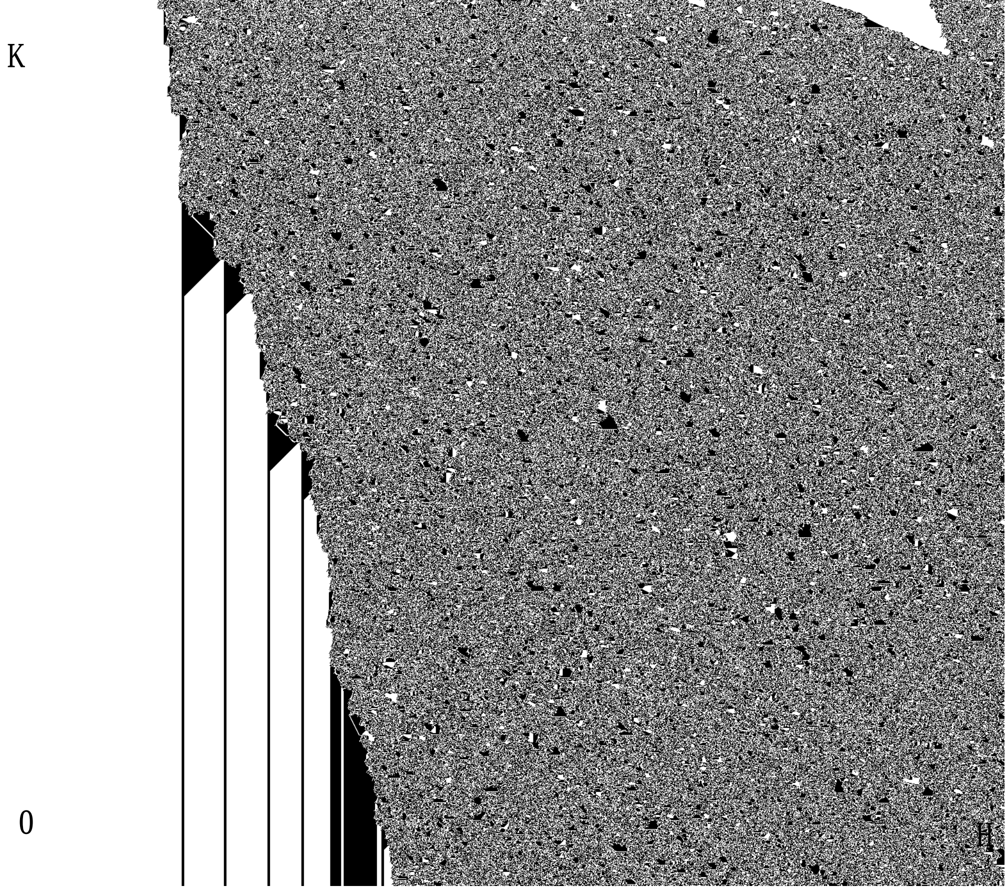
and this allows us to rewrite (14) in the more familar form

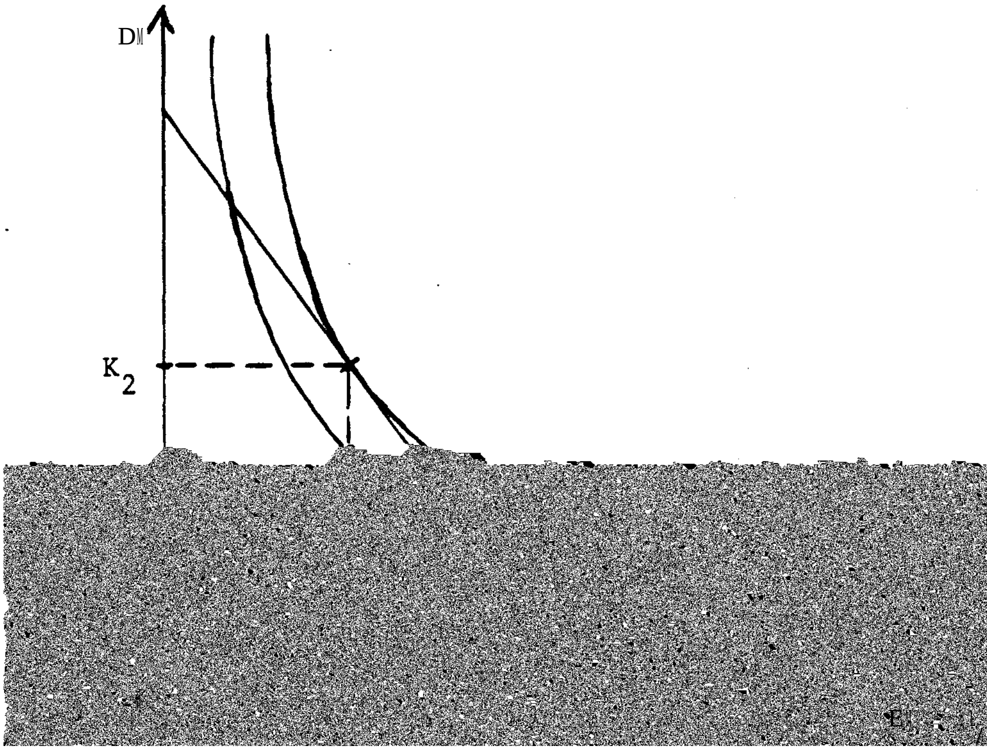
(19) [illegible]

Clearly the relevant isoquant is in K and M = TH space as shown in Fig. 2 (a). Furthermore, the isoquants contain both a production rate and a utilization rate component, $Q = t \cdot q$. In 2(a) the isoquant is drawn

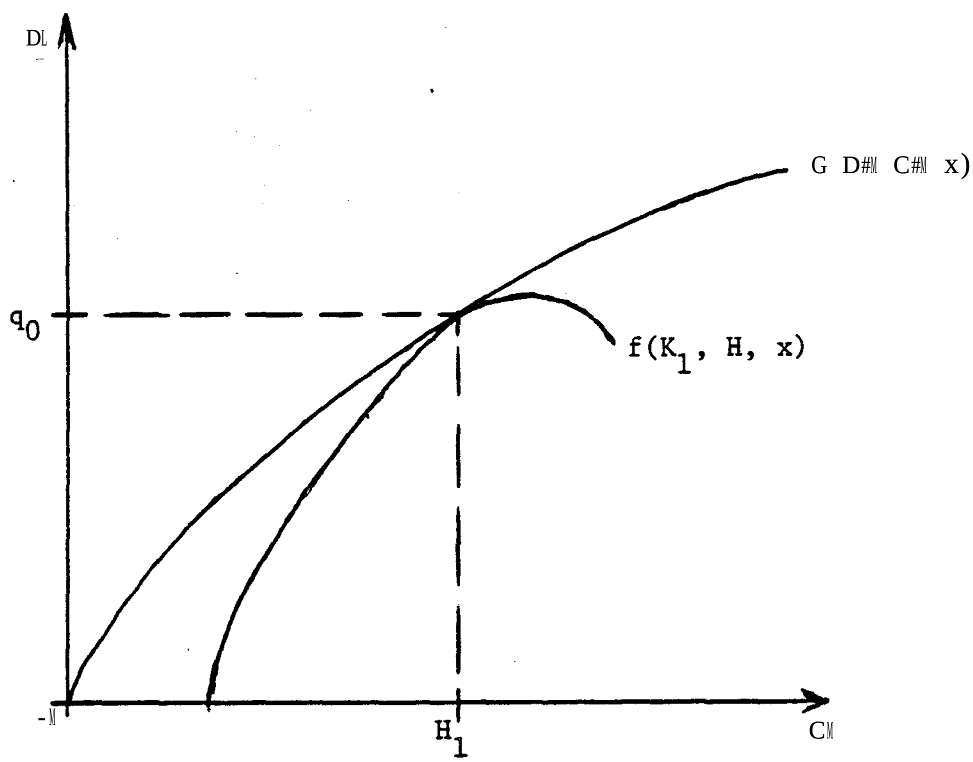


(a)





(b)



(d)

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capital and mving to a higher rate of production. We can see that the deciding factor for the firm is no longer the relative prices of the inputs, but the relative daily cost of the inputs when the length of the working day is a decision variable.

(i) f is a real valued function defined over the non-negative orthant and continuous on this domain.

(ii) f is increasing.

(iii) f is a quasiconcave function.

then given f and positive inputs and outputs, we write the profit mximization problem as

$$(20) \quad \max_{t,q} \{D(Q)Q - C(q, P) : Q = tq > 0, 0 < t \leq 1\}$$

$$= D(Q^*)Q^* - C(q^*, P^*).$$

Q is total daily output, D(Q) is a daily inverse demand function, and C(q, P,) is the cost function dual to f. Solutions to the profit maximization problem are denoted by an asterisk. The first order conditions for a maximum are

$$(21) \quad \begin{aligned} D(Q)q + (\partial D/\partial Q)qQ - C_t &= 0 \\ D(Q)t + (\partial D/\partial Q)tQ - C_q &= 0 \end{aligned}$$

where C_q and C_t are defined by (10). This condition can be restated as

$$(22) \quad C_q/t = MR = C_t/q$$

where MR is marginal revenue in the usual sense. This condition can b interpreted as a variant of the typical neoclassical marginal condition: the firm chooses to operate where the mrginal gain from an increment to time in production is just equal to the marginal gain from an increment to rate of production. C_q/t and

C_t/q are "marginal cost" in the two output dimensions weighted by the reciprocals of t and q. Thus "marginal revenue equals marginal cost" in 4-")M

Input prices that vary by "time of day" can be incorporated in this framework, if the utilization rate can be measured relative to a specific starting time during the day. The proper measurement of t will guarantee that the size of t corresponds to a known set of operating hours such that t and "time of day" are paired. In this context input prices that change over the course of the day can be described by a price function P, P(t) > -$M that depends on utilization rate.

;$L Restrictions on the Production and Cost Functions

We can now make some statements on the firm's optimal choice of q and t by imposing restrictions on the form of f, which imply a form for C. Suppose f is homogeneous of degree l/8 such that

$$24"M \qquad f(X) = [g(X)]^{l/8}$$

where g(X) is linearly homogeneous. Then

$$f(>X) = >^{l/8} f(X) = [g(>X)]^{l/8} = [>g(X)]^{l/8} = J^{/8} [g(X)]^{l/8}$$

and the cost function dual to f decomposes in the following way.

$$M27M) \qquad <(q,P) = q^{8} <(1,P) = q^{8} c(P)$$

where 8 = 1 implies constant returns to scale, 8 < 1 implies increasing returns to scale, 8 > 1 implies decreasing returns to scale, and c(P) is the unit cost function dual to g.

If we impose the neoclassical assumptions of constant returns to scale and perfectly competitive firms, the first order conditions for profit maximization become

$$27"M \qquad c(P) \qquad P_Q^* = \frac{ac(P)}{at}.$$

$$\left.\frac{dA}{dt}\right|_{dQ=0} = [(e-1)q^{e-2} t^{-1}c(P)J(-q/t) + q^{e-1} t^{-1}ac(P)/at - q^{e-1} t^{-2}c(P)$$

which reduces to

$$\left.\frac{dA}{dt}\right|_{dQ=0} = (q^{e-1} t^{-1}) [ac;at - ec(P)/tJ$$

and produces the condition

(30) $$\left.\frac{dA}{dt}\right|_{dQ=0} \gtrless 0 \text{ as } [ac;at - ec(P)/t] \gtrless 0$$

for $q > 0$, $t > 0$.

Therefore, the value of t that minimizes the average cost of producing any quantity Q depends not only on the slope of c in t, but also on the returns to scale parameter e. The

these factors can be illustrated in

For e = 1, (31) is negative over all t and the firm choses t = 1. As e increases (31) declines, implying that the firm is more likely to choose a small value of t when economies of scale are mre pronounced. If we rewrite (31) for the more general case of $P_H = P_h(t)$, p'_h A -% then

$$42" \quad [ac;at - ec(P)/t] = (ap_h/at)H - e;t \, (P_k K + P_h H)$$

and we can derive the following condition

[illegible] >

where $S_H = P_h H/c(P)$ is labor's share of cost, and $e = E_{cq}$ is the elasticity of cost with respect to q. Therefore the optimal t depends on the slope of labor's price in t as well as labor's share of cost and the scale properties of the rate function.

Equation (33) contains the results obtained by Betancourt and Clague in their propositions 2$ 3, and 4, and by Winston for linearly homogeneous rate functions.[10] Yet (33) holds not only for homogenous rate functions of any degree as defined by 25"$ but also for homothetic rate functions. The reader may verify this result using the cost function dual to a homothetic production rate function,

$$C(q,P) = m(q)c(P) \text{ and } A(q,P) = q^{-1} t^{-1} m(q)c(P).$$

where m is a postive, increasing function. It then follows easily that

$$\frac{dA}{dt}\bigg|_{dQ=0} = A[S_H(\partial p_h/\partial t)/p_h - E_{Cq}/t] \gtreqless 0$$

for the two input case.

VI. Conclusion

We have shown that dual cost and production functions exist for the flow-fund production technology derived from a process analysis of factory production. This duality includes the time utilization of the factors of production and asymmetric time patterns of payment of those factors. The usual results associated with the optimal utilization rate for linearly homogeneous rate functions can b extended to include homothetic rate functions.

The relationship between neoclassical production theory and the capital utilization literature is illuminated by the results of section I#. If one ignores the time utilization problem, including the asymmetric variation in .input prices with respect to time, and assumes a linearly homogeneous production function, then the flow-fund dual cost function collapses to the neoclassical cost function. Thus, the flow-fund production and cost relationships contain neoclassical production theory as a special case.

The flow-fund duality mdel encompasses the best attributes of neoclassical production theory, the mathematics of duality theory, and the time characteristics of capital utilization models. The neoclassical isoquant concepts remain useful and valid. The empirical vitality of duality theory is preserved.

The captial utilization problem is incorporated and extended to a wider range of technologies. In this way the unifying goal of this inquiry is achieved.

Furthermore, a general approach to production modeling is suggested. One begins with a process analysis of the system sone ieuis

"From all we know, only [illegible]

or that the ith cost minimizing input demand function cannot slope upward with respect to its own price. Furthermore, twice continuous differentiability of C with respect to P at (q^*,P^*) imp

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